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THE MATH EXPERT

Chapter 1: Real Numbers

Exercise 1.1 (Page 7 of Grade 10 NCERT Textbook)

Q1. Use Euclid's division algorithm to find the HCF of:

(i) 135 and 225 (ii) 196 and 38220 (iii) 867 and 255

Difficulty Level: Easy

What is given /known?

Two different numbers

What is the unknown?

HCF of the given numbers.

Reasoning:

You have to find the HCF of given integers by using Euclid's Division Lemma. It is a technique to compute the highest common factor of two given positive integer. Recall, that the HCF of two positive integers *a* and *b* is the largest positive integer that divides both *a* and *b*.

To obtain the HCF of two positive integers say *a* and *b* with $a > b$, follow the below steps-

Step-I. Apply Euclid's division lemma to *a* and *b*. So, we find whole numbers q and r such that

$$a = bq + r, 0 \leq r < b$$

Step-II. If $r = 0$, b is the HCF of *a* and *b*. If $r \neq 0$, apply the division lemma to *b* and *r*.

Step-III. Continue the process till the remainder is zero. The divisor at this stage will be the required HCF.

Solution:

(i) 135 and 225

In this case 225 > 135. We apply Euclid's division lemma to 135 and 225 and get

$$225 = (135 \times 1) + 90$$

Since, the remainder $r \neq 0$, we apply the division lemma to 135 and 90 to get

$$135 = (90 \times 1) + 45$$

Now, we consider 90 as the divisor and 45 as the remainder and apply the division lemma, to get

$$90 = (45 \times 2) + 0$$

Since, the remainder is zero and the divisor is 45, therefore, the H.C.F of 135 and 225 is 45.

(ii) 196 and 38220

38220 is greater than 196, we apply Euclid's division lemma to 38220 and 196, to get

$$38220 = (196 \times 195) + 0$$

Since, the remainder is zero and the divisor in this step is 195, therefore, the H.C.F of 38220 and 196 is 196.

(iii) 867 and 255

867 is greater than 225 and on applying Euclid's division lemma to 867and 225, we get

$$867 = (255 \times 3) + 102$$

Since, the remainder $r \neq 0$, we apply the division lemma to 225 and 102 and get

$$255 = (102 \times 2) + 51$$

Again, remainder is not zero, we apply Euclid's division lemma 102 and 51 which gives

$$102 = (51 \times 2) + 0$$

Since, the remainder is zero and the divisor is 51, therefore, the H.C.F of 867 and 255 is 51.

Q2. Show that any positive odd integer is of the form $6q+1$, or $6q+3$, or $6q+5$, where q is some integer.

Difficulty Level: Tough

What is given /known?

Some integer q.

What is unknown/to be proved?

That any positive odd integer is of the form $6q+1$, or $6q + 3$, or $6q+5$, where q is some integer.

Reasoning:

To solve this question, first think about the Euclid's division algorithm. Suppose there is any positive integer 'a' and it is of the form $6q + r$, where q is some integer. This means that $0 \leq r < 6$ i.e. $r = 0$ or 1 or 2 or 3 or 4 or 5 but it can't be 6 because r is smaller than 6. So, by Euclid's division lemma, possible values for 'a' can be $6q$ or $6q+1$ or $6q+2$ or $6q+3$, or $6q+4$ or $6q+5$.

Solution:

Let 'a' be any positive integer. Then according to Euclid's algorithm,

$$a = 6q + r \text{ for some integer } q \geq 0$$

and $r = 0, 1, 2, 3, 4, 5$ because $0 \leq r < 6$

Therefore, $a = 6q+0$ or $6q+1$ or $6q+2$ or $6q+3$ or $6q+4$ or $6q+5$.

Now, $6q + 1 = 2 \times 3q + 1 = 2k_1 + 1$ (where k_1 is a positive integer)

$$6q+3 = 6q+2+1 = 2(3q+1)+1 = 2k_2+1$$

(where k_2 is a positive integer)

$$6q+5 = 6q+4+1 = 2(3q+2)+1 = 2k_3+1$$

(where k_3 is a positive integer)

Clearly, $6q+1$, $6q+3$ and $6q+5$ are of the form $2k+1$, where k is an integer.
Therefore, $6q+1$, $6q+3$ and $6q+5$ are not exactly divisible by 2.

