Chapter-3
Exponents and Radicals

W.S-1 In Book

W.S-2

Q1: Simplify

(i) $x^{\frac{1}{2}} \times x^{\frac{5}{2}}$

$x^{\frac{1}{2}+\frac{5}{2}}$ $\quad (\because x^a \times x^b = x^{a+b})$

$= x^{\frac{6}{2}}$

$= x^3$

(ii) $\frac{x^{6/5}}{x^{1/5}}$

$= x^{\frac{6}{5}-\frac{1}{5}}$ $\quad (\because \frac{x^a}{x^b} = x^{a-b})$

$= x^{\frac{5}{5}}$

$= x$

(iii) $(x^7)^0$

$= x^{7\times 0}$ $\quad (\because (x^a)^b = x^{ab})$

$= x^0$ $\quad (x^0 = 1)$

$= 1$

(IV) $5x^{\frac{5}{6}} \times 6x^{\frac{1}{6}}$

$30x^{\frac{5}{6}+\frac{1}{6}}$ $\quad (x^a \times x^b = x^{a+b})$

$30x^{\frac{6}{6}}$

$30x$

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(v) $x^{-\frac{7}{2}} \times 2x^{-\frac{1}{2}}$

$2x^{-\frac{7}{2}+(-\frac{1}{2})}$ $\quad (x^a \times x^b = x^{a+b})$

$2x^{-\frac{7}{2}-\frac{1}{2}}$

$2x^{\frac{-7-1}{2}}$

$2x^{-\frac{8}{2}}$

$2x^{-4} = \frac{2}{x^4}$

Q2: Find the value of:

1. $(512)^{-\frac{2}{9}}$

$(2^9)^{-\frac{2}{9}}$

2^{-2}

$\frac{1}{2^2}$

$\frac{1}{4}$

2. $\left[(216)^{\frac{2}{3}}\right]^{\frac{1}{2}}$

$\left[(6^3)^{\frac{2}{3}}\right]^{\frac{1}{2}}$

$= 6$

3. $10 \div 8^{-\frac{1}{3}}$

$10 \div (2^3)^{-\frac{1}{3}}$

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$10 \div 2^{-1}$

$10 \div \frac{1}{2}$

10×2

$= 20$

4. $16^{\frac{3}{4}}$

$(2^4)^{\frac{3}{4}}$

2^3

$= 2 \times 2 \times 2$

$= 8$

5. $27^{\frac{1}{3}} \times 16^{\frac{-1}{4}}$

$(3^3)^{\frac{1}{3}} \times (2^4)^{\frac{-1}{4}}$

3×2^{-1}

$= \frac{3}{2}$

6. $\frac{1}{[(3^4)^{\frac{1}{2}}]^{-2}}$

$\frac{1}{[3^2]^{-2}}$

$\frac{1}{3^{-4}}$

3^4

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$3 \times 3 \times 3 \times 3$

$= 81$

7. $\dfrac{27^{-\frac{2}{3}} \times 81^{\frac{5}{4}}}{\left(\frac{1}{3}\right)^{-3}}$

$= \dfrac{(3^3)^{-\frac{2}{3}} \times (3^4)^{\frac{5}{4}}}{3^3}$

$= \dfrac{3^{-2} \times 3^5}{3^3}$ $= \dfrac{3^{-2+5}}{3^3}$

$= \dfrac{3^3}{3^3}$

$= 1$

8. $64^{\frac{1}{2}}(64^{\frac{1}{2}}+1)$

$(8^2)^{\frac{1}{2}}(8^{2\cdot\frac{1}{2}}+1)$

$8(8+1)$

8×9

$= 72$

9. $\dfrac{36^{\frac{7}{2}} - 36^{\frac{9}{2}}}{36^{\frac{5}{2}}}$

$= \dfrac{(6^2)^{\frac{7}{2}} - (6^2)^{\frac{9}{2}}}{(6^2)^{\frac{5}{2}}}$

$= \dfrac{6^7 - 6^9}{6^5}$

$$\frac{6^7}{6^5} - \frac{6^9}{6^5}$$

$$= 6^{7-5} - 6^{9-5}$$

$$= 6^2 - 6^4$$

$$= 36 - 1296$$

$$= -1260$$

10. $4 \times 81^{-\frac{1}{2}}\left(81^{\frac{1}{2}} + 81^{\frac{3}{2}}\right)$

$$4 \times (9^2)^{-\frac{1}{2}}\left(9^{2 \times \frac{1}{2}} + 9^{2 \times \frac{3}{2}}\right)$$

