

Chapter - 7

Algebraic Identities

$$1. (a+b)^2 = a^2 + 2ab + b^2$$

$$2. (a-b)^2 = a^2 - 2ab + b^2$$

WS-1

Q1: Find the following by using Identity -1

(i) $(2x+5)^2$

$$\begin{aligned} \text{Using } (a+b)^2 &= a^2 + 2ab + b^2 \\ &= (2x)^2 + 2 \times 2x \times 5 + 5^2 \\ &= 4x^2 + 20x + 25 \end{aligned}$$

(ii) $(8x+3y)^2$

$$\begin{aligned} \text{Using } (a+b)^2 &= a^2 + 2ab + b^2 \\ &= (8x)^2 + 2 \times 8x \times 3y + (3y)^2 \\ &= 64x^2 + 48xy + 9y^2 \end{aligned}$$

(iii) $\left(\frac{3a}{5} + \frac{2b}{3}\right)^2$

$$\begin{aligned} \text{Using } (a+b)^2 &= a^2 + 2ab + b^2 \\ &= \left(\frac{3a}{5}\right)^2 + 2 \times \frac{3a}{5} \times \frac{2b}{3} + \left(\frac{2b}{3}\right)^2 \\ &= \frac{9a^2}{25} + \frac{4ab}{5} + \frac{4b^2}{9} \end{aligned}$$

$$(iv) (7pq + 4ab)^2$$

$$\begin{aligned} \text{Using } (a+b)^2 &= a^2 + 2ab + b^2 \\ &= (7pq)^2 + 2 \times 7pq \times 4ab + (4ab)^2 \\ &= 49p^2q^2 + 56pqab + 16a^2b^2 \end{aligned}$$

$$(v) (0.2x + 1.5y)^2$$

$$\begin{aligned} \text{Using } (a+b)^2 &= a^2 + 2ab + b^2 \\ &= (0.2x)^2 + 2 \times 0.2x \times 1.5y + (1.5y)^2 \\ &= 0.04x^2 + 0.60xy + 2.25y^2 \end{aligned}$$

$$(vi) (2m^2 + 3n^2)^2$$

$$\begin{aligned} \text{Using } (a+b)^2 &= a^2 + 2ab + b^2 \\ &= (2m^2)^2 + 2 \times 2m^2 \times 3n^2 + (3n^2)^2 \\ &= 4m^4 + 12m^2n^2 + 9n^4 \end{aligned}$$

Q2: Evaluate by using identity

$$(i) (101)^2$$

$$= (100+1)^2$$

$$\text{Using } (a+b)^2 = a^2 + 2ab + b^2$$

$$(100)^2 + 2 \times 100 \times 1 + (1)^2$$

$$10201$$

$$(ii) (52)^2$$

$$(50+2)^2$$

$$\text{Using } (a+b)^2 = a^2 + 2ab + b^2$$

$$(50)^2 + 2 \times 50 \times 2 + (2)^2$$

$$2500 + 200 + 4$$
$$= 2704$$

$$(iii) (8.1)^2$$

$$(8 + 0.1)^2$$

$$\text{Using } (a+b)^2 = a^2 + 2ab + b^2$$

$$(8)^2 + 2 \times 8 \times 0.1 + (0.1)^2$$

$$64 + 1.6 + 0.01$$

$$= 65.61$$

$$(iv) (410)^2$$

$$(400 + 10)^2$$

$$\text{Using } (a+b)^2 = a^2 + 2ab + b^2$$

$$(400)^2 + 2 \times 400 \times 10 + (10)^2$$

$$160000 + 80,000 + 100$$

$$= 240100$$

$$(v) (203)^2$$

$$(200 + 3)^2$$

$$\text{Using } (a+b)^2 = a^2 + 2ab + b^2$$

$$= (200)^2 + 2 \times 200 \times 3 + (3)^2$$

$$= 40,000 + 1200 + 9$$

$$= 41209$$

$$(vi) (10.2)^2$$

$$(10 + 0.2)^2$$

$$\text{Using } (a+b)^2 = a^2 + 2ab + b^2$$

$$10^2 + 2 \times 10 \times 0.2 + (0.2)^2$$

$$100 + 4 + 0.04 = 104.04$$

Q1: Find the following

(i) $(x-7)^2$

Using $(a-b)^2 = a^2 - 2ab + b^2$
 $x^2 - 2x \times 7 + 7^2$
 $= x^2 - 14x + 49$

(iii) $\left(\frac{7x}{4} - \frac{2y}{3}\right)^2$

Using $(a-b)^2 = a^2 - 2ab + b^2$

$\left(\frac{7x}{4}\right)^2 - 2 \times \left(\frac{7x}{4}\right) \left(\frac{2y}{3}\right) + \left(\frac{2y}{3}\right)^2$

$= \frac{49x^2}{16} - \frac{7xy}{3} + \frac{4y^2}{9}$

(iv) $(8mn - 3pq)^2$

Using $(a-b)^2 = a^2 - 2ab + b^2$

$(8mn)^2 - 2 \times 8mn \times 3pq + (3pq)^2$
 $64m^2n^2 - 48mnpq + 9p^2q^2$

(v) $(0.1x - 0.5y)^2$

Using $(a-b)^2 = a^2 - 2ab + b^2$

$(0.1x)^2 - 2 \times 0.1x \times 0.5y + (0.5y)^2$
 $= 0.01x^2 - 0.1xy + 0.25y^2$

$$(v) (a^2 - b^2)^2$$

$$\text{Using } (a-b)^2 = a^2 - 2ab + b^2$$

$$= (a^2)^2 - 2 \times a^2 \times b^2 + (b^2)^2$$

$$a^4 - 2a^2b^2 + b^4$$

Q2: Evaluate

$$(i) (48)^2$$

$$(50-2)^2$$

$$\text{Using } (a-b)^2 = a^2 - 2ab + b^2$$

$$(50)^2 - 2 \times 50 \times 2 + (2)^2$$

$$2500 - 200 + 4$$

$$= 2396$$

$$(ii) (9.9)^2$$

$$= (10 - 0.1)^2$$

$$\text{Using } (a-b)^2 = a^2 - 2ab + b^2$$

$$= 10^2 - 2(10)(0.1) + 0.1^2$$

$$= 100 - 2 + 0.01$$

$$= 98.01$$

$$(v) (87)^2$$

$$= (90 - 3)^2$$

$$\text{Using } (a-b)^2 = a^2 - 2ab + b^2$$

$$90^2 - 2(90)(3) + 3^2$$

$$8100 - 540 + 9$$

$$= 7569$$

(vi) $(19.9)^2$

$(20 - 0.1)^2$

Using $(a-b)^2 = a^2 - 2ab + b^2$

$20^2 - 2(20)(0.1) + 0.1^2$

$400 - 4 + 0.01$

$= 396.01$

3. $(a+b)(a-b) = a^2 - b^2$

WS-3

Q1: Find the following

(i) $(2x+7y)(2x-7y)$

Using $(a+b)(a-b) = a^2 - b^2$

$= (2x)^2 - (7y)^2$

$= 4x^2 - 49y^2$

(ii) $(5ab-8c)(5ab+8c)$

Using $(a+b)(a-b) = a^2 - b^2$

$= (5ab)^2 - (8c)^2$

$= 25a^2b^2 - 64c^2$

(iii) $(4p^2+q^2)(4p^2-q^2)$

Using $(a+b)(a-b) = a^2 - b^2$

$$(4p^2)^2 - (q^4)^2$$

$$= 16p^4 - q^4$$

$$(iv) \left(\frac{x-y}{3} \right) \left(\frac{x+y}{3} \right)$$

$$\text{Using } (a+b)(a-b) = a^2 - b^2$$

$$\left(\frac{x}{3}\right)^2 - \left(\frac{y}{3}\right)^2$$

$$= \frac{x^2}{9} - \frac{y^2}{9}$$

$$(v) (0.1m - 0.2n)(0.1m + 0.2n)$$

$$\text{Using } (a+b)(a-b) = a^2 - b^2$$

$$(0.1m)^2 - (0.2n)^2$$

$$0.01m^2 - 0.04n^2$$

$$(vi) (a^3 + b^3)(a^3 - b^3)$$

$$\text{Using } (a+b)(a-b) = a^2 - b^2$$

$$= (a^3)^2 - (b^3)^2$$

$$= a^6 - b^6$$

HW Parts (14/7/22)

