Chapter 12: Areas Related to Circles

Exercise 12.1 (Page 225 of Grade 10 NCERT Textbook)

Q1. The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.

Difficulty Level: Easy

What is the known/given?
Radii of two circles.

What is the unknown?
Radius of 3^{rd} circle.

Reasoning:
Using the formula of circumference of circle $C = 2\pi r$ we find the radius of the circle.

Solution:

Radius (r_1) of 1^{st} circle $= 19$ cm

Radius (r_2) or 2^{nd} circle = cm

Let the radius of 3^{rd} circle be r.

Circumference of 1^{st} circle = $2\pi r_1 = 2\pi(19) = 38\pi$

Circumference of 2^{nd} circle = $2\pi r_2 = 2\pi(9) = 18\pi$

Circumference of 3^{rd} circle = $2\pi r$

Given that,

Circumference of 3^{rd} circle = Circumference of 1^{st} circle + Circumference of 2^{nd} circle

$$2\pi r = 38\pi + 18\pi$$
$$= 56\pi$$
$$r = \frac{56\pi}{2\pi}$$
$$= 28$$

Therefore, the radius of the circle which has circumference equal to the sum of the circumference of the given two circles is 28 cm.

Q2. The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.

Difficulty Level: Easy

What is the known/given?
Radii of two circles.

What is the unknown?
Radius of 3rd circle.

Reasoning:
Using the formula of area of circle $A = \pi r^2$ we find the radius of the circle.

Solution:
Radius of (r_1) 1st circle = 8 cm
Radius of (r_2) 2nd circle = 6 cm
Let the radius of 3rd circle = r.
Area of 1st circle = $\pi r_1^2 = \pi(8)^2 = 64\pi$
Area of 2nd circle = $\pi r_2^2 = \pi(6)^2 = 36\pi$
Given that,
Area of 3rd circle = Area of 1st circle + Area of 2nd circle

$$\pi r^2 = \pi r_1^2 + \pi r_2^2$$
$$\pi r^2 = 64\pi + 36\pi$$
$$\pi r^2 = 100\pi$$
$$r^2 = 100$$
$$r = \pm 10$$

However, the radius cannot be negative. Therefore, the radius of the circle having area equal to the sum of the areas of the two circles is 10 cm.

Q3. Given figure depicts an archery target marked with its five scoring areas from the centre outwards as Gold, Red, Blue, Black and White. The diameter of the region representing Gold score is 21 cm and each of the other bands is 10.5 cm wide. Find the area of each of the five scoring regions. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Difficulty Level: Medium

What is the known/given?
Diameter of the gold region and width of the other regions.

What is the unknown?
Area of each scoring region.

Reasoning:
Area of the region between 2 concentric circles is given by $\pi r_2^2 - \pi r_1^2$.

Solution:
Radius (r_1) of gold region (i.e., 1st circle) $= \frac{21}{2} = 10.5\,\text{cm}$
Given that each circle is 10.5 cm wider than the previous circle.

Therefore,

$$\begin{aligned}
\text{Radius } (r_2) \text{ of } 2^{nd} \text{ circle } &= 10.5 + 10.5 \\
&= 21\,\text{cm} \\
\text{Radius } (r_3) \text{ of } 3^{rd} \text{ circle } &= 21 + 10.5 \\
&= 31.5\,\text{cm} \\
\text{Radius } (r_4) \text{ of } 4^{th} \text{ circle } &= 31.5 + 10.5 \\
&= 42\,\text{cm} \\
\text{Radius } (r_5) \text{ of } 5^{th} \text{ circle } &= 42 + 10.5 \\
&= 52.5\,\text{cm}
\end{aligned}$$

$$\text{Area of gold region} = \text{Area of } 1^{st} \text{ circle} = \pi r_1^2 = \pi(10.5)^2 = 346.5\,\text{cm}^2$$

$$\begin{aligned}
\text{Area of red region} &= \text{Area of } 2^{nd} \text{ circle} - \text{Area of } 1^{st} \text{ circle} \\
&= \pi r_2^2 - \pi r_1^2 \\
&= \pi(21)^2 - (10.5)^2 \\
&= 441\pi - 110.25\pi = 330.75\pi, \\
&= 1039.5\ \text{cm}^2
\end{aligned}$$

$$\begin{aligned}
\text{Area of blue region} &= \text{Area of } 3^{rd} \text{ circle} - \text{Area of } 2^{nd} \text{ circle} \\
&= \pi_{13}^2 - \pi r_1^2 \\
&= \pi(31.5)^2 - \pi(21)^2 \\
&= 992.25\pi - 441\pi = 551.25\pi \\
&= 1732.5\ \text{cm}^2
\end{aligned}$$

$$\begin{aligned}
\text{Area of black region} &= \text{Area of } 4^{th} \text{ circle} - \text{Area of } 3^{rd} \text{ circle} \\
&= \pi r_4^2 - \pi r_3^2 \\
&= \pi(42)^2 - \pi(31.5)^2 \\
&= 1764\pi - 992.25\pi \\
&= 771.75\pi \\
&= 2425.5\ \text{cm}^2
\end{aligned}$$

$$\begin{aligned}
\text{Area of white region} &= \text{Area of } 5^{th} \text{ circle} - \text{Area of } 4^{th} \text{ circle} \\
&= \pi r_5^2 - \pi\pi_4^2 \\
&= \pi(52.5)^2 - \pi(42)^2 \\
&= 2756.25\pi - 1764\pi \\
&= 992.25\pi \\
&= 3118.5\ \text{cm}^2
\end{aligned}$$

Therefore, areas of gold, red, blue, black, and white regions are 346.5 cm^2, 1039.5 cm^2, 1732.5 cm^2, 2425.5 cm^2 and 3118.5 cm^2 respectively.

Q4. The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is traveling at a speed of 66 km per hour?

Difficulty Level: Medium

What is the known/given?
Diameter of the wheel of the car and the speed of the car.

What is the unknown?

Revolutions made by each wheel.

Reasoning:

Distance travelled by the wheel in one revolution is nothing but the circumference of the wheel itself.

Solution:

Diameter of the wheel of the car = 80 cm
Radius (r) of the wheel of the car = 40 cm

Distance travelled in 1 revolution = Circumference of wheel

Circumference of wheel $= 2\pi r$

$$= 2\pi(40) = 80\pi \text{ cm}$$

Speed of car = 66 km/hour

$$= \frac{66 \times 100000}{60} \text{cm/min}$$

$$= 110000 \text{ cm/min}$$

Distance travelled by the car in 10 minutes

$$= 110000 \times 10$$

$$= 1100000 \text{ cm}$$

Let the number of revolutions of the wheel of the car be n.

$$n \times \text{ Distance travelled in 1 revolution} = \text{ Distance travelled in 10 minutes}$$

$$n \times 80\pi = 1100000$$

$$n = \frac{1100000 \times 7}{80 \times 22}$$

$$= \frac{35000}{8}$$

$$= 4375$$

Therefore, each wheel of the car will make 4375 revolutions.

Q5. Tick the correct answer in the following and justify your choice: If the perimeter and the area of a circle are numerically equal, then the radius of the circle is (A) 2 units (B) π units (C) 4 units (D) 7 units

Difficulty Level: Easy

What is the known/given?
Perimeter and area of the circle are numerically equal.

What is the unknown?
Radius of circle.

Reasoning:
Given that Perimeter and area of the circle are numerically equal. We get $2\pi r = \pi r^2$.
Using this relation we find the radius.

Solution:
Let the radius of the circle = *r*.
Circumference of circle $= 2\pi r$
Area of circle $= \pi r^2$
Given that, the circumference of the circle and the area of the circle are equal.
This implies $2\pi r = \pi r^2$

$$2 = r$$

Therefore, the radius of the circle is 2 units. Hence, the correct answer is A.

Chapter 12: Areas Related to Circles

Exercise 12.2 (Page 230 of Grade 10 NCERT Textbook)

Q1. Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°.

Difficulty Level: Easy

What is the known/given?

A circle with radius = 6 cm, angle of the sector $= 60^{\circ}$

What is the unknown?

Area of sector of a circle.

Reasoning:

The formula for Area of the sector of angle θ

$$= \frac{\theta}{360^{\circ}} \times \pi r^2$$

Solution:

$$\theta = 60^{\circ} \text{ Radius } = 6cm$$

$$\begin{aligned} \text{Area of the sector } &= \frac{\theta}{360^{\circ}} \times \pi r^2 \\ &= \frac{60^{\circ}}{360^{\circ}} \times \frac{22}{7} \times 6 \times 6cm^2 \\ &= \frac{132}{7} cm^2 \\ &= 18\frac{6}{7} cm^2 \end{aligned}$$

Q2. Find the area of a quadrant of a circle whose circumference is 22 cm.

Difficulty Level: Medium

What is the known/given?

A circle whose circumference is 22 cm.