$$4 \times 9^{-1}(9 + 9^3)$$

$$\frac{4}{9}(9 + 729)$$

$$\frac{4 \times 328^{82}}{9_1}$$

$$= 328$$

Q3: Evaluate

(i) $(0.04)^{\frac{3}{2}}$

$$\left[(0.2)^2\right]^{\frac{3}{2}}$$

$$= (0.2)^3$$

$$0.2 \times 0.2 \times 0.2$$

$$= 0.008$$

(ii) $(6.25)^{\frac{3}{2}}$

$= (2.5^2)^{\frac{3}{2}}$

$= 2.5^3$

$= 2.5 \times 2.5 \times 2.5$

$= 15.625$

(iii) $(0.03125)^{-\frac{2}{5}}$

$\left[(0.5)^5\right]^{-\frac{2}{5}}$

$= (0.5)^{-2}$

$= \frac{1}{(0.5)^2}$

$= \frac{1}{0.25}$

$\frac{100}{25} \; 4$
$\quad 1$

$= 4$

(iv) $(0.008)^{\frac{2}{3}}$

$\left[(0.2)^3\right]^{\frac{2}{3}}$

0.2^2

$= 0.04$

Q4: Evaluate

(i) $(6^2+8^2)^{\frac{1}{2}}$

$(36+64)^{\frac{1}{2}}$

$= (100)^{\frac{1}{2}}$

$= (10^2)^{\frac{1}{2}}$

$= 10$

(ii) $\left[5(8^{\frac{1}{3}}+27^{\frac{1}{3}})^3\right]^{\frac{1}{4}}$

$= \left[5(2^{3\times\frac{1}{3}}+3^{3\times\frac{1}{3}})^3\right]^{\frac{1}{4}}$

$= \left[5(2+3)^3\right]^{\frac{1}{4}}$

$= \left[5(5)^3\right]^{\frac{1}{4}}$ $(\because x^a \times x^b = x^{a+b})$

$= (5^4)^{\frac{1}{4}}$

$= 5$

(iii) $(17^2-8^2)^{\frac{1}{2}}$

$= (289-64)^{\frac{1}{2}}$

$= (225)^{\frac{1}{2}}$

$= (15^2)^{\frac{1}{2}}$

$= 15$

(iv) $(1^3+2^3+3^3)^{-\frac{5}{2}}$

$= (1+8+27)^{-\frac{5}{2}}$

$= (36)^{-\frac{5}{2}}$

$= (6^2)^{-\frac{5}{2}}$

$= 6^{-5}$

$= \frac{1}{6^5}$ $= \frac{1}{7776}$

Q5: Simplify

(i) $2x^{\frac{1}{6}} \times 2x^{\frac{-7}{6}}$

$4x^{\frac{1}{6}+(\frac{-7}{6})}$ $\left(\because x^a \times x^b = x^{a+b}\right)$

$= 4x^{\frac{1}{6}-\frac{7}{6}}$

$= 4x^{\frac{-6}{6}}$

$= 4x^{-1}$

$= \frac{4}{x}$

(ii) $\left[\sqrt[3]{\left(\frac{1}{x}\right)^{-12}}\right]^{-\frac{2}{3}}$

$= \left[\left[\left(\frac{1}{x}\right)^{-12}\right]^{\frac{1}{3}}\right]^{-\frac{2}{3}}$

$= \left[\left(\frac{1}{x}\right)^{-8}\right]^{-\frac{2}{8}}$

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$= \left(\frac{1}{x}\right)^{+2} = \frac{1}{x^2}$