Hence, these expressions of numbers are odd numbers and therefore any odd integers can be expressed in the form $6q+1$ or $6q+3$ or $6q+5$.

Q3. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Difficulty Level: Easy

What is given /known?

We are told that there is an army contingent of 616 members and an army band of 32 members. The two groups are to march in the same number of columns

What is the unknown?

The maximum number of columns in which they can march.

Reasoning:

Here, we have to pay attention to the point that the army band members and army contingent members have to march in the **same number of columns** and that the number of columns must be the **maximum** possible. The definition of HCF states – HCF is the **highest** number that can be **divided exactly into each of two or more numbers**. In other words, HCF of two numbers is the highest number (maximum) that divides both the numbers. Thus, we have the find the HCF of the members in the army band and the army contingent.

Solution:

HCF (616, 32) will give the maximum number of columns in which they can march.
We use Euclid's algorithm to find the H.C.F:

$$616 = (32 \times 19) + 8$$
$$32 = (8 \times 4) + 0$$

The HCF (616, 32) is 8. Therefore, they can march in 8 columns each.

Q4. Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.
[Hint: Let x be any positive integer then it is of the form $3q$, $3q + 1$ or $3q + 2$. Now square each of these and show that they can be rewritten in the form 3m or 3m + 1.]

Difficulty Level: Medium

To Prove:

The square of any positive integer is either of the form 3m or 3m + 1 for some integer m (using the Euclid's division lemma).

Reasoning:

Suppose that there is a positive integer 'a'. By Euclid's lemma, we know that for positive integers a and b, there exist unique integers q and r, such that $a = bq + r,\ 0 \le r < b$

If we keep the value of $b = 3$, then $0 \le r < 3$ i.e. $r = 0$ or 1 or 2 but it can't be 3 because r is smaller than 3. So, the possible values for $a = 3q$ or $3q + 1$ or $3q + 2$. Now, find the square of all the possible values of a. If q is any positive integer, then its square (let's call it as "m") will also be a positive integer. Now, observe carefully that the square of all the positive integers is either of the form 3m or 3m + 1 for some integer m.

Solution:

Let "a" be any positive integer and $b = 3$.

Then, $a = 3q + r$ for some integer $q \ge 0$ and $r = 0,1,2$ because $0 \le r < 3$.

Therefore, $a = 3q$ or $3q + 1$ or $3q + 2$ or

$$(a)^2 = (3q)^2 \text{ or } (3q+1)^2 \text{ or } (3q+2)^2$$
$$a^2 = 3(3q^2) \text{ or } (9q^2 + 6q + 1) \text{ or } (9q^2 + (12q + 4))$$
$$a^2 = 3\left(3q^2\right) \text{ or } 3\left(3q^2 + 2q\right) + 1 \text{ or } 3\left(3q^2 + 4q + 1\right) + 1$$
$$= \text{m or } 3\text{m} + 1$$

Where m is any positive integer. Hence it can be said that the square of any positive integer is either of the form 3m or 3m + 1.

Q5. Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8.

Difficulty Level: Medium

To Prove:

The cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8.

Reasoning:

Suppose that there is a positive integer 'a'. By Euclid's lemma, we know that for positive integers a and b, there exist unique integers q and r, such that $a = bq + r,\ 0 \le r < b$

If we keep the value of $b = 3$, then $0 \le r < 3$ i.e. $r = 0$ or 1 or 2 but it can't be 3 because r is smaller than 3. So, the possible values for $a = 3q$ or $3q + 1$ or $3q + 2$. Now, find the cube of all the possible values of a. If q is any positive integer, then its cube (let's call it as "m") will also be a positive integer. Now, observe carefully that the cube of all the positive integers is either of the form 9m or 9m + 1 or 9m + 1 for some integer m.