What is the unknown?

Area of quadrant of a circle.

cuemath
THE MATH EXPERT

Reasoning:

Find the radius of the circle (r) from the circumference (c), $C = 2\pi r$

Therefore, $r = \dfrac{c}{2\pi}$

Since quadrant means one of the four equal parts. Using unitary method, since four quadrants corresponds to Area of a circle

Therefore, Area of a quadrant $= \dfrac{1}{4} \times$ Area of a circle

$$= \frac{1}{4} \times \pi r^2$$

Solution:

$$\text{Circumference } (c) = 22 \text{ cm}$$

$$\text{Therefore, radius } (r) = \frac{C}{2\pi}$$

$$= \frac{22}{2 \times \frac{22}{7}}$$

$$= \frac{22 \times 7}{2 \times 22}$$

$$= \frac{7}{2}\, cm$$

Therefore, area of a quadrant $= \dfrac{1}{4} \times \pi r^2$

$$= \frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} cm^2$$

$$= \frac{77}{8} cm^2$$

Q3. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

Difficulty Level: Medium

What is the known/given?

Length of the minute hand of the clock

What is the unknown?

Area swept by the minute hand in 5 minutes.

Reasoning:

Since the minute hand completes one rotation in 1 hour or 60 minutes
Therefore,
Area swept by the minute hand in 60 minutes = Area of the Circle with radius equal to the length of the minute hand.

$$= \pi r^2$$

Using Unitary method

Area swept by minute hand in 1 minute: $\frac{\pi r^2}{60}$

Area swept by minute hand in 5 minutes: $\frac{\pi r^2}{60} \times 5 = \frac{\pi r^2}{12}$

Solution:

Length of the minute hand (r) = 14 cm
We know that the minute hand completes one rotation in 1 hour or 60 minutes

Therefore, Area swept by the minute hand in 60 minutes $= \pi r^2$

Therefore, Area swept by the minute hand in 5 minutes $= \frac{5}{60}\pi r^2 \Rightarrow \frac{1}{12}\pi r^2$

$$= \frac{1}{12} \times \frac{22}{7} \times 14 \times 14 cm^2$$

$$= \frac{154}{3} cm^2$$

Q4. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding:

(i) minor segment (ii) major sector. (Use $\pi = 3.14$)

Difficulty Level: Medium

What is the known/given?

Radius of the circle and angle subtended by the chord at the center.

What is the unknown?

(i) Area of minor segment
(ii) Area of major segment

Reasoning:

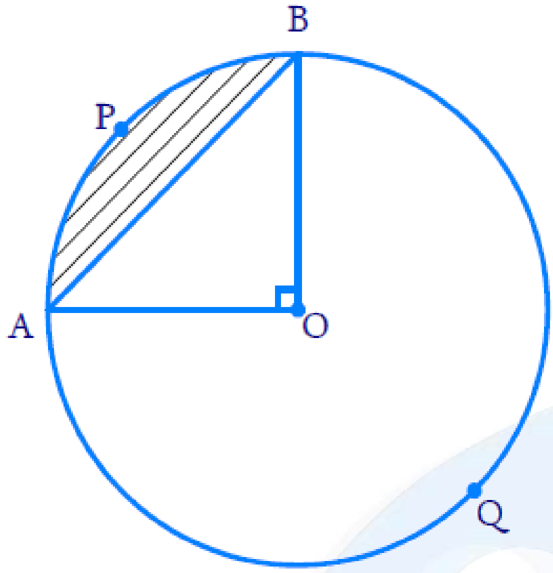
In a circle with radius *r* and angle at the centre with degree measure θ;

(i) Area of the sector $= \frac{\theta}{360^o} \times \pi r^2$

(ii) Area of the segment = Area of the sector – Area of the corresponding triangle

Area of the right triangle $= \frac{1}{2} \times base \times height$

Draw a figure to visualize the area to be found out.



Here, Radius, $r = 10\,cm$ $\quad \theta = 90^{\circ}$

Visually it's clear from the figure that;

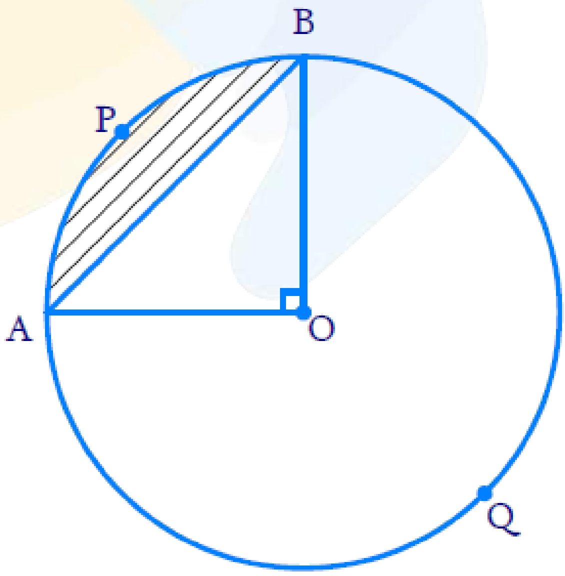
AB is the chord subtends a right angle at the center.

(i) Area of minor segment APB = Area of sector OAPB – Area of right ΔAOB

(ii) Area of major segment AQB $= \pi r^2 -$ Area of minor segment APB

Area of the right triangle $\Delta AOB = \frac{1}{2} \times OA \times OB$

Solution:



(i) Area of minor segment APB = Area of sector OAPB − Area of right ΔAOB

$$= \frac{\theta}{360^\circ} \times \pi r^2 - \frac{1}{2} \times OA \times OB$$

$$= \frac{90^\circ}{360^\circ} \times \pi r^2 - \frac{1}{2} \times r \times r$$

$$= \frac{1}{4}\pi r^2 - \frac{1}{2} r^2$$

$$= r^2\left(\frac{1}{4}\pi - \frac{1}{2}\right)$$

$$= (10cm)^2 \times \left(\frac{3.14 - 2}{4}\right)$$

$$= 100cm^2 \times \left(\frac{1.14}{4}\right)$$

$$= 28.5cm^2$$

(ii) Area of major segment AQB $= \pi r^2 -$ Area of minor segment APB

$$= \pi r^2 - 28.5cm^2$$

$$= 3.14 \times (10cm)^2 - 28.5cm^2$$

$$= 314cm^2 - 28.5cm^2$$

$$= 285.5cm^2$$

Q5. In a circle of radius 21 cm, an arc subtends an angle of 60° at the center. Find:

(i) the length of the arc
(ii) area of the sector formed by the arc
(iii) area of the segment formed by the corresponding chord

Difficulty Level: Hard

What is the known/given?
Radius of the circle and angle subtended by the arc at the center.

What is the unknown?
(i) Length of the arc
(ii) Area of the sector formed by the arc
(iii) Area of the segment formed by the corresponding chord

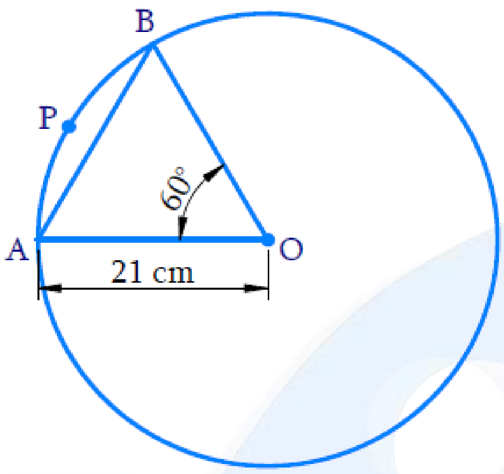
Reasoning:
In a circle with radius *r* and angle at the center with degree measure θ;

(i) Length of the Arc $= \frac{\theta}{360^{o}} \times 2\pi r$

(ii) Area of the sector $= \frac{\theta}{360^{o}} \times \pi r^{2}$

(iii) Area of the segment = Area of the sector – Area of the corresponding triangle

Draw a figure to visualize the problem

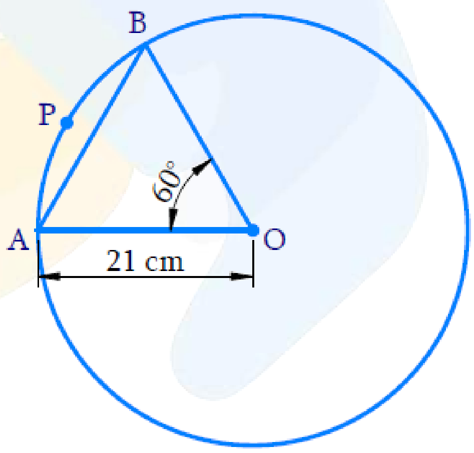


Here, $r = 21cm \qquad \theta = 60^{o}$

Visually it's clear from the figure that;

Area of the segment $=$ Area of sector AOPB – Area of ΔAOB

Solution:



