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THE MATH EXPERT

Chapter - 10: Circles

Exercise 10.1 (Page 209 of Grade 10 NCERT Textbook)

Q1. How many tangents can a circle have?

Difficulty Level: Easy

Unknown:

Number of tangents a circle can have.

Reasoning:

A tangent to a circle is a line that intersects the circle at only one point. On every point on the circle, one tangent can be drawn.

Solution:

As per the above reasoning, a circle can have infinitely many tangents.

Q2. Fill in the blanks:

Difficulty Level: Easy

Solution:

(i) A tangent to a circle intersects it in _________ point (s).

Reasoning:

A tangent to a circle is a line that intersects the circle at only one point.

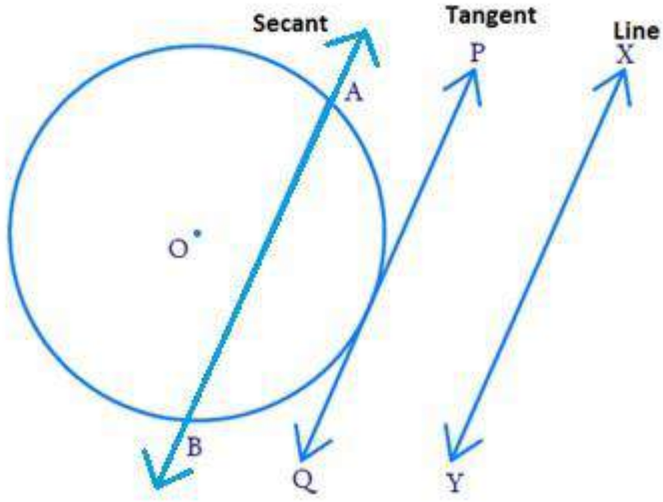
Answer:

One

(ii) A line intersecting a circle in two points is called a ____________.

Reasoning:

Secant is a line that intersects the circle in two points.



Answer:

Secant

(iii) A circle can have _________ parallel tangents at the most.

Reasoning:

Tangent at any point of a circle is perpendicular to the radius through the point of contact. Extended radius is a diameter which has two end points and hence two tangents which are parallel to themselves and perpendicular to the diameter.

Center O, diameter AB, tangents PQ, RS and PQ $\parallel$ RS
A and B are called as point of contact.

Answer: **Two**

(iv) The common point of a tangent to a circle and the circle is called _________ .

Reasoning:

A tangent to a circle is a line that intersects the circle at only one point and that point is called as point of contact.

Answer:

Point of contact

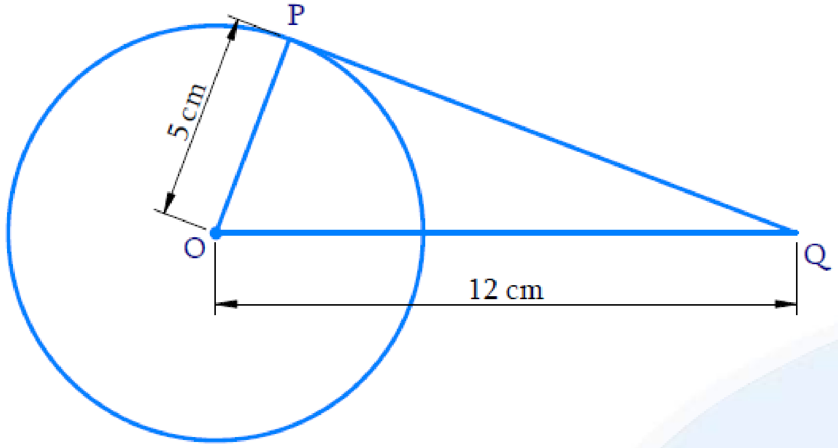
Q3. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the center O at a point Q so that OQ = 12 cm. Length PQ is:

(A) 12 cm (B) 13 cm (C) 8.5 cm (D) $\sqrt{119}$ cm.

Difficulty Level: Easy

Known:

Radius OP = 5 cm
OQ = 12 cm



Unknown:

Length of the tangent PQ

Reasoning:

ΔOPQ is a right-angle triangle according to Theorem 10.1: The tangent at any point of a circle is perpendicular to the radius through the point of contact.

Solution:

By Pythagoras theorem

$$OQ^2 = OP^2 + PQ^2$$
$$12^2 = 5^2 + PQ^2$$
$$144 = 25 + PQ^2$$
$$PQ^2 = 119$$
$$PQ = \sqrt{119}$$

Answer:

Option D

Q4. Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.

Difficulty Level:

Easy

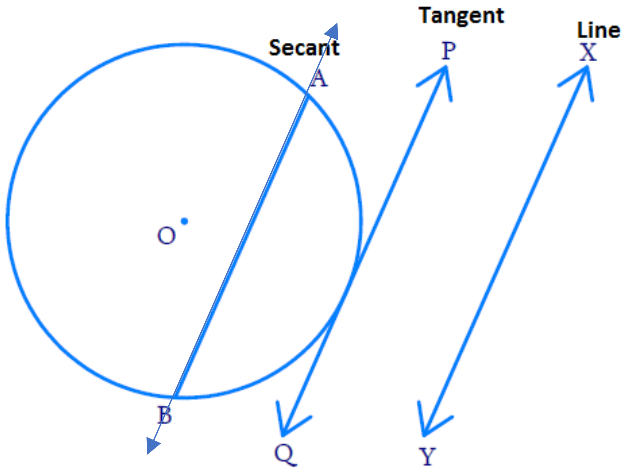
Known:

(i) To draw a circle
(ii) Draw one tangent and one secant to the circle parallel to the given line.