WS-2

Q1: (ii) $(5a-4b)^2$

Using $(a-b)^2 = a^2 - 2ab + b^2$

$$(5a)^2 - 2 \times 5a \times 4b + (4b)^2$$
$$25a^2 - 40ab + 16b^2$$

Q2: (iii) $(299)^2$

$(300-1)^2$

Using $(a-b)^2 = a^2 - 2ab + b^2$

$$(300)^2 - 2 \times 300 \times 1 + (1)^2$$
$$90000 - 600 + 1$$
$$= 89401$$

(iv) $(98)^2$

$(100-2)^2$

Using $(a-b)^2 = a^2 - 2ab + b^2$

$$100^2 - 2 \times 100 \times 2 + 2^2$$
$$10000 - 400 + 4$$
$$= 9604$$

WS-3

WS-3

Q2: Evaluate

(i) $(81)^2 - (19)^2$

Using $(a+b)(a-b) = a^2 - b^2$

$$= (81+19)(81-19)$$

$$= 100 \times 62$$

$$= 6200$$

(ii) $(290)^2 - (210)^2$

Using $(a+b)(a-b) = a^2 - b^2$

$$(290+210)(290-210)$$

$$= 500 \times 80$$

$$= 40000$$

(iii) $(58)^2 - (12)^2$

Using $(a+b)(a-b) = a^2 - b^2$

$$(58+12)(58-12)$$

$$= 70 \times 46$$

$$= 3220$$

(iv) $(176)^2 - (24)^2$

Using $(a+b)(a-b) = a^2 - b^2$

$$(176+24)(176-24)$$

$$200 \times 152$$

$$= 30400$$

$$(v) (367)^2 - 33^2$$

$$\text{Using } a^2 - b^2 = (a+b)(a-b)$$

$$(367+33)(367-33)$$

$$400 \times 334$$

$$= 133600$$

$$(vi) (545)^2 - (445)^2$$

$$\text{Using } a^2 - b^2 = (a+b)(a-b)$$

$$= (545+445)(545-445)$$

$$= 900 \times 100$$

$$= 99000$$

Q3: Simplify

$$(i) 107 \times 93 = (100+7)(100-7)$$

$$\text{Using } (a+b)(a-b) = a^2 - b^2$$

$$(100)^2 - (7)^2$$

$$= 10000 - 49$$

$$= 9951$$

$$(ii) 211 \times 189$$

$$(200+11)(200-11)$$

$$\text{Using } (a+b)(a-b) = a^2 - b^2$$

$$(200)^2 - (11)^2$$

$$40,000 - 121$$

$$= 39879$$

(iii) 195×205

$$(200-5)(200+5)$$

Using $(a+b)(a-b) = a^2 - b^2$

$$(200)^2 - (5)^2$$

$$= 40,000 - 25$$

$$= 39975$$

(iv) 308×292

$$(300+8)(300-8)$$

Using $(a+b)(a-b) = a^2 - b^2$

$$= (300)^2 - (8)^2$$

$$= 90,000 - 64$$

$$= 89936$$

(v) 12.4×11.6

$$(12+0.4)(12-0.4)$$

Using $(a+b)(a-b) = a^2 - b^2$

$$12^2 - 0.4^2$$

$$= 144 - 0.16$$

$$= 143.84$$

(vi) 30.9×29.1

$$(30+0.9)(30-0.9)$$

Using

$$(30)^2 - (0.9)^2$$

$$= 900 - 0.81$$

$$= 899.19$$

Date: / / 20

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

WS-4

Q1: Expand

(i) $(x-2y+3z)^2$

$$[x+(-2y)+3z]^2$$

Using $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$$\begin{aligned} &= x^2 + (-2y)^2 + (3z)^2 + 2(x)(-2y) + 2(-2y)(3z) + 2(3z)(x) \\ &= x^2 + 4y^2 + 9z^2 - 4xy - 12yz + 6xz \end{aligned}$$

(ii) $(-5x+2y+z)^2$

Using $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$$\begin{aligned} &(-5x)^2 + (2y)^2 + z^2 + 2(-5x)(2y) + 2(2y)(z) + 2(-5x)(z) \\ &25x^2 + 4y^2 + z^2 - 20xy + 4yz - 10xz \end{aligned}$$

(iii) $(4x+y-3z)^2$

$$[4x+y+(-3z)]^2$$

Using $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$$\begin{aligned} &(4x)^2 + y^2 + (-3z)^2 + 2(4x)(y) + 2(y)(-3z) + 2(-3z)(4x) \\ &16x^2 + y^2 + 9z^2 + 8xy - 6yz - 24xz \end{aligned}$$

$$(IV) (3a - 5b - 7c)^2$$

$$[3a + (-5b) + (-7c)]^2$$

$$\text{Using } (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$(3a)^2 + (-5b)^2 + (-7c)^2 + 2(3a)(-5b) + 2(-5b)(-7c) + 2(-7c)(3a)$$

$$9a^2 + 25b^2 + 49c^2 - 30ab + 70bc - 42ac$$

$$(V) (-2a - b + 3c)^2$$

$$[-2a + (-b) + 3c]^2$$

$$\text{Using } (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$(-2a)^2 + (-b)^2 + (3c)^2 + 2(-2a)(-b) + 2(-b)(3c) + 2(3c)(-2a)$$

$$4a^2 + b^2 + 9c^2 + 4ab - 6bc - 12ac$$

$$(VI) (-a + 6b - 2c)^2$$

$$[-a + 6b + (-2c)]^2$$

$$\text{Using } (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$(-a)^2 + (6b)^2 + (-2c)^2 + 2(-a)(6b) + 2(6b)(-2c) + 2(-2c)(-a)$$

$$a^2 + 36b^2 + 4c^2 - 12ab - 24bc + 4ac$$

$$(VII) (1 + 2x - 3y)^2$$

$$[1 + 2x + (-3y)]^2$$

$$\text{Using } (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$(1)^2 + (2x)^2 + (-3y)^2 + 2(1)(2x) + 2(2x)(-3y) + 2(-3y)(1)$$

$$1 + 4x^2 + 9y^2 + 4x - 12xy - 6y$$

(viii) $(2x - 4y - 1)^2$

$$[2x + (-4y) + (-1)]^2$$

Using $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$$(2x)^2 + (-4y)^2 + (-1)^2 + 2(2x)(-4y) + 2(-4y)(-1) + 2(-1)(2x)$$