Radius, $r = 21cm \qquad \theta = 60^{o}$

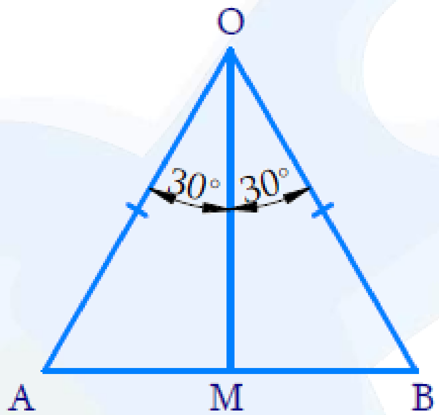
(i) Length of the Arc, $APB = \frac{\theta}{360^{o}} \times 2\pi r$

$$= \frac{60^{o}}{360^{o}} \times 2 \times \frac{22}{7} \times 21cm$$
$$= 22\,cm$$

(ii) Area of the sector, $AOBP = \frac{\theta}{360^{o}} \times \pi r^{2}$

$$= \frac{60^{o}}{360^{o}} \times \frac{22}{7} \times 21 \times 21cm^{2}$$
$$= 231cm^{2}$$

(iii) Area of the segment = Area of the sector AOBP – Area of the triangle AOB
To find area of the segment, we need to find the area of ΔAOB



In ΔAOB

Draw $OM \perp AB$

Consider ΔOAM and ΔOMB

$OA = OB$ (radii of the circle)

$OM = OM$ (common)

$\angle OMA = \angle OMB = 90^{o}$ (Since $OM \perp AB$)

Therefore, $\Delta OMB \cong \Delta OMA$ (By RHS Congruency)

So, AM = MB (Corresponding parts of congruent triangles are equal)

$$\angle OMB = \angle OMA = \frac{1}{2} \times 60^{o} = 30^{o}$$

In ΔAOM

$$\cos 30^\circ = \frac{OM}{OA} \qquad \sin 30^\circ = \frac{AM}{OA}$$

$$\frac{\sqrt{3}}{2} = \frac{OM}{r} \qquad \frac{1}{2} = \frac{AM}{r}$$

$$OM = \frac{\sqrt{3}}{2} r \qquad AM = \frac{1}{2} r$$

$$\text{AB} = 2\text{AM}$$

$$AB = 2 \times \frac{1}{2} r$$

$$AB = r$$

Therefore,

$$\text{Area of } \Delta AOB = \frac{1}{2} \times AB \times OM$$

$$= \frac{1}{2} \times r \times \frac{\sqrt{3}}{2} r$$

$$= \frac{1}{2} \times 21cm \times \frac{\sqrt{3}}{2} \times 21cm$$

$$= \frac{441\sqrt{3}}{4} cm^2$$

Area of the segment $= \left(231 - \frac{441\sqrt{3}}{4}\right) cm^2$

Q6. A chord of a circle of radius 15 cm subtends an angle of 60° at the Centre. Find the areas of the corresponding minor and major segments of the circle.
(Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

Difficulty Level: Hard

What is the known/given?
A chord of a circle with radius (r) = 15 cm subtends an angle $(\theta) = 60^\circ$ at the centre.

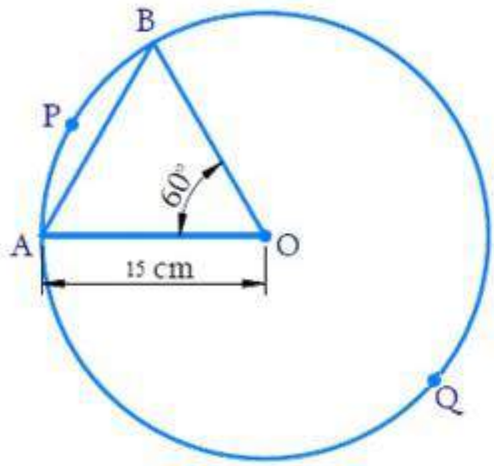
What is the unknown?
Area of minor and major segments of the circle.

Reasoning:
In a circle with radius *r* and angle at the centre with degree measure θ;

(i) Area of the sector $= \frac{\theta}{360^\circ} \times \pi r^2$

(ii) Area of the segment = Area of the sector – Area of the corresponding triangle

Draw a figure to visualize the area to be found out.



Here, Radius, $r = 15cm \qquad \theta = 60^{o}$

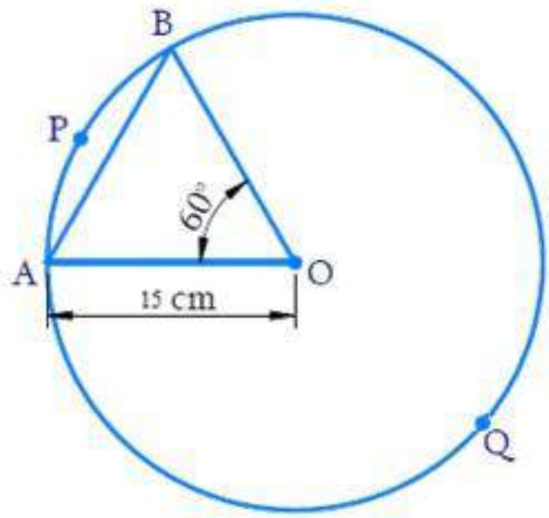
Visually it's clear from the figure that;

AB is the chord subtends 60^{o} angle at the centre.

(i) Area of minor segment APB = Area of sector OAPB – Area of ΔAOB

(ii) Area of major segment AQB $= \pi r^2$ – Area of minor segment APB

Solution:



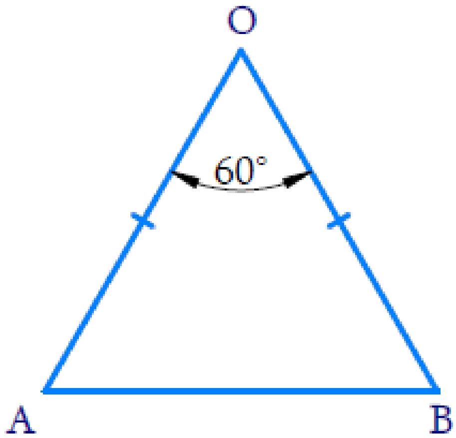
Here, Radius, $r = 15cm \qquad \theta = 60^{o}$

Area of the sector OAPB $= \dfrac{\theta}{360^{o}} \times \pi r^2$

$$= \frac{60^{o}}{360^{o}} \times 3.14 \times 15 \times 15 cm^2$$

$$= 117.75 cm^2$$

In Δ AOB,



$OA = OB = r$ (radii of the circle)

$\angle OBA = \angle OAB$ (Angles opposite equal sides in a triangle are equal)

$$\angle AOB + \angle OBA + \angle OAB = 180^\circ \text{ (Angle sum of a triangle)}$$
$$60^\circ + \angle OAB + \angle OAB = 180^\circ - 60^\circ$$
$$2\angle OAB = 120^\circ$$
$$\angle OAB = 60^\circ$$

$\therefore \Delta$AOB is an equilateral triangle (as all its angles are equal)

$$\Rightarrow AB = OA = OB = r$$

$$\text{Area of } \Delta AOB = \frac{\sqrt{3}}{4}(side)^2$$
$$= \frac{\sqrt{3}}{4}r^2$$
$$= \frac{\sqrt{3}}{4} \times (15cm)^2$$
$$= \frac{1.73}{4} \times 225cm^2$$
$$= 97.3125cm^2$$

(i)Area of minor segment APB = Area of sector OAPB – Area of ΔAOB

$$= 117.75cm^2 - 97.3125cm^2$$
$$= 20.4375cm^2$$

(ii)Area of major segment AQB = Area of the circle − Area of minor segment APB

$$= \pi \times (15cm)^2 - 20.4375cm^2$$
$$= 3.14 \times 225cm^2 - 20.4375cm^2$$
$$= 706.5cm^2 - 20.4375cm^2$$
$$= 686.0625cm^2$$

Q7. A chord of a circle of radius 12 cm subtends an angle of $120°$ at the centre. Find the area of the corresponding segment of the circle.
$\left(\text{Use } \pi = 3.14 \text{ and } \sqrt{3} = 1.73\right)$

Difficulty Level: Hard

What is the known/given?

A chord of a circle with radius (r) = 12 cm subtends an angle (θ) = $120°$ at the centre.

What is the unknown?

Area of segment of the circle.