Unknown:

To draw a circle as per known details.

Solution:



XY is the given line.

AB is the secant parallel to XY, AB ∥ XY

AQ is the tangent parallel to XY, PQ ∥ XY

Chapter - 10: Circles

Exercise 10.2 (Page 213 of Grade 10 NCERT Textbook)

In, Q1 to Q3, choose the correct option and give justification.

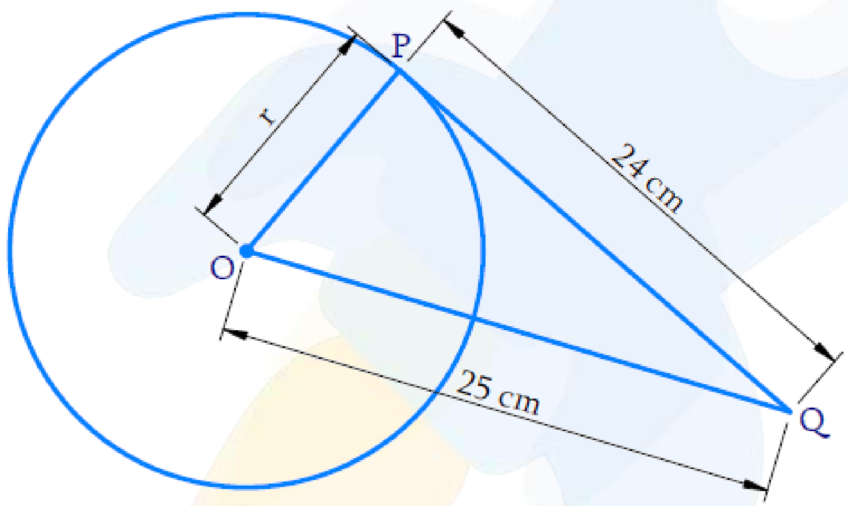
Q1. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the center is 25 cm. The radius of the circle is

(A) 7 cm (B) 12 cm
(C) 15 cm (D) 24.5 cm

Difficulty Level: Easy

Known:

(i) Length of tangent from a point Q, i.e. PQ is 24cm
(ii) Distance of Q from center, i.e. OQ is 25cm.



Unknown:

Radius of the circle

Reasoning:

Tangent at any point of a circle is perpendicular to the radius through the point of contact.

Solution:

$\therefore$ OPQ is a right-angled triangle

By Pythagoras theorem,

$$OQ^2 = OP^2 + PQ^2$$
$$25^2 = r^2 + 24^2$$
$$r^2 = 25^2 - 24^2$$
$$r^2 = 625 - 576$$
$$r^2 = 49$$
$$r = \pm 7$$

Radius cannot be a negative value, $\therefore r = +7$ cm.

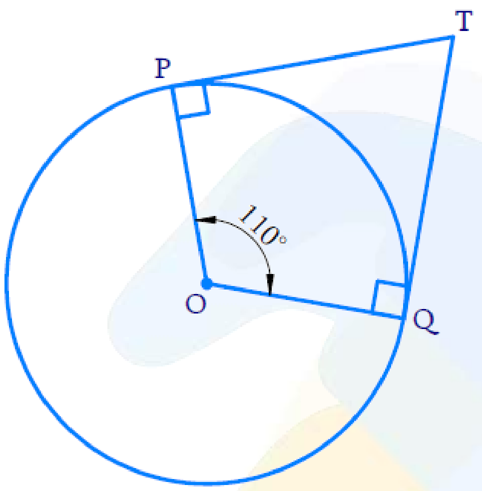
Answer:

Option A

Q2. In the given figure, if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110°$, then $\angle PTQ$ is equal to

(A) 60° (B) 70°

(C) 80° (D) 90°



Difficulty Level: Medium

Known:

(i) TP and TQ are tangents to a circle with Centre O

(ii) $\angle POQ = 110^0$

Unknown:

$\angle PTQ$

Reasoning:

- Tangent at any point of a circle is perpendicular to the radius through the point of contact.
- In the above figure OPTQ is a quadrilateral and $\angle P$ and $\angle Q$ are 90^o
- Sum of the angles of a quadrilateral is 360^o

Solution:

$\therefore$ In OPTQ,

$$\begin{aligned} \angle Q + \angle P + \angle POQ + \angle PTQ &= 360^0 \\ 90^0 + 90^0 + 110^0 + \angle PTQ &= 360^0 \\ 290^0 + \angle PTQ &= 360^0 \\ \angle PTQ &= 360^0 - 290^0 \\ \angle PTQ &= 70^0 \end{aligned}$$

Answer:

Option B

Q3. If tangents PA and PB from a point P to a circle with center O are inclined to each other at angle of 80°, then $\angle$POA is equal to