$$4x^2 + 16y^2 + 1 - 16xy + 8y - 4x$$

(ix) $(6p - 5q - 4x)^2$

$$[6p + (-5q) + (-4x)]^2$$

Using $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$$(6p)^2 + (-5q)^2 + (-4x)^2 + 2(6p)(-5q) + 2(-5q)(-4x) + 2(-4x)(6p)$$

$$36p^2 + 25q^2 + 16x^2 - 60pq + 40qx - 48px$$

5) $(x+a)(x+b) = x^2 + (a+b)x + ab$

WS-5

Q1: Find the product

(i) $(x+5)(x+4)$

Using $(x+a)(x+b) = x^2 + (a+b)x + ab$

$$x^2 + (5+4)x + 5 \times 4$$

$$x^2 + 9x + 20$$

$$(ii) (a+3)(a+6)$$

$$\text{Using } (x+a)(x+b) = x^2 + (a+b)x + ab$$

$$a^2 + (3+6)a + 3 \times 6$$

$$a^2 + 9a + 18$$

$$(iii) (x-9)(x+7)$$

$$\text{Using } (x+a)(x+b) = x^2 + (a+b)x + ab$$

$$x^2 + (-9+7)x + (-9) \times 7$$

$$x^2 - 2x - 63$$

$$(iv) (x+8)(x-5)$$

$$\text{Using } (x+a)(x+b) = x^2 + (a+b)x + ab$$

$$x^2 + (8+(-5))x + 8 \times (-5)$$

$$x^2 + 3x - 40$$

$$(v) (z-3)(z-1)$$

$$\text{Using } (x+a)(x+b) = x^2 + (a+b)x + ab$$

$$z^2 + (-3+(-1))z + (-3) \times (-1)$$

$$z^2 - 4z + 3$$

$$(vi) (p-5)(p-4)$$

$$\text{Using } (x+a)(x+b) = x^2 + (a+b)x + ab$$

$$p^2 + (-5+(-4))p + (-5) \times (-4)$$

$$p^2 - 9p + 20$$

$$(vii) (y-1)(y+2)$$

$$\text{Using } (x+a)(x+b) = x^2 + (a+b)x + ab$$

$$y^2 + (-1+2)y + (-1) \times 2$$

$$y^2 + 1y - 2$$

(viii) $(z+3)(z-7)$
 Using $(x+a)(x+b) = x^2 + (a+b)x + ab$
 $z^2 + (3+(-7))z + 3 \times (-7)$
 $z^2 - 4z - 21$

(ix) $(p+8)(p-3)$
 Using $(x+a)(x+b) = x^2 + (a+b)x + ab$
 $p^2 + (8+(-3))p + 8 \times (-3)$
 $p^2 + 5p - 24$

(x) $(z+6)(z-5)$
 Using $(x+a)(x+b) = x^2 + (a+b)x + ab$
 $z^2 + (6-5)z + 6 \times (-5)$
 $z^2 + 1z - 30$

(xi) $(x-6)(x-9)$
 Using $(x+a)(x+b) = x^2 + (a+b)x + ab$
 $x^2 + (-6-9)x + (-6) \times (-9)$
 $x^2 - 15x + 54$

(xii) $(x-10)(x+9)$
 Using $(x+a)(x+b) = x^2 + (a+b)x + ab$
 $x^2 + (-10+9)x + (-10) \times 9$
 $x^2 - 1x - 90$

(xiii) $(y-4)(y+4)$
 Using $(x+a)(x+b) = x^2 + (-~~a~~+4)(a+b)x + ab$
 $y^2 + (-4+4)y + (-4) \times 4$
 $y^2 - 16$

$$(xiv) (x-4)(x-14)$$

$$\text{Using } (x+a)(x+b) = x^2 + (a+b)x + ab$$

$$x^2 + (-4-14)x + (-4)(-14)$$

$$x^2 - 18x + 56$$

$$(xv) (x-8)(x-2)$$

$$\text{Using } (x+a)(x+b) = x^2 + (a+b)x + ab$$

$$x^2 + (-8-2)x + (-8)(-2)$$

$$x^2 - 10x + 16$$

WS-4

$$(x) (p+5q+2)^2$$

$$\text{Using } (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$p^2 + (5q)^2 + 2^2 + 2(p)(5q) + 2(5q)(2) + 2(2)(p)$$

$$p^2 + 25q^2 + 4 + 10pq + 20q + 4p$$

$$(xi) (3x-4y-5)^2$$

$$[3x + (-4y) + (-5)]^2$$

$$\text{Using } (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$(3x)^2 + (-4y)^2 + (-5)^2 + 2(3x)(-4y) + 2(-4y)(-5) + 2(-5)(3x)$$

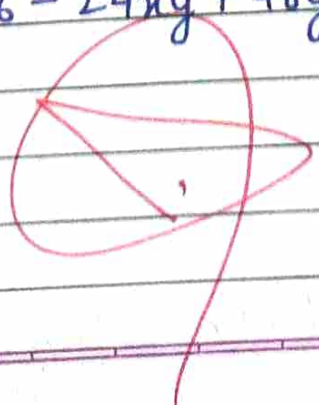
$$9x^2 + 16y^2 + 25 - 24xy + 40y - 30x$$

$$(xii) (-2x+6y+4)^2$$

$$\text{Using } (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$(-2x)^2 + (6y)^2 + 4^2 + 2(-2x)(6y) + 2(6y)(4) + 2(4)(-2x)$$

$$4x^2 + 36y^2 + 16 - 24xy + 48y - 16x$$



~~19/7/22~~

WS-5

Q 2: Evaluate

(i) 102×104
 $(100+2)(100+4)$

Using $(x+a)(x+b) = x^2 + (a+b)x + ab$
 $100^2 + (2+4) \times 100 + 2 \times 4$
 $10,000 + 600 + 8$
 10608

(ii) 105×103
 $= (100+5)(100+3)$

Using $(x+a)(x+b) = x^2 + (a+b)x + ab$
 $100^2 + (5+3) \times 100 + 5 \times 3$
 $10,000 + 800 + 15$
 $= 10815$

(iii) 206×205
 $= (200+6)(200+5)$

Using $(x+a)(x+b) = x^2 + (a+b)x + ab$
 $= 200^2 + (6+5) \times 200 + 6 \times 5$
 $= 40,000 + 2200 + 30$
 $= 42230$

(iv) 98×96
 $= (100-2)(100-4)$

Using $(x+a)(x+b) = x^2 + (a+b)x + ab$

$$\begin{aligned}
 &= 100^2 + (-2-4) \times 100 + (-2) \times (-4) \\
 &= 10,000 - 600 + 8 \\
 &= 9408
 \end{aligned}$$

(v) 87×85

$$(100-13)(100-15)$$

Using $(x+a)(x+b) = x^2 + (a+b)x + ab$

$$\begin{aligned}
 &100^2 + (-13-15) \times 100 + (-13) \times (-15) \\
 &10,000 - 2800 + 325 \\
 &=
 \end{aligned}$$

(vi) ~~104×95~~

$$(100+4)(100-5)$$

Using $(x+a)(x+b) = x^2 + (a+b)x + ab$

$$\begin{aligned}
 &= 100^2 + (4-5) \times 100 + 4 \times (-5) \\
 &= 10,000 - 100 - 20 \\
 &= 9880
 \end{aligned}$$