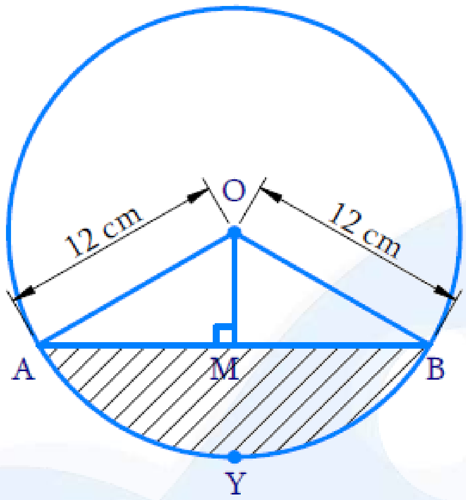
Reasoning:

In a circle with radius r and angle at the centre with degree measure θ;

(i) Area of the sector $= \dfrac{\theta}{360^\circ} \times \pi r^2$

(ii) Area of the segment = Area of the sector – Area of the corresponding triangle

Draw a figure to visualize the problem



Here, Radius, $r = 12cm$ $\quad\theta = 120^\circ$

Visually it's clear from the figure that;

AB is the chord subtends 120° angle at the centre.

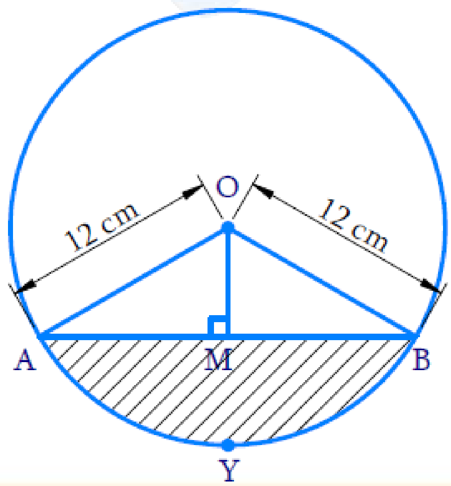
To find area of the segment AYB, we have to find area of the sector OAYB and area of the ΔAOB

(i) Area of sector OAYB $= \dfrac{\theta}{360^\circ} \times \pi r^2$

(ii) Area of $\Delta AOB = \dfrac{1}{2} \times base \times height$

For finding area of ΔAOB, draw $OM \perp AB$ then find base AB and height OM.

Solution:



Here, Radius, $r = 12cm \qquad \theta = 120^{o}$

$$\text{Area of the sector OAYB } = \frac{120^{\circ}}{360^{\circ}} \times \pi r^2$$
$$= \frac{1}{3} \times 3.14 \times (12cm)^2$$
$$= 150.72cm^2$$

Draw a perpendicular OM from O to chord AB

In Δ AOM and Δ BOM

$AO = BO = r$ (radii of circle)

$OM = OM$ (common)

$\angle OMA = \angle OMB = 90^{\circ}$ (drawn)

$\therefore \Delta AOM \cong \Delta BOM$ (By RHS Congruency)

$\Rightarrow \angle AOM = \angle BOM$ (By CPCT)

Therefore, $\angle AOM = \angle BOM = \frac{1}{2}\angle AOB = 60^{\circ}$

In ΔAOM

$$\frac{AM}{OA} = \sin 60^{o}$$
$$\frac{AM}{12cm} = \frac{\sqrt{3}}{2}$$
$$AM = \frac{\sqrt{3}}{2} \times 12cm$$
$$AM = 6\sqrt{3}cm$$

$$\frac{OM}{OA} = \cos 60^{o}$$
$$\frac{OM}{12cm} = \frac{1}{2}$$
$$OM = \frac{1}{2} \times 12cm$$
$$OM = 6cm$$

$$\Rightarrow AB = 2AM$$
$$= 2 \times 6\sqrt{3}cm$$
$$= 12\sqrt{3}cm$$

$$\text{Area of } \Delta AOB = \frac{1}{2} \times AB \times OM$$
$$= \frac{1}{2} \times 12\sqrt{3}cm \times 6cm$$
$$= 36 \times 1.73cm^2$$
$$= 62.28cm^2$$

Area of segment AYB = Area of sector OAYB – Area of Δ AOB

$$= 150.72cm^2 - 62.28cm^2$$
$$= 88.44cm^2$$

Q8. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see given figure). Find

(i) the area of that part of the field in which the horse can graze.
(ii) the increase in the grazing area if the rope were 10 m long instead of 5 m.
$(\text{Use } \pi = 3.14)$

Difficulty Level: Medium

What is the known/given?
Length of side of the square grass field = 15 m and rope of length = 5 m by which a horse is tied to a peg at one corner of the field

What is the unknown?
(i) Area of the field the horse can graze
(ii) Increase in grazing area if the rope were 10 m long instead of 5 m

Reasoning:
(i) From the figure it's clear that the horse can graze area of a sector of a circle with radius (r) 5m and angle with degree measure 90° (as the peg is at corner of a square and angle of square $=90^\circ$ and length of the rope = 5m)

Area of the field horse can graze = Area of the sector

$$= \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{90^\circ}{360^\circ} \times \pi r^2$$

$$= \frac{1}{4}\pi r^2$$

(ii) Similar to the first part
Area that can be grazed by the horse when length of the rope is 10m, is area of a sector of a circle with radius (r_1) 10m and angle with degree measure 90°

Area of the field horse can graze = Area of the sector

$$= \frac{90^\circ}{360^\circ}\pi r_1^2$$

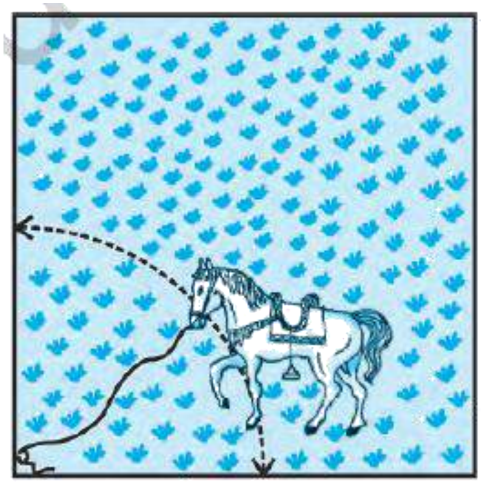
$$= \frac{1}{4}\pi r_1^2$$

Increase in grazing area

$$= \frac{1}{4}\pi r_1^2 - \frac{1}{4}\pi r^2$$

$$= \frac{1}{4}\pi\left(r_1^2 - r^2\right)$$

Solution:



(i) Area of the field the horse can graze = Area of sector of $90°$ in a circle of radius 5 m

$$= \frac{90^\circ}{360^\circ} \times \pi \times (5m)^2$$
$$= \frac{25}{4} \times 3.14 m^2$$
$$= 19.625 m^2$$

(ii) Area that can be grazed by the horse when rope is 10 m long.

$$= \frac{90^\circ}{360^\circ} \times \pi \times (10m)^2$$
$$= \frac{1}{4} \times 3.14 \times 100 m^2$$
$$= 78.5 m^2$$

Increase in grazing area $= 78.5m^2 - 19.625m^2 = 58.875m^2$

Q9. A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in the figure. Find:

(i) The total length of the silver wire required.

(ii) The area of each sector of the brooch.

Difficulty Level: Medium

What is the known/given?

A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divides the circle into 10 equal sectors.

What is the unknown?

(i) The total length of silver wire required
(ii) Area of each sector of the brooch

Reasoning:

(i) Since the silver wire is used in making the 5 diameters and perimeter of the circular brooch.

$\therefore$ Total length of silver were required

$=$ Circumference of circle + $5\times$ diameter

$= \pi d + 5d$ (Where d is diameter of the brooch)

$= d\,(\pi + 5)$

(ii) To find area of each sector of the brooch, we first find the angle made by each sector at the centre of the circle.

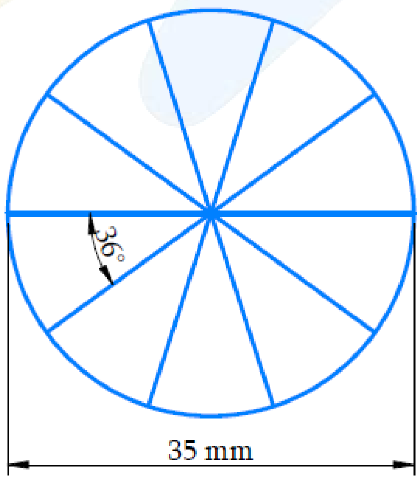
Since the wire divides into 10 equal sectors

$$\therefore \text{ Angle of sector } (\theta) = \frac{360^\circ}{10^\circ} = 36^\circ$$

$$\text{Radius of the brooch, } r = \frac{d}{2}$$

$$\therefore \text{ Area of each sector of brooch} = \frac{\theta}{360^\circ} \times \pi r^2$$

Solution:



(i) Diameter of the brooch (d) = 35 mm

Total length of silver wire required

$$= \text{ Circumference of brooch } + 5\times \text{diameter}$$
$$= \pi d + 5d$$
$$= (\pi + 5)\times 35mm$$
$$= \left(\frac{22+35}{7}\right)\times 35mm$$
$$= 57\times 5mm$$
$$= 285\ mm$$