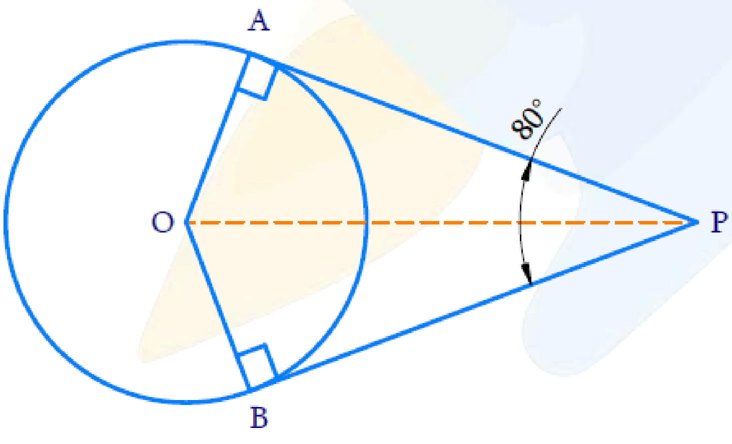
(A) 50° (B) 60°

(C) 70° (D) 80°

Difficulty Level: Medium

Known:

PA and PB are the tangents from P to a circle with center O. Tangents are inclined to each other at angle of 80°



Unknown:

$\angle POA$

Reasoning:

- The lengths of tangents drawn from an external point to a circle are equal.
- Tangent at any point of a circle is perpendicular to the radius through the point of contact.

Solution:

In ΔOAP and in ΔOBP

OA = OB (radii of the circle are always equal)
AP = BP (length of the tangents)
OP = OP (common)

Therefore, by SSS congruency $\Delta OAP \cong \Delta OBP$

SSS congruence rule: If three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent.
If two triangles are congruent then their corresponding parts are equal.
Hence,

$$\angle POA = \angle POB$$
$$\angle OPA = \angle OPB$$

Therefore, OP is the angle bisector of $\angle APB$ and $\angle AOB$

$$\therefore \angle OPA = \angle OPB = \frac{1}{2}(\angle APB)$$
$$= \frac{1}{2} \times 80^\circ$$
$$= 40^\circ$$

By angle sum property of a triangle, In ΔOAP

$\angle A + \angle POA + \angle OPA = 180^\circ$

From the figure,

OA $\perp$ AP $\left(\begin{array}{l}\text{Theorem 10.1 : The tangent at any point of a circle is perpendicular} \\ \text{to the radius through the point of contact.}\end{array}\right)$

$\therefore \angle A = 90^\circ$

$$90^\circ + \angle POA + 40^\circ = 180^\circ$$
$$130^\cdot + \angle POA = 180$$
$$\angle POA = 180^\circ - 130^\circ$$
$$\angle POA = 50^\circ$$

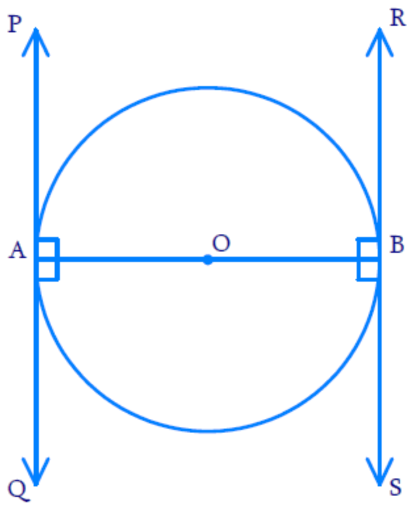
Answer:

Option A

Q4. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Difficulty Level: Medium

Solution:



Given:

• Let AB be a diameter of the circle.
• Two tangents PQ and RS are drawn at points A and B respectively.

To Prove:
Tangents drawn at the ends of a diameter of a circle are parallel.

Reasoning:

- A tangent to a circle is a line that intersects the circle at only one point.
- **Theorem 10.1:** The tangent at any point of a circle is perpendicular to the radius through the point of contact.

We know that, according to **Theorem 10.1,** Radius is perpendicular to the tangent at the point of contact
Thus, OA ⊥ PQ and OB ⊥ Z

Since the Tangents are Perpendicular to the Radius,

$\angle PAO = 90^{\circ}$
$\angle RBO = 90^{\circ}$
$\angle OAQ = 90^{\circ}$
$\angle OBS = 90^{\circ}$

Here $\angle OAQ$ & $\angle OBR$ and $\angle PAO$ & $\angle OBS$ are two pairs of alternate interior angles and they are equal.

If the Alternate interior angles are equal, then lines PQ and RS should be parallel.

We know that PQ & RS are the tangents drawn to the circle at the ends of the diameter A B.

Hence, it is proved that Tangents drawn at the ends of a diameter of a circle are parallel.

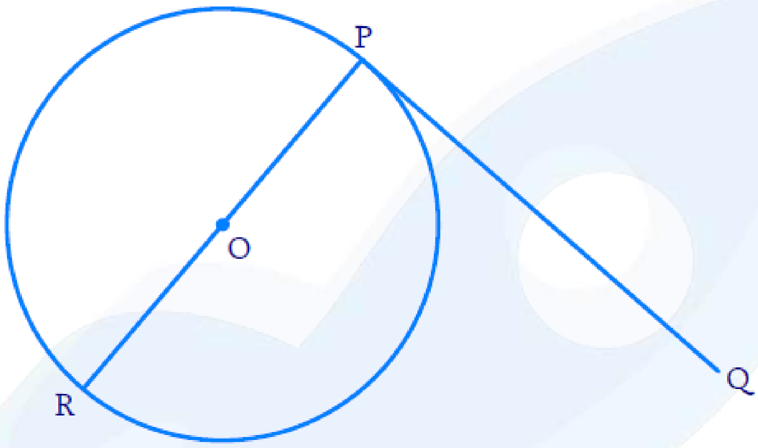
Q5. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the center.