(vii) ~~97×102~~

$$(100-3)(100+2)$$

Using $(x+a)(x+b) = x^2 + (a+b)x + ab$

$$\begin{aligned}
 &= 100^2 + (-3+2) \times 100 + (-3) \times 2 \\
 &= 10,000 - 100 - 6 \\
 &= 9894
 \end{aligned}$$

(viii) 203×198

$(200+3)(200-2)$

Using $(x+a)(x+b) = x^2 + (a+b)x + ab$

$= 200^2 + (3-2) \times 200 + 3 \times (-2)$

$= 40,000 + 200 - 6$

$= 40194$

(ix) 35×37

$(40-5)(40-3)$

Using $(x+a)(x+b) = x^2 + (a+b)x + ab$

$= 40^2 + (-5-3) \times 40 + (-5) \times (-3)$

$= 1600 - 320 + 15$

$= 1295$

(x) 106×93

$(100+6)(100-7)$

Using $(x+a)(x+b) = x^2 + (a+b)x + ab$

$= 100^2 + (6-7) \times 100 + 6 \times (-7)$

$= 10,000 - 100 - 42$

$= 9858$

Q3: Evaluate

(i) $(x^2+3)(x^2+4)$

Using $(x+a)(x+b) = x^2 + (a+b)x + ab$

$(x^2)^2 + (3+4)x^2 + 3 \times 4$

$= x^4 + 7x^2 + 12$

$$(ii) \left[\frac{x+4}{3} \right] \left[\frac{x+1}{3} \right]$$

Using $(x+a)(x+b) = x^2 + (a+b)x + ab$

$$x^2 + \left[\frac{4+1}{3} \right] x + \frac{4 \times 1}{3 \times 3}$$

$$x^2 + \frac{5}{3} x + \frac{4}{9}$$

$$(iii) \left[\frac{x-3}{5} \right] \left[\frac{x-1}{2} \right]$$

Using $(x+a)(x+b) = x^2 + (a+b)x + ab$

$$x^2 + \left[\left(\frac{-3}{5} \right) + \left(\frac{-1}{2} \right) \right] x + \left(\frac{-3}{5} \right) \left(\frac{-1}{2} \right)$$

$$x^2 - \frac{11}{10} x + \frac{3}{10}$$

$$(iv) (y^2-6)(y^2+7)$$

Using $(x+a)(x+b) = x^2 + (a+b)x + ab$

$$(y^2)^2 + (-6+7)y^2 + (-6) \times 7$$
$$= y^4 + 1y^2 - 42$$

$$(v) (z^2+4) \left(\frac{z^2-1}{4} \right)$$

Using $(x+a)(x+b) = x^2 + (a+b)x + ab$

$$(z^2)^2 + \left[\frac{4-1}{4} \right] z^2 + 4 \times \left(\frac{-1}{4} \right)$$

$$= z^4 + \frac{15z^2 + 1}{4}$$

$$(VI) (y^2 - 3)(y^2 - 1)$$

$$\text{Using } (x+a)(x+b) = x^2 + (a+b)x + ab$$

$$(y^2)^2 + [(-3) + (-1)]y^2 + (-3) \times (-1)$$

$$y^4 - 4y^2 + 3$$

$$(VII) (x^3 + 5)(x^2 + 2)$$

$$\text{Using } (x+a)(x+b) = x^2 + (a+b)x + ab$$

$$= (x^3)^2 + (5+2)x^3 + 5 \times 2$$

$$= x^6 + 7x^3 + 10$$

$$(VIII) \left[\frac{p^2 - 1}{4} \right] \left[\frac{p^2 + 1}{8} \right]$$

$$\text{Using } (x+a)(x+b) = x^2 + (a+b)x + ab$$

$$= (p^2)^2 + \left[\frac{-1}{4} + \frac{1}{8} \right] p^2 + \left(\frac{-1}{4} \right) \times \frac{1}{8}$$

$$= p^4 - \frac{1}{8} p^2 - \frac{1}{32}$$

$$(IX) \left[\frac{z+1}{6} \right] [z+6]$$

$$\text{Using } (x+a)(x+b) = x^2 + (a+b)x + ab$$

$$= z^2 + \left[\frac{1+6}{6} \right] z + \frac{1 \times 6}{6}$$

$$= z^2 + \frac{37z}{6} + 1$$

WS-6

Factorise

1. $x^2 + z^2 - 2xz$

$$\begin{aligned} \text{Using } a^2 + b^2 - 2ab &= (a-b)^2 \\ &= (x-z)^2 \\ &= (x-z)(x-z) \end{aligned}$$

2. $4x^2 + 9y^2 - 12xy$

$$(2x)^2 + (3y)^2 - 2 \times 2x \times 3y$$

$$\text{Using } a^2 + b^2 - 2ab = (a-b)^2$$

$$= (2x-3y)^2$$

$$= (2x-3y)(2x-3y)$$

3. $64a^2 + 49b^2 + 112ab$

$$(8a)^2 + (7b)^2 + 2 \times 8a \times 7b$$

$$\text{Using } a^2 + b^2 + 2ab = (a+b)^2$$

$$= (8a+7b)^2$$

$$= (8a+7b)(8a+7b)$$

4. $121p^2 + 16q^2 - 88pq$

$= (11p)^2 + (4q)^2 - 2 \times 11p \times 4q$

Using $a^2 + b^2 - 2ab = (a - b)^2$
 $= (11p - 4q)^2$
 $= (11p - 4q)(11p - 4q)$

5. $9x^2y - 24xy^2 + 16y^3$

$= y(9x^2 - 24xy + 16y^2)$
 $= y[(3x)^2 - 2(3x)(4y) + (4y)^2]$
Using $a^2 - 2ab + b^2 = (a - b)^2$
 $= y(3x - 4y)^2$
 $= y(3x - 4y)(3x - 4y)$