(ii) Radius of brooch $(r) = \frac{35}{2}mm$

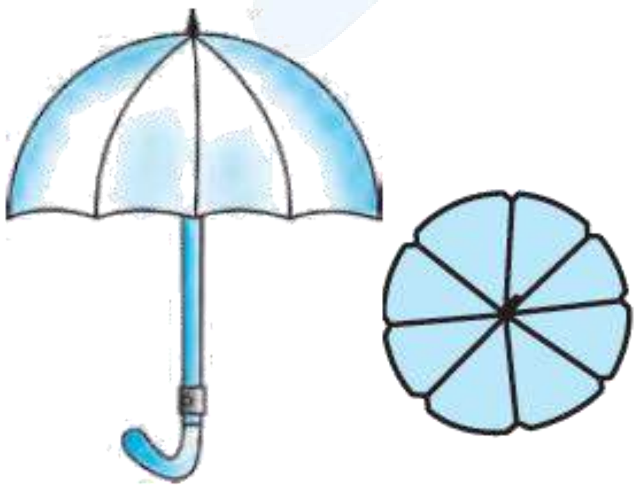
Since the wire divides the brooch into 10 equal sectors

$$\therefore \text{ Angle of sector } \theta = \frac{360^\circ}{10^\circ} = 36^\circ$$

$\therefore$ Area of each sector of the brooch

$$= \frac{36^\circ}{360^\circ}\times \pi r^2$$
$$= \frac{1}{10}\times\frac{22}{7}\times\frac{35}{2}mm\times\frac{35}{2}mm$$
$$= \frac{385}{4}mm^2$$
$$= 96.25mm^2$$

Q10. An umbrella has 8 ribs which are equally spaced (see given figure). Assuming umbrella to be a flat circle of radius 45cm, find the area between the two consecutive ribs of the umbrella.



Difficulty Level: Medium

What is the known/given?

An umbrella has 8 ribs which are equally spaced. Assume umbrella to be a flat circle of radius = 45 cm.

What is the unknown?

The area between the 2 consecutive ribs of the umbrella

Reasoning:

Since there are 8 equal spaced ribs in an umbrella and the umbrella is assumed to be a flat circle.

$\therefore$ Angle between 2 consecutive ribs at the centre $= \dfrac{360^\circ}{8} = 45^\circ$

Area between 2 consecutive ribs of the umbrella = Area of a sector with angle 45°

$$= \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{45^\circ}{360^\circ} \times \pi r^2$$

$$= \frac{1}{8} \pi r^2$$

Solution:

Since there are 8 equally spaced ribs in the umbrella

$\therefore$ Angle between 2 consecutive ribs (θ)

$$= \frac{360^\circ}{8}$$

$$= 45^\circ$$

Area between 2 consecutive ribs of umbrella

$$= \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{45^\circ}{360^\circ} \times \frac{22}{7} \times 45cm \times 45cm$$

$$= \frac{1}{8} \times \frac{22}{7} \times 45cm \times 45cm$$

$$= \frac{22275}{28} cm^2$$

$$= 795.535 \ cm^2$$

Q11. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

Difficulty Level: Medium

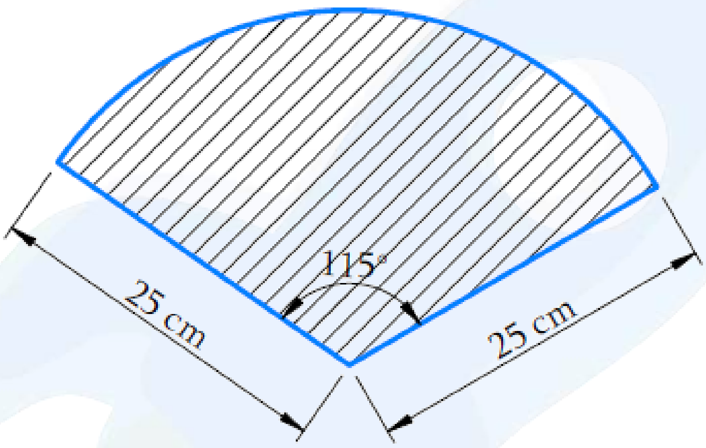
What is the known/given?

A car has 2 wipers which do not overlap. Each wiper has a blade length = 25cm and sweeps through an angle $(\theta) = 115^\circ$

What is the unknown?

Total area cleaned at the sweep of the blade of the 2 wipers.

Reasoning:



Visually it is clear that -

Area cleaned at the sweep of blades of each wiper = Area of the sector with angle $115°$ at the centre and radius of the circle 25cm

Since there are 2 wipers of same blade length and same angle of sweeping. Also there is no area of overlap for the wipers.

$\therefore$ Total area cleaned at each sweep of the blades = $2 \times$ Area cleaned at the sweep of each wiper.

Solution:

Area cleaned at the sweep of blades of each wiper = Area of the sector of a circle with radius 25 cm and of angle $115°$

$$= \frac{\theta}{360^\circ} \times \pi r^2$$
$$= \frac{115^\circ}{360^\circ} \times \pi \times 25 \times 25$$
$$= \frac{23}{72} \times 625\pi$$

Since there are 2 identical blade length wipers

$\therefore$ Total area cleaned at each sweep of the blades $= 2\times\frac{23}{72}\times 625\pi$

$$= 2\times\frac{23}{72}\times\frac{22}{7}\times 625$$

$$= \frac{23\times 11\times 625}{18\times 7}$$

$$= \frac{158125}{126}\,cm^2$$

$$= 1254.96\ cm^2$$

Q12. To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. $(\text{Use } \pi = 3.14)$

Difficulty Level: Medium

What is the known/given?

A lighthouse spreads a red coloured light over a sector of angle of $80°$ to a distance of 16.5 km to warn ships for underwater rock. $(\text{Use } \pi = 3.14)$

What is the unknown?

Area of the sea over which the ships are warned.

Reasoning:

Since the lighthouse spreads red coloured light over a sector of a circle with radius = 16.5 km and angle with degree measure 80°

Area of sea over which the ships are warned = area of the sector of the circle with radius 16.5 km and angle with degree measure 80°

$$= \frac{\theta}{360^{\circ}}\times\pi r^2$$

$$= \frac{80^{\circ}}{360^{\circ}}\times\pi r^2$$

$$= \frac{2}{9}\pi r^2$$

Solution:

Area of sea over which the ships are warned = area of the sector of the circle with radius, $r = 16.5km$ and angle with degree measure 80°

$$= \frac{80^\circ}{360^\circ} \times \pi r^2$$

$$= \frac{2}{9} \times 3.14 \times 16.5km \times 16.5km$$

$$= 189.97\ km^2$$

Q13. A round table cover has six equal designs as shown in figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of ₹ 0.35 per cm^2. $\left(\text{Use}\sqrt{3} = 1.7\right)$

Difficulty Level: Hard

What is the known/given?

A round table cover has 6 equal designs as shown in the figure. The radius of the cover = 28 cm and the rate of making design is ₹ 0.35 per cm^2

What is the unknown?

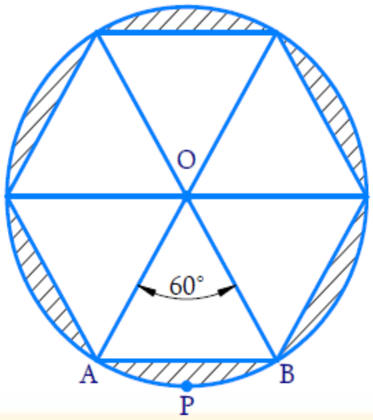
The cost of making the design.

Reasoning:

In a circle with radius r and angle at the centre with degree measure θ;

(i) Area of the sector $= \frac{\theta}{360^o} \times \pi r^2$

(ii) Area of the segment = Area of the sector – Area of the corresponding triangle



(i) Visually it is clear that the designs are segments of the circle

$\therefore$ Area of the design = Area of 6 segments of the circle.

(ii) Since the table cover has 6 equal design therefore angle of each segment at the center $= \frac{360^o}{6} = 60^o$

(iii) Consider segment APB. Chord AB subtends an angle of 60^o at the centre.

$\therefore$ Area of segment APB = Area of sector AOPB – Area of ΔAOB

(iv) To find area of ΔAOB

In ΔAOB

OA = OB (radii of the circle)

$$\angle OAB = \angle OBA \text{(angles oppositeequal sides of atriangle are equal)}$$
$$\angle AOB + \angle OAB + \angle OBA = 180^\circ \text{(Using angle sum property of a triangle)}$$
$$\angle AOB + 2\angle OAB = 180^\circ$$
$$2\angle OAB = 180^\circ - 60^\circ$$
$$= 120^\circ$$
$$\angle OAB = \frac{120}{2}$$
$$= 60^\circ$$
$$= \angle OBA$$

Since all angles of a triangle are of measure 60^o

$\therefore$ ΔAOB is an equilateral triangle.