(iii) $a^{\frac{4}{7}} \div a^{\frac{10}{7}}$

$= a^{\frac{4}{7}-\frac{10}{7}}$ $(\because x^a \div x^b = x^{a-b})$

$= a^{\frac{4-10}{7}}$

$= a^{\frac{-6}{7}}$

$= \frac{1}{a^{\frac{6}{7}}}$

Q6 Verify

$\left[(729)^{-\frac{5}{3}}\right]^{-\frac{1}{2}} = (729)^{-\frac{5}{3}\left(-\frac{1}{2}\right)}$

$\Rightarrow \left[(9^3)^{\frac{5}{3}}\right]^{-\frac{1}{2}} = (9^3)^{-\frac{5}{3}\left(-\frac{1}{2}\right)}$

$\Rightarrow (9)^{\frac{5}{2}} = (9)^{\frac{5}{2}}$

LHS = RHS

So Verified

Q7 Solve

(i) $(\sqrt{6})^{x-2} = 1$

$= (\sqrt{6})^{x-2} = (\sqrt{6})^0$ $[\because x^0 = 1]$

Bases are same so power will be equal

$x - 2 = 0$

$x = 0 + 2$

$\Rightarrow x = 2$

(ii) $3^{4x} = \frac{1}{81}$

$\Rightarrow 3^{4x} = \frac{1}{3^4}$

$\Rightarrow 3^{4x} = 3^{-4}$

Bases are same so power will be equal

$4x = -4$

$x = \frac{-4}{4} \quad x = -1$

(iii) $(\sqrt{2})^x = 2^8$

$= (\sqrt{2})^x = (\sqrt{2})^8 \quad = 2^{\frac{1}{2}x} = 2^8$

Bases are same so power will be equal

$\frac{1}{2}x = 8 \quad \Rightarrow x = 8 \times 2$

$\Rightarrow x = 16$

(iv) $2^{2x+1} = 4^{2x+1}$

$2^{2x+1} = (2^2)^{2x-1}$

$2^{2x+1} = 2^{4x-2}$

Bases are same so power will be equal

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$2x + 1 = 4x - 2$

$2x - 4x = -2 - 1$

$-2x = -3$

$x = \frac{-3}{-2} \Rightarrow x = \frac{3}{2}$

Value Based Questions

(i) Dimension of the sheet = 75 cm × 40 cm

No. of sections = 9

Area of sheet given to 9 sections = $9 \times 75 \times 40\ cm^2$

$= 3 \times 3 \times 3 \times 5 \times 5 \times 2 \times 2 \times 2 \times 5$

$= 2^3 \times 3^3 \times 5^3$

$= (2 \times 3 \times 5)^3$

$= (30)^3\ cm^2$

(ii) Planting of more trees

Water conservation

Reducing Deforestation.