Solution:

Let "a" be any positive integer and $q = 3.$

Then, $a = 3q + r$ for some integer $q \ge 0$ and $0 \le r < 3$

Therefore, $a = 3q$ or $3q + 1$ or $3q + 2$

Case – I. When $a = 3q$

$$(a)^3 = (3q)^3 = 27q^3 = 9\left(3q^3\right) = 9\text{m}$$

Where m is an integer such that $\text{m} = 3q^3$

Case – II. When $a = 3q + 1$

$$(a)^3 = (3q + 1)^3$$
$$(a)^3 = 27q^3 + 27q^2 + 9q + 1$$
$$(a)^3 = 9\left(3q^3 + 3q^2 + q\right) + 1$$
$$(a)^3 = 9\text{m} + 1$$

Where m is an integer such that $\text{m} = 3q^3 + 3q^2 + q$

Case – III. When $a = 3q + 2$

$$(a)^3 = (3q+2)^3$$

$$(a)^3 = 27q^3 + 54q^2 + 36q + 8$$

$$(a)^3 = 9(3q^3 + 6q^2 + 4q) + 8$$

$$(a)^3 = 9\text{m} + 8$$

Where m is an integer such that $\text{m} = 3q^3 + 6q^2 + 4q$

Thus, we can see that the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8.

Chapter 1: Real Numbers

Exercise 1.2 (Page of Grade 10 NCERT Textbook)

Q1. Express each number as a product of its prime factors:

(i) 140 (ii) 156 (iii) 3825 (iv) 5005 (v) 7429

Difficulty Level: Easy

What is given /known?

A number.

What is the unknown?
The expression of the given number as a product of its prime factors.

Reasoning:
Find the prime factors of the given numbers by prime factorization method and then multiply the obtained prime numbers to get the product of the prime numbers.

Solution:

(i) 140

$$\begin{aligned}\text{Prime factors of } 140 &= 2,\ 2,\ 5,\ 7\\ &= 2^2 \times 5 \times 7\end{aligned}$$

(ii) 156

$$\begin{aligned}\text{Prime factors of } 156 &= 2\times 2\times 3\times 13\\ &= 2^2 \times 3\times 13\end{aligned}$$

(iii) 3825

$$\begin{aligned}\text{Prime factors of } 3825 &= 3\times 3\times 5\times 5\times 17\\ &= 3^2 \times 5^2 \times 17\end{aligned}$$

(iv) 5005

$$\text{Prime factors of } 5005 = 5 \times 7 \times 11 \times 13$$

(v) 7429

$$\text{Prime factors of } 7429 = 17 \times 19 \times 23$$

Q2. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.

(i) 26 and 91 (ii) 510 and 92 (iii) 336 and 54

Difficulty Level: Easy

What is given /known?
Pairs of numbers.

What is the unknown?
The LCM and HCF of the pairs of integers and to verify that LCM × HCF = Product of the two numbers.

Reasoning:

- To find the LCM and HCF of the given pairs of the integers, first find the prime factors of the given pairs of integers.
- Then, find the product of smallest power of each common factor in the numbers. This will be the HCF.
- Then find the product of greatest power of each prime factor in the number. This would be the LCM.
- Now, you have to verify LCM × HCF = product of the two numbers, find the product of LCM and HCF and also the two given numbers. If LHS is equal to the RHS then it will be verified.

Solution:

(i) 26 and 91

$$\text{Prime factors of } 26 = 2 \times 13$$
$$\text{Prime factors of } 91 = 7 \times 13$$
$$\text{HCF of } 26 \text{ and } 91 = 13$$

$$\begin{aligned}\text{LCM of 26 and 91} &= 2 \times 7 \times 13 \\ &= 14 \times 17 \\ &= 182\end{aligned}$$

$$\begin{aligned}\text{Product of two numbers} &= 26 \times 91 \\ &= 2366\end{aligned}$$

$$\begin{aligned}\text{LCM} \times \text{HCF} &= 182 \times 13 \\ &= 2366\end{aligned}$$