6. $2a^3 + 4a^2b + 2ab^2$

$= 2a(a^2 + 2ab + b^2)$

Using Identity $a^2 + 2ab + b^2 = (a + b)^2$

$= 2a(a + b)^2$

$= 2a(a + b)(a + b)$

7. $50a^2 + 98b^2 + 140ab$

$= 2(25a^2 + 49b^2 + 70ab)$

$= 2[(5a)^2 + (7b)^2 + 2 \times 5a \times 7b]$

Using Identity $a^2 + b^2 + 2ab = (a + b)^2$

$= 2(5a + 7b)^2$

$= 2(5a + 7b)(5a + 7b)$

$$8. \quad 64x^2 - 81y^2$$

$$= (8x)^2 - (9y)^2$$

Using Identity $a^2 - b^2 = (a+b)(a-b)$

$$= (8x+9y)(8x-9y)$$

$$9. \quad 25p^2 - 9q^2$$

$$= (5p)^2 - (3q)^2$$

Using Identity $a^2 - b^2 = (a+b)(a-b)$

$$(5p+3q)(5p-3q)$$

$$10. \quad 16a^2b - 64b^3$$

$$= 16b(a^2 - 4b^2)$$

$$= 16b[(a)^2 - (2b)^2]$$

Using Identity $a^2 - b^2 = (a+b)(a-b)$

$$= 16b(a+2b)(a-2b)$$

~~$$10. \quad 16a^2b - 64b^2$$~~

$$11. \quad 25x^3y^3 - 49xy$$

$$xy(25x^2y^2 - 49)$$

$$xy[(25xy)^2 - 7^2]$$

Using $a^2 - b^2 = (a+b)(a-b)$

$$xy(5xy+7)(5xy-7)$$

$$12. \quad p^4 - 256$$

$$= (p^2)^2 - (16)^2$$

Using $a^2 - b^2 = (a+b)(a-b)$

$$(p^2+16)(p^2-16)$$

$$(p^2+16)(p^2-4^2)$$

$$(p^2+16)(p+4)(p-4)$$

13. $a^2-(b-c)^2$

using $a^2-b^2=(a+b)(a-b)$

$$= (a+b-c)[a-(b-c)]$$

$$= (a+b-c)(a-b+c)$$

14. $25m^2-(4n+3l)^2$

$$= (5m)^2-(4n+3l)^2$$

using $a^2-b^2=(a+b)(a-b)$

$$= (5m+4n+3l)[5m-(4n+3l)]$$

$$= (5m+4n+3l)(5m-4n-3l)$$

15. $(2a+3b)^2-4c^2$

$$(2a+3b)^2-(2c)^2$$

using $a^2-b^2=(a+b)(a-b)$

$$(2a+3b+2c)(2a+3b-2c)$$

16. $(64m^2-144mn+81n^2)-25p^2$

$$= [(8m)^2-2 \times 8m \times 9n+(9n)^2]- (5p)^2$$

using $a^2-2ab+b^2=(a-b)^2$

$$= (8m-9n)^2-(5p)^2$$

using $a^2-b^2=(a+b)(a-b)$

$$= (8m-9n+5p)(8m-9n-5p)$$

17. $16x^2+9y^2+4z^2+24xy+12yz+16zx$

$$= (4x)^2+(3y)^2+(2z)^2+2 \times (4x)(3y)+2 \times (3y)(2z)+2 \times (2z)(4x)$$

$$= 16x^2+9y^2+4z^2+24xy+12yz+16zx$$

using $a^2+b^2+c^2+2ab+2bc+2ca$

$$(4x+3y+2z)^2$$

$$(4x + 3y + 2z)(4x + 3y + 2z)$$

$$18. \quad x^2 + 4y^2 + 9z^2 - 4xy + 12yz - 6zx$$

$$= (-x)^2 + (2y)^2 + (3z)^2 + 2(-x)(2y) + 2(2y)(3z) + 2(3z)(-x)$$

$$\text{Using } a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2$$

$$= (-x + 2y + 3z)^2$$

$$= (-x + 2y + 3z)(-x + 2y + 3z)$$

$$19. \quad 4a^2 + b^2 + 25c^2 - 4ab - 10bc + 20ca$$

$$= (2a)^2 + (-b)^2 + (5c)^2 + 2(2a)(-b) + 2(-b)(5c) + 2(5c)(2a)$$

$$\text{Using } a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2$$

$$= (2a - b + 5c)^2$$

$$= (2a - b + 5c)(2a - b + 5c)$$

$$20. \quad a^2 + \frac{b^2}{4} + \frac{c^2}{9} + ab + \frac{bc}{3} + \frac{2ca}{3}$$

$$= a^2 + \left(\frac{b}{2}\right)^2 + \left(\frac{c}{3}\right)^2 + 2\left(a\right)\left(\frac{b}{2}\right) + 2\left(\frac{b}{2}\right)\left(\frac{c}{3}\right) + 2\left(a\right)\left(\frac{c}{3}\right)$$

$$\text{Using } a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2$$

$$= \left(\frac{a + \frac{b}{2} + \frac{c}{3}}{2}\right)^2$$

$$= \left(\frac{a + \frac{b}{2} + \frac{c}{3}}{2}\right)\left(\frac{a + \frac{b}{2} + \frac{c}{3}}{2}\right)$$

1. $x^2 + 14x + 33$

$$= x^2 + 11x + 3x + 33$$

$$= x(x+11) + 3(x+11)$$

$$= (x+11)(x+3)$$

2. $x^2 + x - 72$

$$= x^2 + 9x - 8x - 72$$

$$= x(x+9) - 8(x+9)$$

$$= (x+9)(x-8)$$

3. $y^2 - y - 6$

$$y^2 - 3y + 2y - 6$$

$$= y(y-3) + 2(y-3)$$

$$= (y-3)(y+2)$$

4. $x^2 + 12x + 27$

$$x^2 + 9x + 3x + 27$$

$$x(x+9) + 3(x+9)$$

$$= (x+9)(x+3)$$

5. $y^2 + y - 132$

$$y^2 + 12y - 11y - 132$$

$$y(y+12) - 11(y+12)$$

$$= (y+12)(y-11)$$

$$\begin{aligned}
 6. \quad & x^2 + 11x + 30 \\
 & x^2 + 6x + 5x + 30 \\
 & x(x+6) + 5(x+6) \\
 & = (x+6)(x+5)
 \end{aligned}$$

$$\begin{aligned}
 7. \quad & x^2 - 11x - 42 \\
 & x^2 - 14x + 3x - 42 \\
 & x(x-14) + 3(x-14) \\
 & = (x-14)(x+3)
 \end{aligned}$$

$$\begin{aligned}
 8. \quad & z^2 - 12z + 27 \\
 & z^2 - 9z - 3z + 27 \\
 & z(z-9) - 3(z-9) \\
 & = (z-9)(z-3)
 \end{aligned}$$

$$\begin{aligned}
 9. \quad & p^2 - 5p - 6 \\
 & p^2 - 6p + p - 6 \\
 & p(p-6) + 1(p-6) \\
 & = (p+1)(p-6)
 \end{aligned}$$

$$\begin{aligned}
 10. \quad & p^2 + p - 56 \\
 & p^2 + 8p - 7p - 56 \\
 & p(p+8) - 7(p+8) \\
 & = (p+8)(p-7)
 \end{aligned}$$

$$\begin{aligned}
 11. \quad & x^2 - 8x - 65 \\
 & x^2 - 13x + 5x - 65 \\
 & x(x-13) + 5(x-13) \\
 & = (x-13)(x+5)
 \end{aligned}$$

$$\begin{aligned}
 12. \quad & x^2 + 8x + 15 \\
 & x^2 + 5x + 3x + 15 \\
 & x(x+5) + 3(x+5) \\
 & = (x+5)(x+3)
 \end{aligned}$$

$$\begin{aligned}
 13. \quad & x^2 + 2x - 24 \\
 & x^2 + 6x - 4x - 24 \\
 & x(x+6) - 4(x+6) \\
 & = (x+6)(x-4)
 \end{aligned}$$

$$\begin{aligned}
 14. \quad & x^2 - 3x - 54 \\
 & x^2 - 9x + 6x - 54 \\
 & x(x-9) + 6(x-9) \\
 & = (x-9)(x+6)
 \end{aligned}$$

$$\begin{aligned}
 15. \quad & a^2 - 7a + 12 \\
 & a^2 - 3a - 4a + 12 \\
 & = a(a-3) - 4(a-3) \\
 & = (a-3)(a-4)
 \end{aligned}$$

$$\begin{aligned}
 16. \quad & p^2 - 3pq + 2q^2 \\
 & p^2 - 2pq - 1pq + 2q^2 \\
 & = p(p-2q) - q(p-2q) \\
 & = (p-2q)(p-q)
 \end{aligned}$$

Value Based Questions

$$\begin{aligned}
 1. \quad & \text{No. of CD's Gautam purchased} = x^2 + 8x - 20 \\
 & \text{No. of CD's Sanyam purchased} = (x+4)^2 \\
 & = x^2 + 4^2 + 2 \times x \times 4
 \end{aligned}$$

$$= x^2 + 16 + 8x$$

$$= x + 8x + 16$$

(i) Sanyam purchased more CD's by 36

$$x^2 + 8x + 16 - (x^2 + 8x - 20)$$

$$x^2 + 8x + 16 - x^2 - 8x + 20$$

$$= 36$$

(ii) Advantage: Improves Concentration

Disadvantage: Distraction in Study.