Using area of equilateral triangle = $\frac{\sqrt{3}}{4}(side)^2$

$$\text{Area of } \Delta AOB = \frac{\sqrt{3}}{4} r^2$$

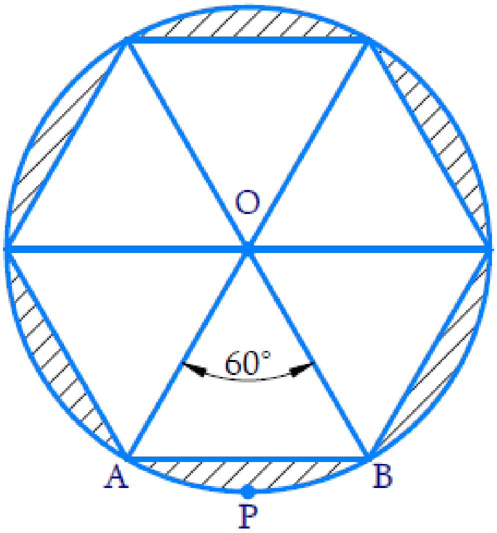
$$\text{Area of sector } AOBP = \frac{60^o}{360^o} \pi r^2$$

$$\text{Area of segment } APB = \frac{60^o}{360^o} \pi r^2 - \frac{\sqrt{3}}{4} r^2$$

$$\text{Area of designs } = 6 \times \left(\frac{60^o}{360^o} \pi r^2 - \frac{\sqrt{3}}{4} r^2 \right)$$

Since we know cost of making 1cm^2 of designs we can use unitary method to find cost of designs.

Solution:



From the figure we observe the designs are made in the segments of a circle.

Since the table cover has 6 equal designs

$\therefore$ angle subtended by each chord (bounding the segment) at the center $= \dfrac{360^\circ}{6} = 60^\circ$

Consider ΔAOB

$$\Delta OAB = \Delta OBA \left(QOB = OA \text{ , angles opposite equal sides in a triangle are equal} \right)$$

$$\Delta AOB + \Delta OAB + \Delta OBA = 180^\circ \left(\text{angle sum of a triangle} \right)$$

$$2\Delta OAB = 180^\circ - 60^\circ$$

$$\Delta OAB = \frac{120^\circ}{2}$$

$$= 60^\circ$$

$\therefore$ ΔAOB is an equilateral triangle

$$\text{Area of } \Delta AOB = \frac{\sqrt{3}}{4}(\text{side})^2$$

$$= \frac{\sqrt{3}}{4} \times (28)^2$$

$$= \sqrt{3} \times 7 \times 28$$

$$= 196\sqrt{3}$$

$$= 196 \times 1.7$$

$$= 333.2 \text{cm}^2$$

$$\text{Area of sector OAPB} = \frac{60^\circ}{360^\circ} \times \pi r^2$$

$$= \frac{1}{6} \times \frac{22}{7} \times 28 \times 28$$

$$= \frac{11 \times 4 \times 28}{3}$$

$$= \frac{1232}{3} \text{cm}^2$$

Area of segment APB = Area of sector OAPB − Area of ΔAOB

$$= \left(\frac{1232}{3} - 333.2\right) cm^2$$

Area of designs = 6 Area of segment

$$= 6 \times \left(\frac{1232}{3} - 333.2\right)$$

$$= 2464 - 1999.2\, cm^2$$

$$= 464.8\, cm^2$$

Cost of making $1 cm^2$ of designs = ₹ 0.35

∴ Cost of making $464.8 cm^2$ of designs

= ₹ 0.35 × 464.8

= ₹ 162.68

Q14. Tick the correct answer in the following:

Area of a sector of angle p (in degrees) of a circle with radius R is

(A) $\frac{P}{180^o} \times 2\pi R$, (B) $\frac{P}{180^o} \times 2\pi R^2$ (C) $\frac{P}{720^o} \times 2\pi R$ (D) $\frac{P}{720^o} \times 2\pi R^2$

Difficulty Level: Easy

What is the known/given?

A sector of angle p (in degree) of a circle with radius R.

What is the unknown?

Area of a sector.

Reasoning:

Consider

Area of the sector of angle $\theta = \frac{\theta}{360} \times \pi r^2$

where r is the radius of the circle

Here $\theta = p$ and $r = R$

∴ Substituting above values in formula we get

$$\text{Area of the sector} = \frac{p}{360^o} \times \pi R^2$$

Multiplying numerator and denominator of formulas obtained above by 2 we get

$$\text{Area of the sector } = \frac{p}{720^o} \times 2\pi R^2$$

Solution:

If radius of a circle = R

We know, $\text{Area of sector of angle}\,\theta = \frac{\theta}{360^o} \times \pi R^2$

$$\therefore \text{ Area of sector of angle p} = \frac{p}{360^o} \times \pi R^2$$

$$= \frac{2}{2}\left(\frac{p}{360^o} \times \pi R^2\right)$$

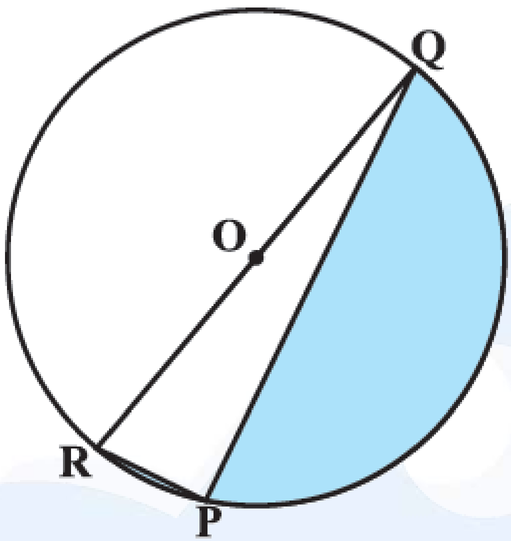
$$= \frac{p}{720^o} \times 2\pi R^2$$

Hence D is the correct answer.

Chapter 12: Areas Related to Circles

Exercise 12.3 (Page 234 of Grade 10 NCERT Textbook)

Q1. Find the area of the shaded region in the figure, if PQ = 24 cm, PR = 7 cm and O is the center of the circle.



Difficulty Level: Medium

What is the known/given?
PQ = 24 cm, PR = 7 cm, O is the center the circle.

Use $\pi = \dfrac{22}{7}$

What is the unknown?
Area of the shaded region in figure.

Reasoning:

(i) Visually it's clear that

$$\text{Area of the shaded region} = \text{Area of semicircle RPQ - Areaof } \Delta RPQ$$
$$= \frac{1}{2}\pi \times (OR)^2 - \text{Areaof } \Delta RPQ$$

Since we don't know RQ (diameter) OR and PQ (radii of circle), we are unable to find either area of semicircle and we don't know RQ so we can't find area of ΔRPQ using heron's formula (since either 2 sides are known) or by $\frac{1}{2} \times \text{base} \times \text{height}$ as we have only 2 sides and that their respective heights.

So, we need to find RQ.

(ii) Using the knowledge:

Angle subtended by an arc at any point on the circle is half of the angle subtended by it at the center.

$$\because \angle ROQ = 180^\circ$$

$$\therefore \angle RPQ = \frac{1}{2} \times 180^\circ = 90^\circ$$

(OR) angle in a semicircle $= 90^\circ$

We get ΔRQP is a right-angle triangle.

Using Pythagoras theorem

$$RQ^2 = PR^2 + PQ^2$$

$$\text{We get } RQ = \sqrt{PR^2 + PQ^2}$$

$$\because \text{radius} = \frac{1}{2} RQ.$$

We can find the area of semicircle.

And since $\angle RPQ = 90^\circ$

$\therefore$ RP is the height for PQ or vice versa

So, using the formula.

Area of $\Delta = \frac{1}{2} \times \text{base} \times \text{height}$

Area of $\Delta RPQ = \frac{1}{2} \times PQ \times RP$

Solution:

PQ = 24 cm PR = 7cm

Hence the angle in a semicircle is a right angle

$$\therefore \angle RPQ = 90^\circ$$

$\Rightarrow \Delta RQP$ is a right angled triangle.