So, product of two numbers $= \text{LCM} \times \text{HCF}$

(ii) 510 and 92

$$\text{Prime factors of } 510 = 2\times3\times5\times17$$
$$\text{Prime factors of } 92 = 2\times2\times23$$
$$\text{HCF of two numbers } = 2$$

$$\text{LCM of two numbers} = 2\times2\times3\times5\times17\times23 = 23460$$

$$\text{Product of two numbers } = 510\times92 = 46920$$

$$\text{LCM}\times\text{HCF} = 2\times23460 = 46920$$

Product of two numbers = LCM × HCF

(iii) 336 and 54

$$\text{Prime factors of } 336 = 2\times2\times2\times2\times3\times7$$
$$\text{Prime factors of } 54 = 2\times3\times3\times3$$
$$\text{HCF of two numbers } = 6$$

$$\text{LCM of two numbers} = 2\times2\times2\times2\times3\times3\times3\times7 = 2^4\times3^3\times7 = 3024$$

$$\text{Product of two numbers} = 336\times54 = 18144$$

$$\text{LCM}\times\text{HCF} = 3024\times\underline{6} = 18144$$

Product duct of two numbers = LCM × HCF

Q3. Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 12, 15 and 21 (ii) 17, 23 and 29 (iii) 8, 9 and 25

Difficulty Level: Easy

What is given /known?

(i) 12, 15 and 21 (ii) 17, 23 and 29 (iii) 8, 9 and 25

What is the unknown?

The LCM and HCF of the given integers by applying the prime factorisation method.

Reasoning:

To solve this question, follow these steps:

- First find the prime factors of the given integers.
- Find the HCF of the given pair of integers i.e. product of smallest power of each prime factor, involved in the number.
- Lastly, Find the LCM of the given pair of integers i.e. product of greatest power of each prime factor, involved in the number.

Solution:

(i) 12, 15 and 21

$$\text{Prime factors of } 12 = 2\times2\times3$$
$$= 2^2\times3$$
$$\text{Prime factors of } 15 = 3\times5$$
$$\text{Prime factors of } 21 = 2\times2\times3$$
$$\text{HCF of } 12{,}15 \text{ and } 21 = 3$$
$$\text{LCM of } 12{,}15 \text{ and } 21 = 2^2\times3\times5\times7$$
$$= 420$$

(ii) 17, 23 and 29

Prime factors of 17 = 17×1
Prime factors of 23 = 23×1
Prime factors of 29 = 29×1
HCF of 17, 23 and 29 = 1

$$\text{LCM of } 17{,}23 \text{ and } 29 = 17\times23\times29$$
$$= 11339$$

(iii) 8, 9 and 25

$$\text{Prime factors of } 8 = 2\times2\times2\times1$$
$$= 2^3\times1$$
$$\text{Prime factors of } 9 = 3\times3\times1$$
$$= 3^2\times1$$
$$\text{Prime factors of } 25 = 5\times5\times1$$
$$= 5^2\times1$$

HCF of 8, 9 and 25 = 1

$$\text{LCM of } 8{,}9 \text{ and } 25 = 2\times2\times2\times3\times3\times5\times5$$
$$= 1800$$

Q4. Given that HCF (306, 657) = 9, find LCM (306, 657).

Difficulty Level: Easy

What is given /known?

HCF of two numbers (306, 657) = 9

What is the unknown?

LCM of the given numbers.

Reasoning:

We know that $\text{LCM} \times \text{HCF} = \text{product of two given integers}$

We have the given numbers as 306 and 657 and we can find the product of 306 and 657. The HCF of this two numbers is 9. Put the values in the above property and find the value of unknown i.e. HCF.

Solution:

Given, HCF (306, 657) = 9.

We have to find,

LCM (306, 657) = ?

We know that

$$\text{LCM} \times \text{HCF} = \text{Product of two numbers}$$

$$\text{LCM} \times 9 = 306 \times 657$$

$$\text{LCM} = \frac{306 \times 657}{9}$$

$$\text{LCM} = 34 \times 657$$

$$\text{LCM} = 22338$$

Q5. Check whether 6^n can end with the digit 0 for any natural number n.

Difficulty Level: Medium

What is the unknown?

Whether 6^n can end with the digit 0 for any natural number n.

Reasoning:

If any number ends with the digit 0 that means it should be divisible by 5. That is, if 6^n ends with the digit 0, then the prime factorization of 6^n would contain the prime 5.

Solution:

Prime factors of $6^n = (2 \times 3)^n = (2)^n (3)^n$

You can observe clearly, 5 is not in the prime factors of 6^n.

That means 6^n will not be divisible by 5.

Therefore, 6^n cannot end with the digit 0 for any natural number n.

Q6. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Difficulty Level: Medium

What is the unknown?