Difficulty Level: Medium

To prove:

Perpendicular at the point of contact to the tangent to a circle passes through the centre.

Reasoning and Solution:



By theorem, tangent at any point of a circle is perpendicular to the radius through the point of contact.
Radius will always pass through the center of the circle.
Hence proved that perpendicular PR of tangent PQ passes through center O.

Q6. The length of a tangent from a point A at distance 5 cm from the center of the circle is 4 cm. Find the radius of the circle.

Difficulty Level: Easy

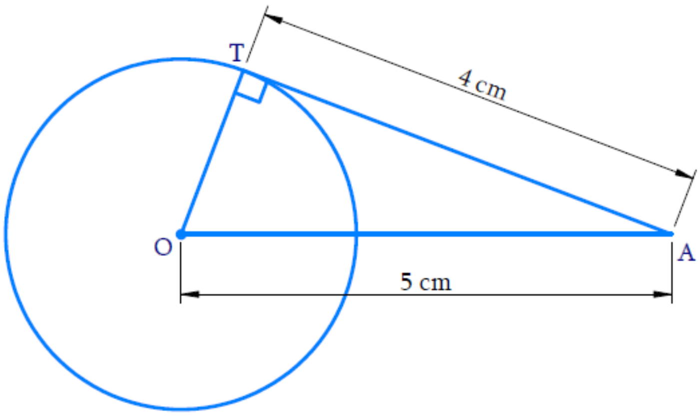
Known:

Length of tangent from point A = 4 cm.
Distance between the centre of the circle and point A is 5 cm.

Unknown:

Radius of the circle

Reasoning:



Tangent at any point of a circle is perpendicular to the radius through the point of contact.

Therefore, $\angle OTA = 90^\circ$ and ΔOTA is right angled triangle.

By Pythagoras theorem

$$\begin{aligned} OA^2 &= OT^2 + AT^2 \\ 5^2 &= OT^2 + 4^2 \\ OT^2 &= 25 - 16 \\ OT^2 &= 9 \\ OT &= \pm 3 \end{aligned}$$

Radius OT cannot be negative. Hence, the radius is 3 cm.

Q7. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

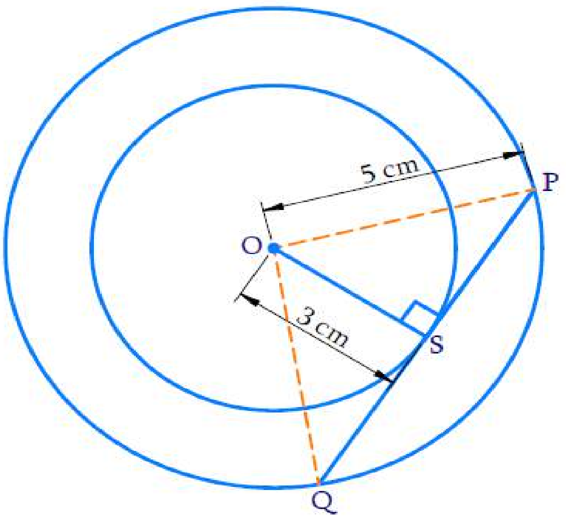
Difficulty Level: Medium

Known:
Two concentric circles are of radii 5cm and 3cm.

Unknown:
The length of the chord of the larger circle which touches the smaller circle.

Reasoning:
Chord of the larger circle is a tangent to the smaller circle.

Solution:



PQ is chord of a larger circle and tangent of a smaller circle.
Tangent is perpendicular to the radius at the point of contact S.
$\therefore \angle OSP = 90^{o}$

In ΔOSP (Right angled triangle)

By the Pythagoras Theorem,

$$OP^2 = OS^2 + SP^2$$
$$5^2 = 3^2 + SP^2$$
$$SP^2 = 25 - 9$$
$$SP^2 = 16$$
$$SP = \pm 4$$

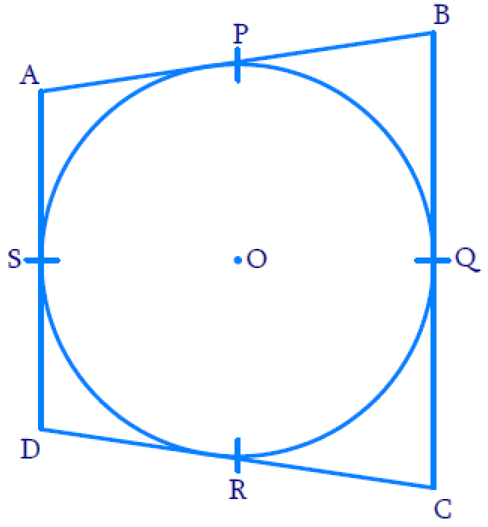
SP is length of tangent and cannot be negative
$\therefore SP = 4cm.$
QS = SP (Perpendicular from center bisects the chord considering the larger circles)

Therefore, $QS = SP = 4cm$

Length of the chord $PQ = QS + SP$
$= 4 + 4$
$PQ = 8cm$

Therefore, the length of the chord of the larger circle is 8 cm.