$\therefore$ Using Pythagoras theorem.

$$PQ^2 = PR^2 + PQ^2$$

$$RQ = \sqrt{7^2 + 24^2}$$

$$= \sqrt{49 + 576}$$

$$= \sqrt{625}$$

$$= 25\text{cm}$$

$$\therefore \text{ Radius (r)} = \frac{25}{2}\text{cm}$$

Area of shaded region = Area of semicircle RPQ – Area of ΔRQP

$$= \frac{1}{2} \times \pi r^2 - \frac{1}{2} \times PQ \times RP$$

$$= \frac{1}{2} \times \left[\frac{22}{7} \times \frac{25}{2} \times \frac{25}{2} - 24 \times 7\right]$$

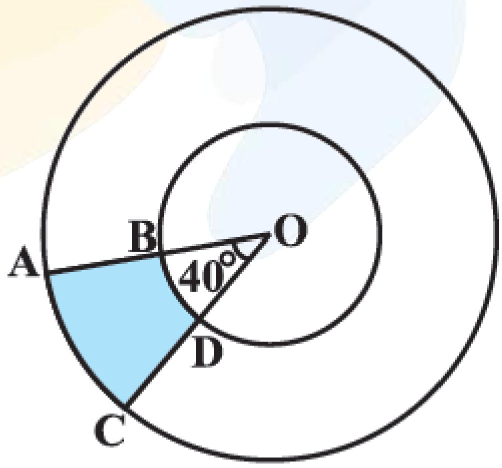
$$= \frac{1}{2} \times \left[\frac{6875}{14} - 168\right]$$

$$= \frac{1}{2} \times \left[\frac{6875 - 2352}{14}\right] = \frac{1}{2} \times \frac{4523}{14}$$

$$= \frac{4523}{28} cm^2$$

$$= 161.54 \ cm^2 \left(approximately\right)$$

Q2. Find the area of the shaded region in the given figure, if radii of the two concentric circles with center O are 7 cm and 14 cm respectively and $\angle AOC = 40^\circ$.



Difficulty Level: Medium

What is the known/given?

Radius of 2 concentric circles with center O are 7cm and 14 cm and $\angle AOC = 40^\circ$.

What is the unknown?
Area of the shaded region.

Reasoning:
In a circle with radius r and angle at the centre with degree measure θ;

$$\text{Area of the sector} = \frac{\theta}{360^\circ} \times \pi r^2$$

Area of the shaded region can be calculated by subtracting the area of the sector of smaller circle from the area of the sector of the larger circle.

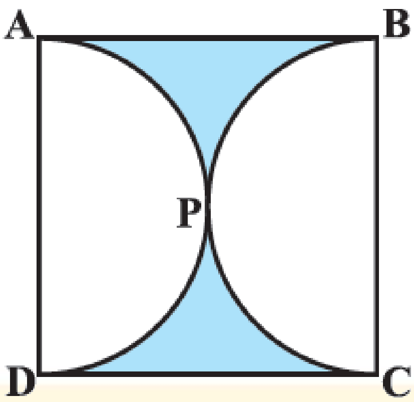
Area of shaded region ABDC = Area of sector ACO – Area of sector BDO

Solution:
Radius of the larger circle, $R = OA = 14cm$
Radius of the smaller circle, $r = OB = 7cm$
Angle at the centre, $\theta = 40^\circ$

Area of shaded region ABDC = Area of sector ACO – Area of sector BDO

$$\begin{aligned}
&= \frac{\theta}{360^\circ} \times \pi R^2 - \frac{\theta}{360^\circ} \times \pi r^2 \\
&= \frac{\theta}{360^\circ} \pi \left(R^2 - r^2\right) \\
&= \frac{\theta}{360^\circ} \pi (R + r)(R - r) \\
&= \frac{40^\circ}{360^\circ} \times \frac{22}{7} \times (14 + 7) \times (14 - 7) \\
&= \frac{1}{9} \times \frac{22}{7} \times 21 \times 7 \\
&= \frac{22 \times 7}{3} \\
&= \frac{154}{3} cm^2
\end{aligned}$$

Q3. Find the area of the shaded region in the given figure, if ABCD is a square of side 14 cm and APD and BPC are semicircles.



Difficulty Level: Medium

What is the known/given?

ABCD is a square of side (l) 14cm. APD and BPC are semicircles.

What is the unknown?

Area of the shaded region.

Reasoning:

(i) From figure it is clear that diameter of both semicircles = side of square =14 cm

$\therefore$ Radius of each semicircle $(\text{r}) = \frac{14}{2} = 7\text{cm}$

(ii) Visually, it is clear

Area of shaded region = Area of square ABCD- (Area of semicircle APD + Area of semicircle BPC)

$$= (side)^2 - \left(\frac{\pi r^2}{2} + \frac{\pi r^2}{2}\right)$$

$$= (side)^2 - \pi r^2$$

Solution:

Since semicircles APD and BPC are drawn using sides AD and BC respectively as diameter.

$\therefore$ Diameter of each semicircle =14 cm

Radius of each semicircle $(\text{r}) = \frac{14}{2} = 7\text{cm}$

Area of shaded region

$$= \text{Area of square ABCD-}(\text{Area of semicircle APD + Area of semicircle BPC})$$

$$= (side)^2 - \left(\frac{1}{2}\pi r^2 + \frac{1}{2}\pi r^2\right)$$

$$= (side)^2 - \pi r^2$$

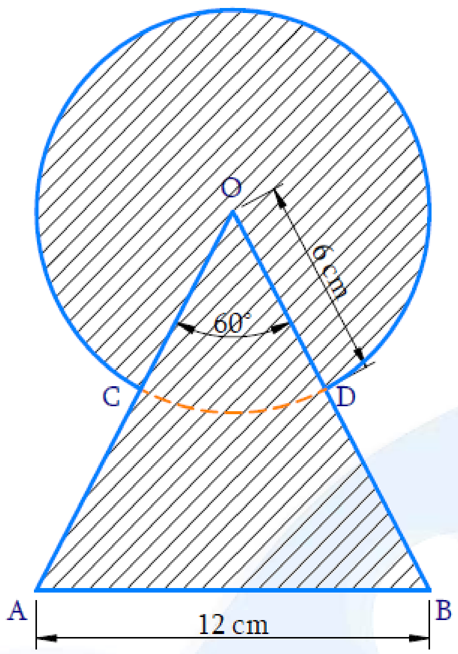
$$= (14cm)^2 - \pi \times (7cm)^2$$

$$= 196cm^2 - \frac{22}{7} \times 7cm \times 7cm$$

$$= 196cm^2 - 154cm^2$$

$$= 42cm^2$$

Q4. Find the area of the shaded region in the given figure, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.



Difficulty Level: Medium

What is the known/given?

A circular arc of radius = 6 cm is drawn with vertex O of an equilateral ΔOAB of side 12 cm as centre.

What is the unknown?

Area of the shaded region.

Reasoning:

From the figure it is clear that the shaded region has an overlap region of area of a sector of angle 60° (since each angle of an equilateral Δ is of measure 60° area of a circle with radius 6 cm and area of triangle OAB with side 12 cm.

$\therefore$ Area of shaded region = Area of circle with radius 6 cm + Area of ΔOAB - Area of sector of angle 60°

$$= \pi r^2 + \frac{\sqrt{3}}{4}(side)^2 - \frac{\theta}{360^0} \times \pi r^2$$

Using formula, Area of an equilateral $\Delta = \frac{\sqrt{3}}{4}(side)^2$

Area of the sector of angle $\theta = \frac{\theta}{360^\circ} \times \pi r^2$

Where r is the radius of the circle

Solution:

Radius of circle (r) = 6 cm

Side of equilateral $\Delta OAB, (s) = 12cm$

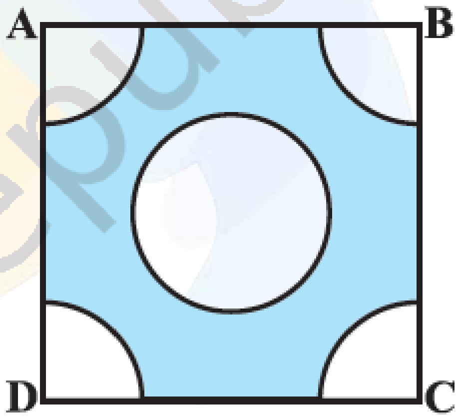
We know each interior angle of equilateral $\Delta = 60^\circ$

Since they overlap, part of area of a sector OCD is in area of the circle and triangle.

$\therefore$ Area of shaded region = Area of circle + Area of ΔOAB − Area of sector OCD

$$= \pi(6cm)^2 + \frac{\sqrt{3}}{4}(12cm)^2 - \frac{60}{360} \times \pi(6cm)^2$$
$$= 36\pi cm^2 + 36\sqrt{3}cm^2 - 6\pi cm^2$$
$$= \left(30\pi + 36\sqrt{3}\right)cm^2$$
$$= \left(30 \times \frac{22}{7} + 36\sqrt{3}\right)cm^2$$
$$= \left(\frac{660}{7} + 36\sqrt{3}\right)cm^2$$

Q5. From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in the given figure. Find the area of the remaining portion of the square.



Difficulty Level: Medium

What is the known/given?

From each corner of a square of side = 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm.

What is the unknown?

Area of remaining portion of the square.

Reasoning:

(i) Since diameter of circle which is cut out = 2 cm

$\therefore$ Radius of this circle (r) = 1 cm

(ii) Logically since all quadrants cut out are of same radius.