Whether $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Reasoning:

To solve this question, recall that:

- Prime numbers are whole numbers whose only factors are 1 and itself.
- Composite number are the positive integers which has factors other than 1 and itself.

Now, simplify $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$. On simplifying them, you will find that both the numbers have more than two factors. So, if the number has more than two factors, it will be composite.

Solution:

It can be observed that,

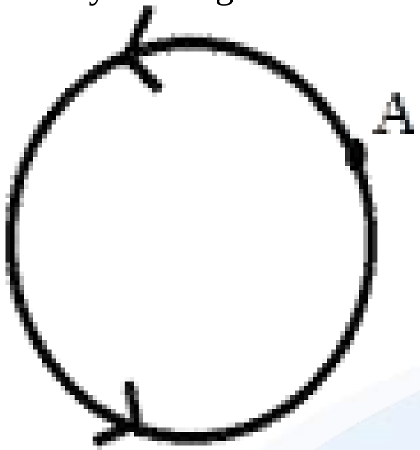
$$\begin{aligned} 7 \times 11 \times 13 + 13 &= 13(7 \times 11 + 1) \\ &= 13(77 + 1) \\ &= 13 \times 78 \\ &= 13 \times 13 \times 6 \times 1 \\ &= 13 \times 13 \times 2 \times 3 \times 1 \end{aligned}$$

The given number has 2,3,13 and 1 as its factors. Therefore, it is a composite number.

$$\begin{aligned} 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 &= 5 \times (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1) \\ &= 5 \times (1008 + 1) \\ &= 5 \times 1009 \times 1 \end{aligned}$$

1009 cannot be factorised further. Therefore, the given expression has 5,1009 and 1 as its factors. Hence, it is a composite number.

Q7. There is a circular path around a sport field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time and go in the same direction. After how many minutes will they meet again at the starting point?



Difficulty Level: Medium

What is known/given?

- Sonia takes 18 minutes to drive one round of the field.
- Ravi takes 12 minutes for the same.
- They both start at the same point and at the same time and go in the same direction.

What is the unknown?

After how many minutes will they meet again at the same point.

Reasoning:

Time taken by Sonia is more than Ravi to complete one round. Now, you have to find after how many minutes will they meet again at the same point. For this, there will be a number which is a divisible by both 18 and 12 and that will be the time when both meet again at the starting point. To find this you have to take LCM of both the numbers.

Solution:

LCM of 18 and 12,

$$18 = 2 \times 3 \times 3$$

$$12 = 2 \times 2 \times 3$$

$$\text{LCM of } 12 \text{ and } 18 = 2 \times 2 \times 3 \times 3$$

$$= 36$$

Therefore, Ravi and Sonia will meet together at starting point after 36 minutes.

Chapter 1: Real Numbers

Exercise 1.3 (Page 14 of Grade 10 NCERT Textbook)

Q1. Prove that $\sqrt{5}$ is irrational.

Difficulty Level: Medium

What is unknown/to be proved:
$\sqrt{5}$ is irrational

Reasoning:
In this question you have to prove that $\sqrt{5}$ is irrational. This question can be solved with the help of contradiction method. Suppose that $\sqrt{5}$ is rational. If $\sqrt{5}$ is rational that means it can be written in the form of $\frac{p}{q}$, where *p* and *q* integers and $q \neq 0$.

Now, *p* and *q* have common factors, when you cancel them you will get $\frac{a}{b}$ where *a* and *b* are co-primes and have no common factor other than 1.

Now square both the sides, if a^2 is divisible by 5 that means a is also divisible by 5 (Let *p* be a prime number. If p divides a^2, then p divides a, where a is a positive integer). So, you can write a = 5c. Again on squaring you will get the value of a^2 substitute the value of a^2 in the above equation, you will get $\frac{b^2}{5} = c^2$, this means b^2 is divisible by 5 and so *b* is also divisible by 5. Therefore, *a* and *b* have 5 as a common factor.

But this contradicts the fact that *a* and *b* are coprime. This contradiction has arisen because of our incorrect assumption that $\sqrt{5}$ is a rational number. So, we conclude that $\sqrt{5}$ is an irrational number.

Solution:
Let us assume, to the contrary that $\sqrt{5}$ is a rational number. Let *p* and *q* have common factors, so by cancelling them we will get $\frac{a}{b}$, where *a* and *b* are co-primes.