Q8. A quadrilateral ABCD is drawn to circumscribe a circle (see given Figure).
Prove that: AB + CD = AD + BC



Difficulty Level: Medium

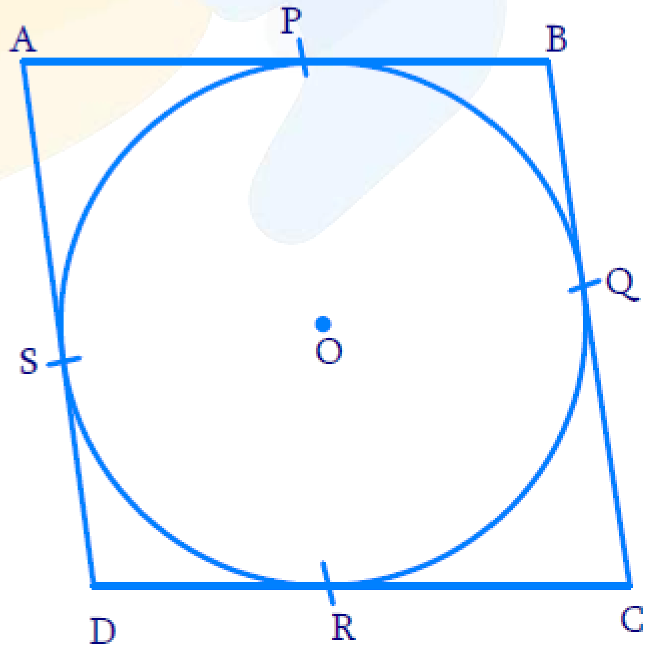
To prove:

AB + CD = AD + BC

Reasoning:

- A tangent to a circle is a line that intersects the circle at only one point.
- **Theorem 10.1:** The tangent at any point of a circle is perpendicular to the radius through the point of contact.
- **Theorem 10.2:** The lengths of tangents drawn from an external point to a circle are equal.

Solution:



We know that the length of tangents drawn from an external point of the circle are equal according to Theorem 10.2.

Therefore,
BP = BQ (Tangents from point B) ----(i)
CR = CQ (Tangents from point C) ----(ii)
DR = DS (Tangents from point D) ----(iii)
AP = AS (Tangents from point A) ----(iv)

Adding all the four equations, (i) +(ii) + (iii)+ (iv) we get,

$$BP + CR + DR + AP = BQ + CQ + DS + AS$$

On re-grouping them,

$$(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ) --- (v)$$

From the figure, we can see that:

$AP + BP = AB$

$CR + DR = CD$

$AS + DS = AD$

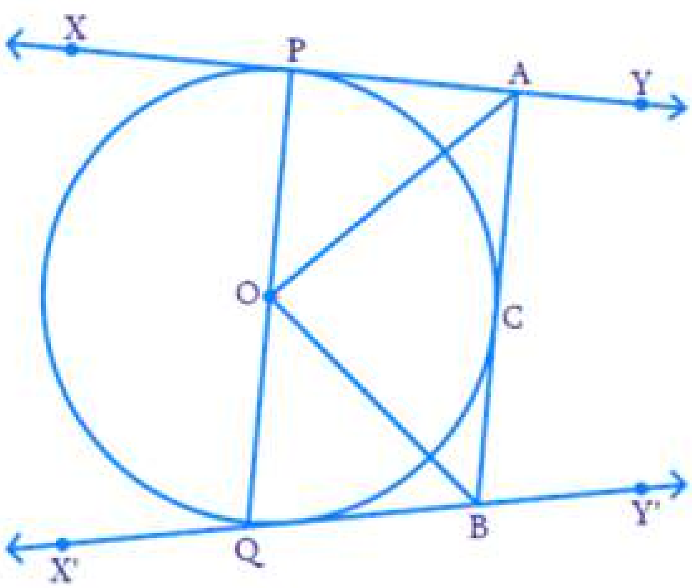
$BQ + CQ = BC$

On substituting the above values in (v), we get

$$AB + CD = AD + BC$$

Hence Proved.

Q9. In Figure, XY and X′Y′ are two parallel tangents to a circle with center O and another tangent AB with point of contact C intersecting XY at A and X′Y′ at B. Prove that $\angle AOB = 90°$.



Difficulty Level: Medium

Given:

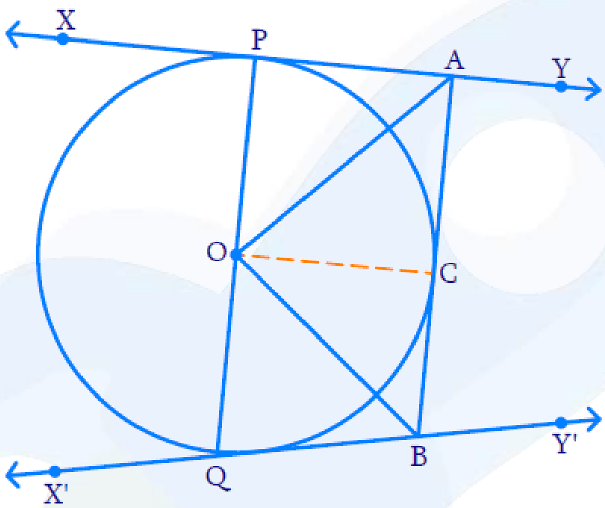
• O is the centre of the circle.
• XY and X'Y' are the two parallel tangents to the circle.
• AB is another tangent with point of contact C, intersecting XY at A and X'Y' at B.