2. Base of flower pot = $x^2 + 6x + 9$

Area of rectangular lane = $(x+3)(7x+21)$

$$\text{No. of pots placed} = \frac{(x+3)(7x+21)}{x^2 + 6x + 9}$$

$$= \frac{(x+3)(7x+21)}{x^2 + 3x + 3x + 9}$$

$$= \frac{(x+3)(7x+21)}{x(x+3) + 3(x+3)}$$

$$= \frac{(x+3)(7x+21)}{x(x+3) + 3(x+3)}$$

$$= \frac{(x+3)(7x+21)}{(x+3)(x+3)}$$

$$= \frac{(x+3)(7x+21)}{(x+3)(x+3)}$$

$$= \frac{7x+21}{x+3} = \frac{7(x+3)}{x+3}$$

$$= \frac{7x+21}{x+3} = \frac{7(x+3)}{x+3}$$

$$= \underline{\underline{7 \text{ Ans}}}$$

Brain Teasers

$$\begin{aligned}
 \text{Q1: a) } & 1 - 6z + 9z^2 \\
 &= 1^2 - 2 \times 1 \times 3z + (3z)^2 \\
 &= \text{Using } a^2 - 2ab + b^2 \\
 &= (1 - 3z)^2 \\
 &= (1 - 3z)(1 - 3z)
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } & y^2 + 3y + y + 3 \\
 &= y(y+3) + 1(y+3) \\
 &= (y+3)(y+1)
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } & p^2 - 17p - 38 \\
 &= \cancel{p^2} = p^2 - 19p + 2p - 38 \\
 &= p(p-19) + 2(p-19) \\
 &= (p+2)(p-19)
 \end{aligned}$$

$$\begin{aligned}
 \text{e) Area of Square} &= 4x^2 + 12x + 9 \\
 \text{Side}^2 &= (2x)^2 + 2 \times 2x \times 3 + 3^2 \\
 &= \text{Using } a^2 + 2ab + b^2 = (a+b)^2 \\
 &= \text{Side}^2 = (2x+3)^2 \\
 \text{Side} &= 2x+3
 \end{aligned}$$

(B) Answer the following

$$\begin{aligned}
 \text{a) } & (a+3)(a-3)(a^2+9) \\
 & \text{Using } (a+b)(a-b) = a^2 - b^2 \\
 &= (a^2 - 3^2)(a^2 + 3^2) \\
 &= (a^2 - 9)(a^2 + 9) \\
 & \text{Using } (a+b)(a-b) = a^2 - b^2 \\
 &= (a^2)^2 - 9^2 \\
 &= a^4 - 81
 \end{aligned}$$

b) $\frac{x^2+1}{x^2} = 6$ find $\frac{x^4+1}{x^4}$

$$\frac{x^2+1}{x^2} = 6$$

Squaring both sides

$$\left[\frac{x^2+1}{x^2} \right]^2 = 6^2$$

$$= (a+b)^2 = a^2 + b^2 + 2ab$$

$$= (x^2)^2 + \left(\frac{1}{x^2} \right)^2 + 2 \times x^2 \times \frac{1}{x^2} = 36$$

$$= \frac{x^4+1}{x^4} + 2 = 36$$

$$= \frac{x^4+1}{x^4} = 36 - 2$$

$$= \frac{x^4+1}{x^4} = 34$$

c) xy^2k find $= xy^2k = (4xy+3y)^2 - (4xy-3y)^2$

Using $(a^2+b^2) = (a+b)(a-b)$

$$x^2y^2k = [4xy+3y+4xy-3y] [4xy+3y-4xy+3y]$$

$$x^2y^2k = 8xy \cdot 6y$$

$$x^2y^2k = 48xy^2$$

$$k = 48$$

$$d) a^2 + b^2 = 9, ab = 4$$

$$\begin{aligned} & 3(a+b)^2 - 2(a-b)^2 \\ &= 3(a^2 + b^2 + 2ab) - 2(a^2 + b^2 - 2ab) \\ &= 3(9 + 2 \times 4) - 2(9 - 2 \times 4) \\ &= 3(9 + 8) - 2(9 - 8) \\ &= 3(17) - 2(1) \\ &= 51 - 2 \\ &= 49 \end{aligned}$$

$$e) \text{ If } a + b + c = 12$$

$$a^2 + b^2 + c^2 = 144$$

Find $ab + bc + ca =$

Using $(a^2 + b^2 + c^2) =$

$$12^2 = 144 + 2(ab + bc + ca)$$

$$144 - 144 = 2(ab + bc + ca)$$

$$0 = 2(ab + bc + ca)$$

$$0 = (ab + bc + ca)$$

Q2: Find the Product

$$\begin{aligned} \text{(i)} \quad (7x - 9y)(7x - 9y) &= (7x)^2 - (9y)^2 \\ &= (7x - 9y)^2 \\ &= 49x^2 - 81y^2 \end{aligned}$$

$$\text{Using } (a-b)^2 = a^2 + b^2 - 2ab$$

$$= (7x)^2 + (9y)^2 - 2 \times 7x \times 9y$$

$$= 49x^2 + 81y^2 - 126xy$$

$$\text{(ii)} \quad (6ax + 5by)(6ax - 5by)$$

$$\text{Using } (a+b)(a-b) = a^2 - b^2$$

$$= (6ax)^2 - (5by)^2$$
$$= 36a^2x^2 - 25b^2y^2$$

$$\text{(iii)} \left[\frac{3}{4}p^2 + \frac{2}{3}q^2 \right] \left[\frac{3}{4}p^2 + \frac{2}{3}q^2 \right]$$

$$\left[\frac{3}{4}p^2 + \frac{2}{3}q^2 \right]^2$$

$$\text{Using } (a+b)^2 = a^2 + b^2 + 2ab$$

$$= \left(\frac{3}{4}p^2 \right)^2 + \left(\frac{2}{3}q^2 \right)^2 + 2 \left(\frac{3}{4}p^2 \right) \left(\frac{2}{3}q^2 \right)$$

$$= \frac{9}{16}p^4 + \frac{4}{9}q^4 + p^2q^2$$

$$\text{(iv)} \left[x + \frac{3}{4}y - 4z \right]^2$$

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$= x^2 + \left(\frac{3}{4}y \right)^2 + (-4z)^2 + 2(x) \left(\frac{3}{4}y \right) + 2 \left(\frac{3}{4}y \right) (-4z) + 2(-4z)(x)$$

$$= x^2 + \frac{9}{16}y^2 + 16z^2 + \frac{3}{2}xy - 6yz - 8xz$$

$$\text{(v)} (x+2y)(x-2y)(x^2+4y^2)$$

$$\text{Using } (a+b)(a-b) = a^2 - b^2$$

$$= (x^2 - 4y^2)(x^2 + 4y^2)$$

$$\text{Using } (a+b)(a-b) = a^2 - b^2$$

$$= (x^2)^2 - (4y^2)^2$$

$$= x^4 - 16y^4$$

$$(vi) \left[\frac{x+1}{x} \right] \left[\frac{x-1}{x} \right] \left[\frac{x^2+1}{x^2} \right] \left[\frac{x^4+1}{x^4} \right]$$