$\frac{\sqrt{5}}{1} = \frac{a}{b}$ (where a and b are co-primes and have no common factor other than 1)

$$\sqrt{5}b = a$$

Squaring both sides,

$$5b^2 = a^2$$

$$b^2 = \frac{a^2}{5} \qquad (1)$$

5 divides a^2,

That means it also divide a,

$$\frac{a}{5} = c$$

$$a = 5c$$

on squaring,

$$a^2 = 25c^2$$

put the value of a^2 in equation (1)

$$5b^2 = 25c^2$$

$$b^2 = 5c^2$$

$$\frac{b^2}{5} = c^2$$

This means b^2 is divisible by 5 and so b is also divisible by 5. Therefore, a and b have 5 as a common factor. But this contradicts the fact that a and b are coprime. This contradiction has arisen because of our incorrect assumption that $\sqrt{5}$ is a rational number. So, we conclude that $\sqrt{5}$ is irrational.

Q2. Prove that $3 + 2\sqrt{5}$ is irrational.

Difficulty Level: Easy

What is unknown/to be proved:

$3 + 2\sqrt{5}$ is irrational

Reasoning:

In this question you have to prove that $3 + 2\sqrt{5}$ is irrational. Solve this question with the help of contradiction method, suppose that that $3 + 2\sqrt{5}$ is rational. If $3 + 2\sqrt{5}$ is rational that means it can be written in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$. Now, p and q have common factors, when you cancel them you will get $\frac{a}{b}$ where a and b are co-primes and have no common factor other than 1. First, find out the value of $\sqrt{5}$ i.e. $\sqrt{5} = \frac{a-3b}{2b}$, where $\frac{a-3b}{2b}$ *is* a rational number and $b \neq 0$. If, $\frac{a-3b}{2b}$ is a rational number that means $\sqrt{5}$ is also a rational number. But, we know that $\sqrt{5}$ is irrational this contradicts the fact that $\sqrt{5}$ is irrational.

Therefore, our assumption was wrong that $3 + 2\sqrt{5} +$ is rational.
So, $3 + 2\sqrt{5} +$ is irrational.

Solution:

Let us assume, to the contrary that $3 + 2\sqrt{5} +$ is a rational number. Let *p* and *q* have common factors, so by cancelling them we will get $\frac{a}{b}$, where *a* and *b* are co-primes.

$$3+2\sqrt{5}=\frac{a}{b}$$

$$b(3+2\sqrt{5})=a \qquad \left(\begin{array}{l}\text{where } a \text{ and } b \text{ are co-primes and have} \\ \text{no common factor other than} 1\end{array}\right)$$

$$3b+2\sqrt{5b}=a$$

$$2\sqrt{5b}=a-3b$$

$$\sqrt{5}=\frac{a-3b}{2b}$$

Since, $\frac{a-3b}{2b}$ is a rational number then $\sqrt{5}$ is also a rational number.
But we know that $\sqrt{5}$ is irrational this contradicts the fact that $\sqrt{5}$ is rational.

Therefore, our assumption was wrong that $3 + 2\sqrt{5}$ is rational. So, $3 + 2\sqrt{5}$ is irrational.

Q3. Prove that the following are irrationals:

(i) $\frac{1}{\sqrt{2}}$ (ii) $7\sqrt{5}$ (iii) $6 + \sqrt{2}$

Difficulty Level: Easy

What is unknown/to be proved:

(i) $\frac{1}{\sqrt{2}}$ (ii) $7\sqrt{5}$ (iii) $6 + \sqrt{2}$ are Irrationals.

Solution:

(i)

Let us assume, to the contrary $\frac{1}{\sqrt{2}}$ that is a rational number.

$$\frac{1}{\sqrt{2}} = \frac{p}{q}$$

Let p and q have common factors, so by cancelling them we will get $\frac{a}{b}$, where a and b are co-primes.

$$\frac{1}{\sqrt{2}} = \frac{a}{b} \qquad (\text{where } a \text{ and } b \text{ are co-primes and have no common factor otherthan} 1)$$

$$(\sqrt{2})\text{a} = \text{b}$$

$$\sqrt{2} = \frac{b}{a}$$

Since, b and a are integers, $\frac{b}{a}$ is rational number and so, $\sqrt{2}$ is rational.