To prove:

$\angle AOB = 90^\circ$

Reasoning and Solution:

Join point O to C.



In ΔOPA and ΔOCA

OP = OC (Radii of the circle are equal)
AP = AC (The lengths of tangents drawn from an external point A to a circle are equal.)
AO = AO (Common)

By SSS congruency, $\Delta OPA \cong \Delta OCA$

SSS congruence rule: If three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent.

Therefore, $\angle POA = \angle AOC$

Similarly, $\Delta OCB \cong \Delta OQB$

Therefore, $\angle COB = \angle BOQ$

PQ is a diameter, hence a straight line and $\angle POQ = 180^\circ$

But $\angle POQ = \angle POA + \angle AOC + \angle COB + \angle BOQ$

$\therefore \angle POA + \angle AOC + \angle COB + \angle BOQ = 180^\circ$

$$2\angle AOC + 2\angle COB = 180^\circ \qquad (\because \angle POA = \angle AOC \text{ and } \angle COB = \angle BOQ)$$

$$\therefore \angle AOC + \angle COB = 90^\circ$$

From the figure, $\angle AOC + \angle COB = \angle AOB$

$$\therefore \angle AOB = 90^\circ$$

Hence Proved

Q10. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the center.

Difficulty Level: Medium

Given:

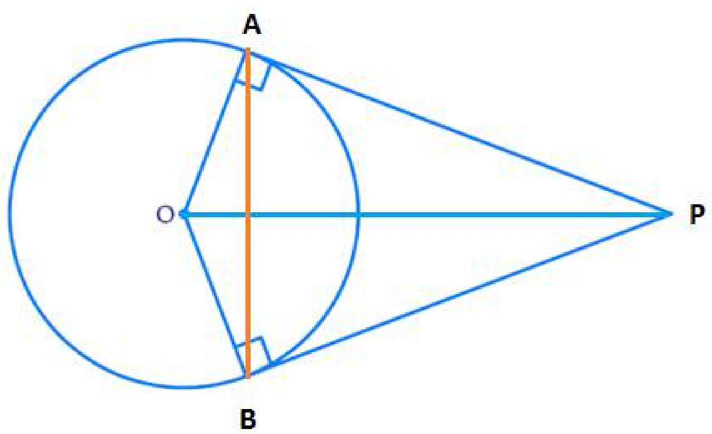
- Let us consider a circle centered at point O.
- Let P be an external point from which two tangents PA and PB are drawn to the circle which are touching the circle at point A and B respectively
- AB is the line segment, joining point of contacts A and B together such that it subtends $\angle AOB$ at center O of the circle.

To prove:

The angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the point of contact at the centre.

i.e. $\angle APB$ is supplementary to $\angle AOB$

Reasoning and Solution:



According to **Theorem 10.1**: The tangent at any point of a circle is perpendicular to the radius through the point of contact.

$$\therefore \angle OAP = \angle OBP = 90^\circ \text{...............Equation (i)}$$

In a quadrilateral, sum of 4 angles is 360°

$\therefore$ In OAPB,

$$\angle OAP + \angle APB + \angle PBO + \angle BOA = 360^{\circ}$$

Using Equation (i), we can write the above equation as

$$90 + \angle APB + 90 + \angle BOA = 360$$

$$\angle APB + \angle BOA = 360^{\circ} - 180^{\circ}$$

$$\therefore \angle APB + \angle BOA = 180^{\circ}$$

Where,
$\angle APB =$ Angle between the two tangents PA and PB from external point P.
$\angle BOA =$ Angle subtended by the line segment joining the point of contact at the centre.
Hence Proved.

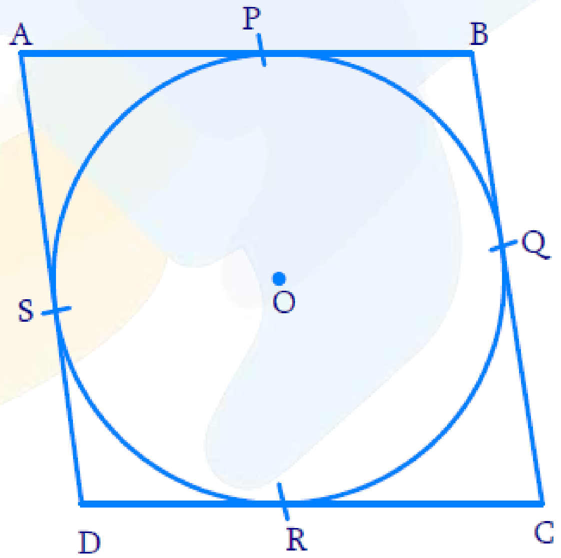
Q11. Prove that the parallelogram circumscribing a circle is a rhombus.

Difficulty Level: Medium

To prove:

The parallelogram circumscribing a circle is a rhombus

Reasoning and Solution:



ABCD is a parallelogram. Therefore, opposite sides are equal.

$$\therefore AB = CD$$
$$BC = AD$$

According to **Theorem 10.2**: The lengths of tangents drawn from an external point to a circle are equal.