Using $(a+b)(a-b) = a^2 - b^2$

$$\left(\frac{x^2-1}{x^2} \right) \left(\frac{x^2+1}{x^2} \right) \left[\frac{x^4+1}{x^4} \right]$$

Using $(a+b)(a-b) = a^2 - b^2$

$$\left[\frac{(x^2)^2 - (1)^2}{x^2} \right] \left[\frac{x^4+1}{x^4} \right]$$

$$= \left[\frac{x^4-1}{x^2} \right] \left[\frac{x^4+1}{x^4} \right]$$

$$= \frac{(x^4)^2 - 1}{(x^4)^2} \Rightarrow \frac{x^8 - 1}{x^8}$$

Q3: $4x^2 + 12xy + 9y^2, x = -2, y = 2$

$$= (2x)^2 + 2 \times 2x \times 3y + (3y)^2$$

Using $a^2 + 2ab + b^2 = (a+b)^2$

$$= (2x + 3y)^2$$

Put $x = -2, y = 2$

$$[2(-2) + 3(2)]^2$$

$$= (-4 + 6)^2$$

$$= 4$$

$$\frac{4m^2 - mn + 25n^2}{25}, m=5, n=8$$

$$\left(\frac{2m}{5}\right)^2 - \frac{2 \times 2m \times 5n}{4} + \left(\frac{5}{4}\right)$$

Using

$$= \left[\frac{2m}{5} - \frac{5n}{4}\right]^2$$

Put $m=5, n=8$

$$= \frac{2 \times 5}{5} - \frac{5 \times 8}{4}$$

$$= 2 - 10$$

$$= -8$$

Q4: Simplify

(i) $(a+b)^2 + (a-b)^2$

$$= a^2 + b^2 + 2ab + a^2 + b^2 - 2ab$$

$$= 2a^2 + 2b^2$$

$$= 2(a^2 + b^2)$$

(ii) $(3x+7y)^2 - (3x-7y)^2$

$$= (3x)^2 + (7y)^2 + 2 \times 3x \times 7y - [(3x)^2 + (7y)^2 - 2 \times 3x \times 7y]$$

$$= 9x^2 + 49y^2 + 42xy - [9x^2 + 49y^2 - 42xy]$$

$$= 9x^2 + 49y^2 + 42xy - 9x^2 - 49y^2 + 42xy$$

$$= 84xy$$

(iii) $(6a+5b+4c)^2 - (6a-5b-4c)^2$

Using $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$$= (6a)^2 + (5b)^2 + (4c)^2 + 2(6a)(5b) + 2(5b)(4c) + 2(4c)(6a) \\ - [(6a)^2 + (-5b)^2 + (-4c)^2 + 2(6a)(-5b) + 2(5b)(-4c) + 2(-4c)(6a)]$$

$$= 36a^2 + 25b^2 + 16c^2 + 60ab + 40bc + 48ca - [36a^2 + 25b^2 + 16c^2 \\ - 60ab + 40bc + 48ca] \\ = 36a^2 + 25b^2 + 16c^2 + 60ab + 40bc + 48ca - 36a^2 - 25b^2 - 16c^2 + 60ab \\ - 40bc + 48ca \\ = 120ab + 96ac$$

$$(iv) (a^2 - b^2)(a^2 + b^2) - (a^2 - b^2)^2$$

$$\text{Using } (a-b)(a+b) = a^2 - b^2 \\ = a^4 - b^4 - [(a^2)^2 + (b^2)^2 - 2a^2b^2] \\ = a^4 - b^4 - a^4 - b^4 + 2a^2b^2 \\ = -2b^4 + 2a^2b^2 \\ = -2b^2(b^2 - a^2)$$

Q5: Evaluate

$$i) 103 \times 97$$

$$(100+3)(100-3)$$

$$\text{Using } (a+b)(a-b) = a^2 - b^2$$

$$= 100^2 - 3^2$$

$$= 10,000 - 9$$

$$= 9991$$

$$(ii) (10.1)^2$$

$$(10+0.1)^2$$

$$\text{Using } (a+b)^2 = a^2 + b^2 + 2ab$$

$$= 10^2 + 0.1^2 + 2 \times 10 \times 0.1$$

$$= 100 + 0.01 + 2$$

$$= 102.01$$

$$(iii) 85^2 - 75^2$$

$$\begin{aligned} \text{Using } a^2 - b^2 &= (a+b)(a-b) \\ &= (85+75)(85-75) \\ &= 160 \times 10 \\ &= 1600 \end{aligned}$$

$$(iv) 291 \times 309$$

$$\begin{aligned} &(300-9)(300+9) \\ \text{Using } (a+b)(a-b) &= a^2 - b^2 \\ &300^2 - 9^2 \\ &= 90,000 - 81 \\ &= 89919 \end{aligned}$$

Q6: Evaluate x

$$(i) 36x = 78^2 - 42^2$$

$$\begin{aligned} \text{Using } a^2 - b^2 &= (a+b)(a-b) \\ \Rightarrow 36x &= (78+42)(78-42) \\ \Rightarrow 36x &= 120 \times 36 \\ \Rightarrow x &= \frac{120 \times 36}{36} \\ \Rightarrow x &= 120 \end{aligned}$$

$$(ii) 6.2x = 8.1 \times 8.1 - 1.9 \times 1.9$$

$$\begin{aligned} 6.2x &= (8.1)^2 - (1.9)^2 \\ \Rightarrow 6.2x &= (8.1+1.9)(8.1-1.9) \\ \Rightarrow 6.2x &= 10 \times 6.2 \\ \Rightarrow x &= \frac{10 \times 6.2}{6.2} \\ \Rightarrow x &= 10 \end{aligned}$$

Q8: Factorise

$$\begin{aligned} \text{(i)} \quad & z^2 - 4z - 77 \\ & z^2 - 11z + 7z - 77 \\ & z^2 - 11z + 7z - 77 \\ & z(z-11) + 7(z-11) \\ & = (z-11)(z+7) \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad & x^2 + 25x + 144 \\ & x^2 + 16x + 9x + 144 \\ & = \cancel{x+16} x(x+16) + 9(x+16) \\ & = (x+9)(x+16) \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad & x^2 + 5x - 104 \\ & x^2 + 13x - 8x - 104 \\ & x(x+13) - 8(x+13) \\ & (x+13)(x-8) \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad & 49x^2 - 64 \\ & = (7x)^2 - (8)^2 \\ & \text{using } a^2 - b^2 = (a+b)(a-b) \\ & = (7x+8)(7x-8) \end{aligned}$$

$$\begin{aligned} \text{(v)} \quad & x^2 - 1 - 2y - y^2 \\ & x^2 - (1 + 2y + y^2) \\ & x^2 - [1^2 + 2 \times 1 \times y + y^2] \\ & x^2 - (1+y)^2 \\ & \text{using } x^2 - y^2 = (x+y)(x-y) \\ & (x+1+y)(x-(y+1)) \\ & (x+1+y)(x-1-y) \end{aligned}$$