Brain Teasers

Q1 a) Value of $\left[(625)^{-\frac{1}{2}}\right]^2$

$= \left[(25^2)^{-\frac{1}{2}}\right]^2$

$= (25)^{-2}$

$= \frac{1}{25^2}$

$= \frac{1}{625}$

OR

$\left[(625)^{-\frac{1}{2}}\right]^2$

$= 625^{-1}$

$= \frac{1}{625}$

b) $5^k = 1$

$5^k = 5^0 \quad (\because k^0 = 1)$

Since bases are same so powers will be equal

$k = 0$

c) $\left[27^{\frac{1}{3}} + 64^{\frac{1}{3}}\right]^2$

$= \left[3^{3 \times \frac{1}{3}} + 4^{3 \times \frac{1}{3}}\right]^2$

$= [3+4]^2 \qquad = 7^2$

$= 49$

d) (iii) $(a \div b)^4$

e) 4.25×10^{-7}

$= 0.0000000425$

8 a) $\left(\frac{13}{21}\right)^{\frac{2}{5}}$

$\sqrt[5]{\left(\frac{13}{21}\right)^2}$

b) $(81)^{-\frac{3}{4}}$

$(3^4)^{-\frac{3}{4}}$

3^{-3}

$= \frac{1}{3^3} = \frac{1}{27}$

c) $5 \times 16^{\frac{3}{4}}$

$5 \times 2^{4 \times \frac{3}{4}}$

5×2^3

$= 5 \times 8$

$= 40$

d) $(0.000064)^{\frac{5}{6}}$

$= (0.2)^{6 \times \frac{5}{6}}$

$= (0.2)5$

$= 0.00032$

e) $3^{x-1} = \frac{1}{27}$

$= 3^{x-1} = \frac{1}{3^3}$

$= 3^{x-1} = 3^{-3}$

Since bases are same so powers will be equal

$x-1 = -3$

$x = -3+1$

$x = -2$

Q2 $\frac{(64)^{-\frac{1}{6}} \times (216)^{-\frac{1}{3}} \times 81^{\frac{1}{4}}}{(512)^{-\frac{1}{3}} \times (16)^{\frac{1}{4}} \times (9)^{-\frac{1}{2}}}$

$= \frac{(2^6)^{-\frac{1}{6}} \times (6^3)^{-\frac{1}{3}} \times (3^4)^{\frac{1}{4}}}{(8^3)^{-\frac{1}{3}} \times (2^4)^{\frac{1}{4}} \times (9^2)^{-\frac{1}{2}}}$

$= \frac{2^{-1} \times 6^{-1} \times 3}{8^{-1} \times 2 \times 3^{-1}}$

$= \frac{3 \times 8 \times 8}{2 \times 6 \times 2}$

$= 3$

Q3: $\left[\frac{\sqrt[3]{x^4y}}{\sqrt[3]{xy^7}}\right]^{-4}$

$= \left[\frac{\sqrt[3]{x^4y}}{\sqrt[3]{xy^7}}\right]^{-4}$

$= \left[\left(\frac{x^4y^1}{x^1y^7}\right)^{\frac{1}{3}}\right]^{-4}$ $= \left[x^{4-1}\ y^{1-7}\right]^{-\frac{4}{3}}$

$= \left[x^3y^{-6}\right]^{-\frac{4}{3}}$

$= (x^3)^{-\frac{4}{3}} \times (y^{-6})^{-\frac{4}{3}}$

$= x^{-4}y^{+8}$

$= \frac{y^8}{x^4}$

Q4: Evaluate

(i) $3 \times 16^{\frac{3}{4}}$

$3 \times 2^{4 \times \frac{3}{4}}$

3×2^3

$= 3 \times 8$

$= 24$

(ii) $2 \times (27)^{-\frac{2}{3}}$

$= 2 \times 3^{3 \times -\frac{2}{3}}$

$= 2 \times 3^{-2}$

$= 2 \times \frac{1}{9}$

$= \frac{2}{9}$

(iii) $2 \times 9^{\frac{3}{2}} \times 9^{-\frac{1}{2}}$

$= 2 \times 9^{\frac{3}{2}} \times 9^{-\frac{1}{2}}$

$= 2 \times (3^2)^{\frac{3}{2}} \times (3^2)^{-\frac{1}{2}}$

$= 2 \times 3^3 \times 3^{-1}$

$= 2 \times 27^{9} \times \frac{1}{3_1}$

$= 2 \times 9$

$= 18$

Q5: $[5^2+12^2]^{-\frac{1}{2}}$

$= [25+144]^{-\frac{1}{2}}$

$= (169)^{-\frac{1}{2}}$

$= [(13)^2]^{-\frac{1}{2}} = \frac{1}{13}$

Q6: Find x

(i) $2^x + 2^x + 2^x = 192$

$2^x(1+1+1) = 192$

$2^x \times 3 = 192$

$2^x = \frac{192}{3} \; 64$

$2^x = 64$

$2^x = 2^6$

Bases are same so power will be equal

$x = 6$

(ii) $8^{255} = 32^x$

$\Rightarrow (2^3)^{255} = (2^5)^x$

$\Rightarrow 2^{765} = 2^{5x}$

Bases are same so powers will be equal

$5x = 765$

$x = \frac{765}{5}$

$x = 153$

(iii) $2^{2x+2} = 4^{2x-1}$

$2^{2x+2} = 2^{2(2x-1)}$

$2^{2x+2} = 2^{4x-2}$

Bases are same so powers will be equal.