But we know that $\sqrt{2}$ is irrational this contradicts the fact that $\sqrt{2}$ is rational.

So, our assumption was wrong. Therefore, $\frac{1}{\sqrt{2}}$ is a rational number.

(ii) $7\sqrt{5}$

Let us assume, to the contrary that $7\sqrt{5}$ is a rational number.

$$7\sqrt{5} = \frac{p}{q}$$

Let p and q have common factors, so by cancelling them we will get $\frac{a}{b}$, where a and b are co-primes.

$$7\sqrt{5} = \frac{a}{b} \qquad \left(\begin{array}{l}\text{where } a \text{ and } b \text{ are co-primes and have} \\ \text{no common factor other than} 1\end{array}\right)$$

$$7\sqrt{5}\text{b} = \text{a}$$

$$\sqrt{5} = \frac{a}{7b}$$

Since, a, 7 and b are integers. So, $\frac{a}{7b}$ is rational number and so, $\sqrt{5}$ is rational. But this contradict the fact that $\sqrt{5}$

So, our assumption was wrong. Therefore, $7\sqrt{5}$ is a rational number.

(iii) $6+\sqrt{2}$

Let us assume, to the contrary that $6+\sqrt{2}$ is a rational number.

$$6+\sqrt{2}=\frac{p}{q}$$

Let p and q have common factors, so by cancelling them we will get $\frac{a}{b}$, where a and b are co-primes.

$$6+\sqrt{2}=\frac{a}{b} \qquad \left(\begin{array}{l}\text{where } a \text{ and } b \text{ are co-primes and have no common factor} \\ \text{other than } 1\end{array}\right)$$

$$\sqrt{2}=\frac{a}{b}-6$$

Since, a, b and 6 are integers. So, $\frac{a}{b}-6$ is rational number and so, $\sqrt{2}$ is also a rational number.

But this contradicts the fact that $\sqrt{2}$ is irrational. So, our assumption was wrong.

Therefore, $6+\sqrt{2}$ is a rational number.

Chapter 1: Real Numbers

Exercise 1.4 (Page 17 of Grade 10 NCERT Textbook)

Q1. Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(i) $\frac{13}{3125}$ (ii) $\frac{17}{8}$ (iii) $\frac{64}{455}$ (iv) $\frac{15}{1600}$

(v) $\frac{29}{343}$ (vi) $\frac{23}{2^3 5^2}$ (vii) $\frac{129}{2^2 5^7 7^5}$ (viii) $\frac{6}{15}$

(ix) $\frac{35}{50}$ (x) $\frac{77}{210}$

Difficulty Level: Medium

Reasoning:

Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is of the form $2^n \times 5^m$, where n, m are non-negative integers. Then x has a decimal expansion which terminates.

Solution:

(i) $\frac{13}{3125}$

The denominator is of the form 5^5.

Hence, the decimal expansion of $\frac{13}{3125}$ is terminating.

(ii) $\frac{17}{8}$

The denominator is of the form 2^3.

Hence, the decimal expansion of $\frac{17}{8}$ is terminating.

(iii) $\frac{64}{455}$

$455 = 5 \times 7 \times 13$

Since the denominator is not in the form $2^m \times 5^n$, and it also contains 7 and 13 as its factors, its decimal expansion will be non-terminating repeating.

(iv) $\frac{15}{1600}$

$$1600 = 2^6 \times 5^2$$

The denominator is of the form $2^m \times 5^n$.

Hence, the decimal expansion of $\frac{15}{1600}$ is terminating.

(v) $\frac{29}{343}$

$$343 = 7^3$$

Since the denominator is not in the form $2^m \times 5^n$, and it has 7 as its factor, the decimal expansion of $\frac{29}{343}$ is non-terminating repeating.

(vi) $\frac{23}{2^3 \times 5^2}$

The denominator is of the form $2^m \times 5^n$.

Hence, the decimal expansion of $\frac{23}{2^3 \times 5^2}$ is terminating.

(vii) $\frac{129}{2^2 \times 5^7 \times 7^5}$

Since the denominator is not of the form $2^m \times 5^n$, and it also has 7 as its factor, the decimal expansion of $\frac{129}{2^2 \times 5^7 \times 7^5}$ is non-terminating repeating.