$$(VI) 4(x-y)^2 - 12(x-y) + 9$$

$$\text{put } x-y = a$$

$$4a^2 - 12a + 9$$

$$= 4a^2 - 6a - 6a + 9$$

$$= 2a(2a-3) - 3a(2a-3)$$

$$= (2a-3)(2a-3)$$

$$= [2(x-y)-3][2(x-y)-3]$$

$$(VII) x^2 + y - xy - x$$

$$= x^2 - xy - x + y$$

$$= x(x-y) - 1(x-y)$$

$$= (x-y)(x-1)$$

$$(VIII) 81(x+1)^2 + 90(x+1)(y+2) + 25(y+2)^2$$

$$\text{Put } x+1 = a, y+2 = b$$

$$81a^2 + 90ab + 25b^2$$

$$= (9a)^2 + 2 \times 9a \times 5b + (5b)^2$$

$$= \text{Using } a^2 + 2ab + b^2 = (a+b)^2$$

$$= (9a + 5b)^2$$

$$= [9(x+1) + 5(y+2)]^2$$

$$= (9x + 9 + 5y + 10)^2$$

$$= (9x + 5y + 19)^2$$

$$Q8: x \frac{1}{x} = 8 \text{ Find } x^2 + \frac{1}{x^2}$$

$$x + \frac{1}{x} = 8$$

Squaring both sides

$$\left(x + \frac{1}{x}\right)^2 = 8^2$$

$$\frac{x^2+1}{x^2} = 64 + 2 \times x \times \frac{1}{x} = 64$$

$$\frac{x^2+1}{x^2} + 2 = 64$$

$$\frac{x^2+1}{x^2} = 64 - 2$$

$$\frac{x^2+1}{x^2} = 62$$

Q9: If $\frac{a-1}{a} = 5$ Find $\frac{a^2+1}{a^2}$

$$\frac{a-1}{a} = 5$$

Squaring both sides

$$\left(\frac{a-1}{a}\right)^2 = 5^2$$

$$\frac{a^2+1}{a^2} = 25$$

$$\frac{a^2+1}{a^2} = 2 \times a \times \frac{1}{a} = 25$$

$$\frac{a^2+1}{a^2} - 2 = 25$$

$$\frac{a^2+1}{a^2} = 27$$

Q10: $\frac{a^2+1}{a^2} = 23$ find $\left[\frac{a+1}{a}\right]$

$$= \left(\frac{a+1}{a} \right)^2 = \frac{a^2+1}{a^2} + 2 \cdot \frac{a}{a} \cdot \frac{1}{a}$$

$$= \left(\frac{a+1}{a} \right)^2 = 23+2$$

$$= \left(\frac{a+1}{a} \right)^2 = 25$$

$$= \left(\frac{a+1}{a} \right)^2 = 5^2$$

$$= \frac{a+1}{a} = 5$$

Q11: $\frac{x^2+1}{x^2} = 51$ find $\frac{x-1}{x}$

$$\left(\frac{x-1}{x} \right)^2 = \frac{x^2+1}{x^2} - 2$$

$$\left(\frac{x-1}{x} \right)^2 = 51 - 2$$

$$\left(\frac{x-1}{x} \right)^2 = 7^2$$

$$\frac{x-1}{x} = 7$$

Q12: $2.3 \times 2.3 - 0.3 \times 0.3$
 $2.3 \times 2.3 - 2 \times 2.3 \times 0.3 + 0.3 \times 0.3$

$$\frac{(2.3)^2 - (0.3)^2}{(2.3 - 0.3)^2} \text{ Using } a^2 - b^2 = (a+b)(a-b)$$

$$(2.3)^2 - 2 \times 2.3 \times 0.3 + (0.3)^2 \text{ Using } (a-b)^2 = a^2 + b^2 - 2ab = (a-b)(a+b)$$

$$\frac{(2.3+0.3)(2.3-0.3)}{(2.3-0.3)^2}$$

$$= \frac{2.6}{2} = 1.3$$

Q13: $3x - 4y = 10, xy = -1$, find $9x^2 + 16y^2$

$$3x - 4y = 10$$

Squaring both sides

$$(3x - 4y)^2 = 10^2$$

$$(3x)^2 + (4y)^2 - 2 \times 3x \times 4y = 100$$

$$9x^2 + 16y^2 - 24xy = 100$$

$$9x^2 + 16y^2 - 24(-1) = 100$$

$$9x^2 + 16y^2 + 24 = 100$$

$$9x^2 + 16y^2 = 100 - 24$$

$$9x^2 + 16y^2 = 76$$

Q14: $5x - 2y = 7, xy = 2$, find $(5x + 2y)^2$

$$(5x + 2y)^2$$

$$(5x - 2y) = 7$$

Squaring both sides

$$(5x)^2 + (2y)^2 - 2 \times 5x \times 2y = 7^2$$

$$25x^2 + 4y^2 - 20xy = 49$$

$$25x^2 + 4y^2 - 20(2) = 49$$

$$25x^2 + 4y^2 - 40 = 49$$

$$25x^2 + 4y^2 = 89$$

$$\begin{aligned} \text{Now } (5x+2y)^2 &= (5x)^2 + (2y)^2 + 2 \times 5x \times 2y \\ &= 25x^2 + 4y^2 + 20xy \\ &= 89 + 20(2) \\ &= 89 + 40 \end{aligned}$$

$$\text{So } (5x+2y)^2 = 129$$

15. $\frac{a+1}{a} = \frac{17}{4}$, find $\left(\frac{a-1}{a}\right)$

Squaring both sides

$$\left(\frac{a+1}{a}\right)^2 = \left(\frac{17}{4}\right)^2$$

$$\frac{a^2+1}{a^2} + 2 = \frac{289}{16}$$

$$\frac{a^2+1}{a^2} = \frac{289-2}{16}$$

$$\frac{a^2+1}{a^2} = \frac{257}{16}$$

$$\left(\frac{a-1}{a}\right)^2 = \frac{a^2+1}{a^2} - 2$$

$$\left(\frac{a-1}{a}\right)^2 = \frac{257-2}{16}$$

$$\left(\frac{a-1}{a}\right)^2 = \frac{257-32}{16}$$

$$\left(\frac{a-1}{a}\right)^2 = \frac{225}{4}$$

$$\frac{a-1}{a} = \frac{15}{2}$$

HOTS

1. No. of observations = $x+3$

Sum of observations = $x^4 - 81$

$$\text{Mean} = \frac{\text{Sum of observations}}{\text{No. of observations}}$$

$$= \frac{x^4 - 81}{x+3} = \frac{(x^2)^2 - 9^2}{x+3}$$

$$= \frac{(x^2+9)(x^2-9)}{x+3}$$

$$= \frac{(x^2+9)(x^2-3^2)}{x+3}$$

$$= \frac{(x^2+9)(x-3)(x+3)}{x+3}$$

$$\text{Mean} = (x^2+9)(x-3)$$

2. Area of circle = $\pi x^2 + 10\pi x + 25\pi$

$$\pi x^2 = \pi(x^2 + 10x + 25)$$

$$x^2 = x^2 + 2x \times 5 + 5^2$$

$$x^2 = (x+5)^2$$

$$x = x+5$$