Therefore,
BP = BQ (Tangents from point B)…… (1)
CR = CQ (Tangents from point C)…… (2)
DR = DS (Tangents from point D)…… (3)
AP = AS (Tangents from point A)……. (4)

Adding (1) + (2) + (3) + (4)

$$BP + CR + DR + AP = BQ + CQ + DS + AS$$

On re-grouping,

$$BP + AP + CR + DR = BQ + CQ + DS + AS$$

$$AB + CD = BC + AD$$

Substitute CD = AB and AD = BC since ABCD is a parallelogram, then

$$AB + AB = BC + BC$$

$$2AB = 2BC$$

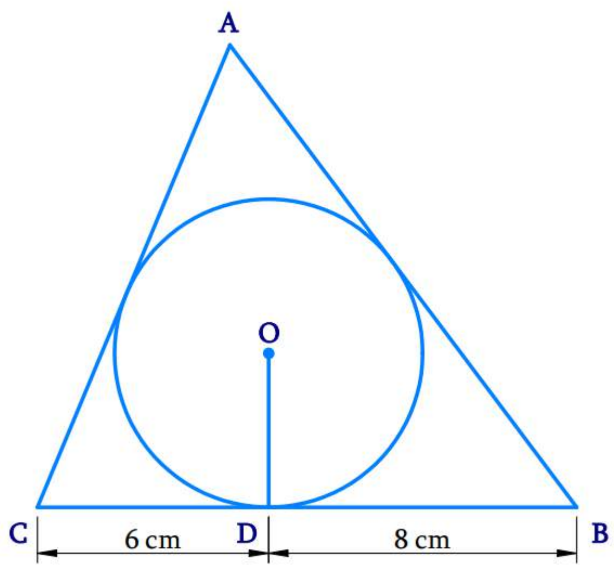
$$AB = BC$$

$\therefore AB = BC = CD = DA$

This implies that all the four sides are equal.

Therefore, the parallelogram circumscribing a circle is a rhombus.

Q12. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see Fig. 10.14). Find the sides AB and AC.



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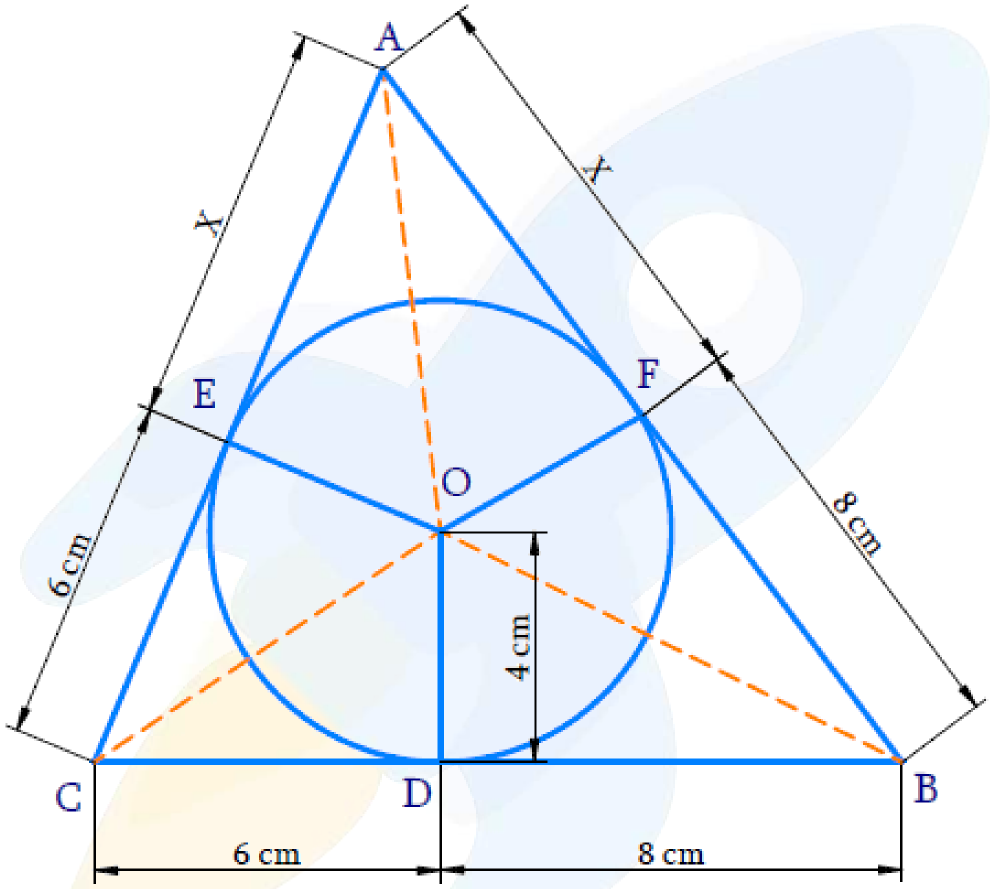
Difficulty Level: Hard

Unknown:

Sides AB and AC

Known:

(i) Triangle ABC is drawn to circumscribe a circle of radius 4cm.



BD=8 cm
CD=6 cm

Reasoning:
Finding the area of triangle ABC in two ways and equating them will result in unknown length. Hence other sides can be calculated.