(viii) $\frac{6}{15}=\frac{2\times 3}{3\times 5}$

$=\frac{2}{5}$

The denominator is of the form 5^{n}.

Hence, the decimal expansion of $\frac{6}{15}$ is terminating.

(ix) $\frac{35}{50}=\frac{7\times 5}{10\times 5}$

$=\frac{7}{10}$

$10=2\times 5$

The denominator is of the form $2^{m}\times 5^{n}$.

Hence, the decimal expansion of $\frac{35}{50}$ is terminating.

(x) $\frac{77}{210}=\frac{11\times 7}{30\times 7}$

$=\frac{11}{30}$

$30=2\times 3\times 5$

Since the denominator is not of the form $2^{m}\times 5^{n}$, and it also has 3 as its factor, the decimal expansion of $\frac{77}{210}$ is non-terminating repeating.

Q2. Write down the decimal expansions of those rational numbers in Question 1 above which have terminating decimal expansions.

Difficulty Level: Medium

Solution:

(i) $\frac{13}{3125} = 0.00416$

```
         0.00416
3125)13.00000
      0
      -----
      130
        0
      -----
      13000
      12500
      -----
      5000
      3125
      -----
      18750
      18750
      -----
      x
      -----
```

(ii) $\frac{17}{8} = 2.125$

```
     2.215
8)17
   16
   -----
   10
    8
   -----
   20
   16
   -----
   40
   40
   -----
   x
   -----
```

(iii) $\frac{64}{455}$ it is non-terminating

(iv) $\frac{15}{1600} = 0.009375$

$$
\begin{array}{r}
0.009375 \\
1600\overline{)15.000000} \\
0 \\
----- \\
150 \\
0 \\
----- \\
1500 \\
0 \\
----- \\
15000 \\
14400 \\
----- \\
6000 \\
4800 \\
----- \\
12000 \\
11200 \\
----- \\
8000 \\
8000 \\
----- \\
x \\

\end{array}
$$

(v) $\frac{29}{343}$ it is non-terminating

(vi) $\frac{23}{2^3 \times 5^2} = \frac{23}{200}$

$= 0.115$

$$\begin{array}{r} 00.115 \\ 200\overline{)23.000} \end{array}$$

0

23

–0

230

–200

300

–200

1000

–1000

0

(vii) $\frac{129}{2^2 \times 5^7 \times 7^5}$ it is non-terminating

(viii) $\frac{6}{15} = \frac{2 \times 3}{3 \times 5} = \frac{2}{5}$

$= 0.4$

$$\begin{array}{r} 0.4 \\ 5\overline{)2.0} \end{array}$$

0

20

20

-----

x

(ix) $\frac{35}{50} = 0.7$

$$\begin{array}{r} 0.7 \\ 50\overline{)35.0} \\ 0 \\ \hline 350 \\ 350 \\ \hline x \\ \hline \end{array}$$

(x) $\frac{77}{210}$ it is non-terminating

Q3. The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form $\frac{p}{q}$, what can you say about the prime factor of q?

(i) 43.123456789

(ii) 0.120120012000120000……

(iii) $43.\overline{123456789}$

Difficulty Level: Medium

Reasoning:

Let x be a rational number whose decimal expansion terminates.

Then x can be expressed in the form $\frac{p}{q}$, where p and q are coprime, and the prime factorisation of q is of the form $2^m \times 5^n$, where n, m are non-negative integers.

Solution:

(i) 43.123456789

Since this number has a terminating decimal expansion, it is a rational number of the form $\frac{p}{q}$ and q is of the form $2^{\mathrm{m}} \times 5^{\mathrm{n}}$

i.e., the prime factors of q will be either 2 or 5 or both.

(ii) $0.120120012000120000\ldots\ldots$

The decimal expansion is neither terminating nor recurring. Therefore, the given number is an irrational number.

(iii) $43.\overline{123456789}$

Since the decimal expansion is non-terminating recurring, the given number is a rational number of the form $\frac{p}{q}$ and q is not of the form $2^m \times 5^n$ i.e., the prime factors of q will also have a factor other than 2 or 5.