$2x + 2 = 4x - 2$

$2x - 4x = -2 - 2$

$-2x = -4$

$x = \frac{4}{2}$ $\quad x = 2$

Q7: $4^x - 4^{x-1} = 24$

$\Rightarrow 4^x - 4^x \times 4^{-1} = 24$

$4^x - \frac{4^x}{4} = 24$

$4^x\left(1 + \frac{1}{4}\right) = 24$

$4^x\left(\frac{4-1}{4}\right) = 24$

$4^x\left(\frac{3}{4}\right) = 24$

$4^x = \frac{8}{24} \times 4$

$4^x = 32$

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$(2^2)^x = 2^5$

$2^{2x} = 5$

So bases are same so power will be equal

$2x = 5$

$x = \frac{5}{2}$

$x = 2\frac{1}{2}$

HOTS

1. $(6^{-1} - 8^{-1})^{-1} + (2^{-1} - 3^{-1})^{-1}$

$= \left(\frac{1}{6} - \frac{1}{8}\right)^{-1} + \left(\frac{1}{2} - \frac{1}{3}\right)^{-1}$

$= \left(\frac{4-3}{24}\right)^{-1} + \left(\frac{3-2}{6}\right)^{-1}$

$= \left(\frac{1}{24}\right)^{-1} + \left(\frac{1}{6}\right)^{-1}$

$24 + 6$

$= 30$

2. $3^{x-1} = 9$, $4^{y+2} = 64$

find $\frac{y}{x} - \frac{x}{y}$

$3^{x-1} = 9$ $\qquad$ $4^{y+2} = 64$

$3^{x-1} = 3^2$ $\qquad$ $4^{y+2} = 4^3$

Bases are same so power will be equal.

$x-1 = 2$ $\qquad$ $y+2 = 3$

$x = 3$ $\qquad$ $y = 1$

Put $x = 3$, $y = 1$

$\frac{1}{3} - \frac{3}{1}$

$\frac{1-9}{3} = \frac{-8}{3}$

Enrichment Questions

1. $9^x \times 3^2 x \left[3^{\frac{-x}{x}}\right]^{-2} = \frac{1}{27}$

$3^{2x} \times 3^2 \times 3^x = \frac{1}{3^3}$

$3^{2x+2+x} = \frac{1}{3^3}$

$3^{3x+2} = \frac{1}{3^3}$

Bases are same so powers will be equal

$3x+2=-3$

$3x=-3-2$

$3x=-5$

$x=\frac{-5}{3}$

Q2: Let req. no. be x

$$\left(\frac{-3}{2}\right)^{-3} \div x = \left(\frac{-8}{27}\right)^{-2}$$

$$\left(\frac{-2}{3}\right)^{3} \times \frac{1}{x} = \left(\frac{-27}{8}\right)^{2}$$

$$\left(\frac{-2}{3}\right) \times \left(\frac{-2}{3}\right) \times \left(\frac{-2}{3}\right) \times \frac{1}{x} = \left(\frac{-27}{8}\right) \times \left(\frac{-27}{8}\right)$$

$$\frac{1}{x} = \left(\frac{-27}{8}\right) \times \left(\frac{-27}{8}\right) \times \left(\frac{-3}{2}\right) \times \left(\frac{-3}{2}\right) \times \left(\frac{-3}{2}\right)$$

$$\frac{1}{x} = -\left(\frac{3}{2}\right)^{3}\left(\frac{3}{2}\right)^{3}\left(\frac{3}{2}\right)^{3}$$

$$\frac{1}{x} = -\left(\frac{3}{2}\right)^{9}$$

$$x = -\left(\frac{2}{3}\right)^{9}$$

So req. no. is $-\left(\frac{2}{3}\right)^{9}$