Let
AE = AF = x (The lengths of tangents drawn from an external point to a circle are equal.)
CD = CE = 6 cm (Tangents from point C)
BD = BF = 8 cm (Tangents from point B)

Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

where $s=\frac{1}{2}(a+b+c)$

a, b and c are sides of a triangle

$a = AB = x + 8$

$b = BC = 8 + 6$

$= 14$

$c = CA = 6 + x$

$$s=\frac{1}{2}(x+8+14+6+x)$$

$$s=\frac{1}{2}(2x+28)$$

$$s=x+14$$

$$\text{Area of } \Delta ABC=\sqrt{(x+14)(x+14-x-8)(x+14-14)(x+14-x-6)}$$

$$=\sqrt{(x+14)\times(6)\times(x)\times(8)}$$

$$=\sqrt{48x(x+14)}$$

$$=\sqrt{48(x^2+14x)} \text{ square units..........(1)}$$

Area of ΔABC =Area of ΔAOC + Area of ΔAOB + Area of ΔBOC

$$=\left[\frac{1}{2}(x+6)\times 4\right]+\left[\frac{1}{2}(x+8)\times 4\right]+\left[\frac{1}{2}\times 14\times 4\right] \quad \left(\because \text{area of } \Delta=\frac{1}{2}\times b\times h\right)$$

$$=2x+12+2x+16+28$$

$$=4x+56$$

$$=4(x+14) \text{ square units.............(2)}$$

Equating (1) and (2)

$$\sqrt{48(x^2+14x)}=4(x+14)$$

Squaring both sides

$$48(x^2+14x)=4^2(x+14)^2$$

$$48(x^2+14x)=16(x^2+28x+196)[\text{Using } (a+b)^2=a^2+2ab+b^2]$$

$$\frac{48}{16}(x^2+14x)=x^2+28x+196$$

$$3x^2+42x=x^2+28x+196$$

$$3x^2-x^2+42x-28x-196=0$$

$$2x^2+14x-196=0(\text{divide this equation by } 2)$$

$$x^2+7x-98=0$$

Solving by factorization method,

$$x^2 + 14x - 7x - 98 = 0$$
$$x(x+14) - 7(x+14) = 0$$
$$(x+14)(x-7) = 0$$

$$x + 14 = 0 \quad \text{and} \quad x - 7 = 0$$
$$x = -14 \qquad\qquad x = 7$$

Since x represents length, it cannot be negative.
$\therefore$ x=7

$$AB = a = x + 8 = 7 + 8 = 15\ cm$$
$$AC = c = 6 + x = 6 + 7 = 13\ cm$$

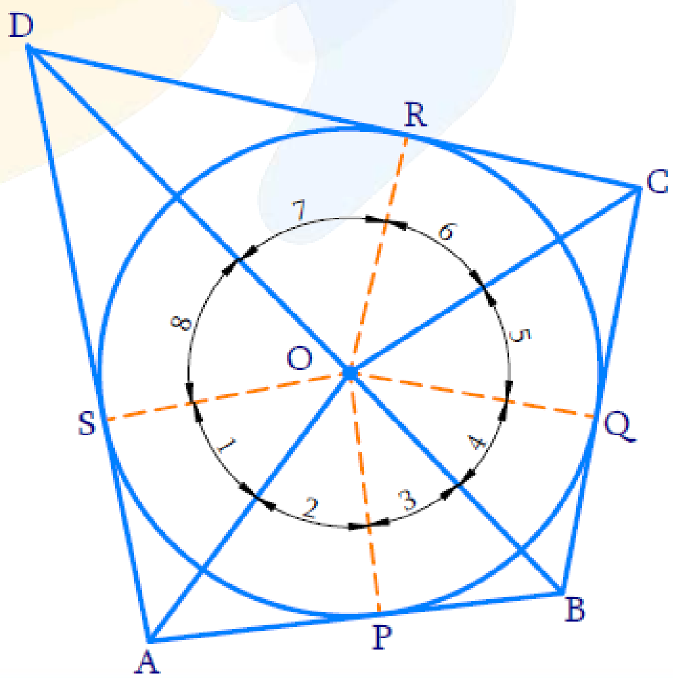
Answer:
AB = 15 cm
AC = 13 cm

Q13. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the center of the circle.

Difficulty Level: Hard

To prove:
Opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Reasoning and Solution:



We know that, tangents drawn from an external point to a circle
subtend equal angles at the centre.

In the above figure, P, Q, R, S are points of contact
AS = AP (The lengths of tangents drawn from an external point A to a circle are equal.)

$$\angle SOA = \angle POA = \angle 1 = \angle 2$$

Tangents drawn from an external point to a circle subtend equal angles at the centre.

Similarly,

$$\angle 3 = \angle 4, \angle 5 = \angle 6, \angle 7 = \angle 8$$

Since complete angle is $360°$,

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$$

$$2(\angle 1 + \angle 8 + \angle 4 + \angle 5) = 360° \quad (or) \quad 2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360$$

$$\angle 1 + \angle 8 + \angle 4 + \angle 5 = 180° \qquad \angle 2 + \angle 3 + \angle 6 + \angle 7 = 180$$

$$\text{from above fig, } \angle AOD + \angle BOC = 180° \qquad \angle AOB + \angle COD = 180$$

$\angle AOD$ and $\angle BOC$ are angles subtended by opposite sides of quadrilateral circumscribing a circle and sum of them is 180°

Hence Proved.