Area of portions cut out of square

$$= \text{Area of the circle} + 4\times(\text{Area of each quadrant})$$
$$= \pi r^2 + 4\left(\frac{90^\circ}{360^\circ}\times \pi r^2\right)$$
$$= \pi r^2 + 4\times\frac{\pi r^2}{4}$$
$$= \pi r^2 + \pi r^2$$
$$= 2\pi r^2$$

(iii) From the figure it is clear that

Area of remaining portion of the square

$$=\text{Area of square} - \text{Area of portion cut out of square}$$
$$= (side)^2 - 2\pi r^2$$

Solution:

Diameter of circle = 2 cm

Radius of circle $(r) = \dfrac{2cm}{2} = 1cm$

Radius of all quadrants cut out $(r) = 1cm$

Area of the portions cut out of the square

$$= \text{Area of the circle} + 4\times(\text{Area of each quadrant})$$
$$= \pi r^2 + 4\left(\frac{90^\circ}{360^\circ}\times \pi r^2\right)$$
$$= \pi r^2 + 4\times\frac{\pi r^2}{4}$$
$$= \pi r^2 + \pi r^2$$
$$= 2\pi r^2$$
$$= 2\times\frac{22}{7}\times(1cm)^2$$
$$= \frac{44}{7}cm^2$$

Area of remaining portion of the square = Area of square – Area of portion cut out

$$= (side)^2 - 2\pi r^2$$

$$= (4cm)^2 - \frac{44}{7}cm^2$$

$$= 16cm^2 - \frac{44}{7}cm^2$$

$$= \frac{112-44}{7}cm^2$$

$$= \frac{68}{7}cm^2$$

Q6. In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in the given figure. Find the area of the design.

Difficulty Level: Hard

What is the known/given?

A circular table cover of radius = 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in figure.

What is the unknown?

The area of the design.

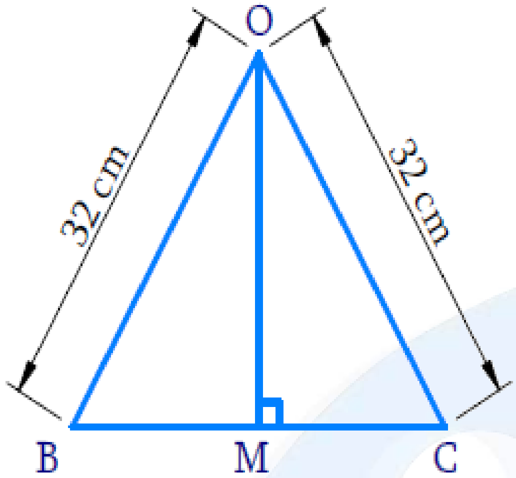
Reasoning:

Mark O as centre of the circle. Join BO and CO.

Since, we know that equal chords of a circle subtends equal angles at the center and all sides of an equilateral triangle are equal.

$\therefore$ Each side of Δ will subtend equal angles at centre

$$\therefore \angle BOC = \frac{360^\circ}{3} = 120^\circ$$



Consider ΔBOC

Drop a perpendicular from OM to BC

Since we know perpendicular from the center of circle to a chord bisects it

$$\Rightarrow BM = MC$$

OB = OC (radii) and OM = OM (common)

$$\therefore \Delta OBM \cong \Delta OCM (\text{ by SSS congruency})$$

$$\Rightarrow \angle BOM = \angle COM \qquad (\text{CPCTC})$$

$$\therefore 2\angle BOM = \angle BOC = 120^\circ$$

$$\angle BOM = \frac{120^\circ}{2} = 60^\circ$$

$$\sin 60^\circ = \frac{BM}{BO} = \frac{\sqrt{3}}{2}$$

$$\therefore BM = \frac{\sqrt{3}}{2} \times BO = \frac{\sqrt{3}}{2} \times 32 = 16\sqrt{3}$$

$$\Rightarrow BC = 2BM = 32\sqrt{3}\text{cm}$$

Using formula of area of equilateral Δ

$$= \frac{\sqrt{3}}{4}(\text{side})^2$$

We can find area of ΔABC since a side BC of Δ is known.

Visually from figure it's clear

Area of the design = Area of circle – Area of ΔABC

$$= \pi r^2 - \frac{\sqrt{3}}{4}(BC)^2$$

This can be solved with ease as both the radius of the circle and BC are known.

Solution:

Let the center of the circle be O. Join BO and CO.

Since equal chords of a circle subtend equal angles as its centre.

$\therefore$ Sides AB, BC and AC of ΔABC will subtend equal angles at the centre of circle

$$\therefore \angle BOC = \frac{360^\circ}{3} = 120^\circ$$

Draw $OM \perp BC$

In ΔBOM and ΔCOM

BO = CO (radii circle)

OM = OM (common)

BM = CM .... (perpendicular drawn from the center of the circle to a chord bisects it)

$\therefore \Delta BOM \cong \Delta COM$ (By SSS Congruency)

$\therefore \angle BOM = \angle COM$ ...... (2) (CPCT)

From figure

$$\angle BOM + \angle COM = \angle BOC$$

$$2\angle BOM = 120^\circ \qquad (\text{Using }(2))$$

$$\angle BOM = \frac{120^\circ}{2}$$

$$= 60^\circ$$

In ΔBOM

$$\sin 60^\circ = \frac{BM}{BO} = \frac{\sqrt{3}}{2}$$

$$\therefore BM = \frac{\sqrt{3}}{2} BO = \frac{\sqrt{3}}{2} \times 32 = 16\sqrt{3}$$

$$BC = BM + CM$$

$$BC = 2BM$$

$$BC = 2 \times 16\sqrt{3}$$

$$BC = 32\sqrt{3}$$

Radius of circle (r) = 32 cm

From figure, we observe

Area of design = Area of circle - Area of ΔABC

$$= \pi r^2 - \frac{\sqrt{3}}{4}(BC)^2$$

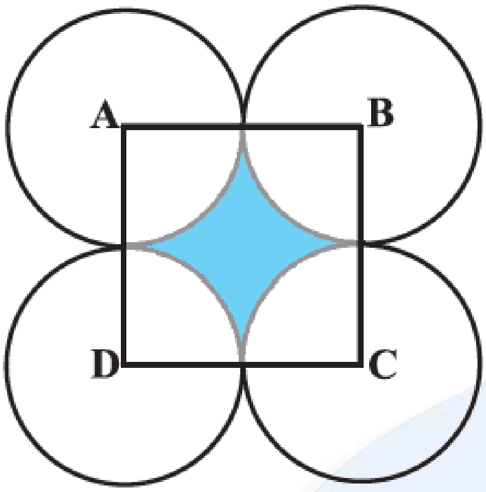
$$= \frac{22}{7} \times (32)^2 - \frac{\sqrt{3}}{4} \times (32\sqrt{3})^2$$

$$= \frac{22}{7} \times 1024 - \frac{\sqrt{3}}{4} \times 1024 \times 3$$

$$= \frac{22528}{7} - 768\sqrt{3}$$

Area of design $= \left(\frac{22528}{7} - 768\sqrt{3}\right) cm^2$

Q7. In the given figure, ABCD is a square of side 14 cm. With Centers A, B, C and D, four circles are drawn such that each circle touch externally two of the remaining three circles. Find the area of the shaded region.



Difficulty Level: Medium

What is the known/given?

ABCD is a square of side = 14 cm.

With centers A, B, C, D four circles are drawn such that each circle touches externally 2 of the remaining 3 circles.

What is the unknown?

Area of the shaded region.

Reasoning:

Since the circles are touching each other externally, visually it is clear that

Radius of each circle $r = \frac{1}{2} \times$ (side of square)

Also, ABCD being a square all angles are of measure 90°

Therefore, all sectors are equal as they have same radii and angle.

$\therefore$ Angle of each sector which is part of the square $(\theta) = 90^\circ$

$$\therefore \text{ Area of each sector } = \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{90^\circ}{360^\circ} \times \pi r^2$$

$$= \frac{\pi r^2}{4}$$

From the figure it is clear that:

Area of shaded region = Area of square – Area of 4 sectors

$$= (\text{side})^2 - 4 \times \text{Area of each sector}$$
$$= (14)^2 - 4 \times \frac{\pi r^2}{4}$$
$$= (14)^2 - \pi r^2$$

Solution:

Area of each of the 4 sectors is equal as each sector subtends an angle of 90° at the center of a circle with radius, $r = \frac{1}{2} \times 14cm = 7cm$

$$\text{Area of each sector} = \frac{\theta}{360^\circ} \times \pi r^2$$
$$= \frac{90^\circ}{360^\circ} \times \pi (7)^2$$
$$= \frac{1}{4} \times \frac{22}{7} \times 7 \times 7$$
$$= \frac{77}{2} cm^2$$

$$\text{Area of shaded region} = \text{Area of square} - 4 \times \text{ Area of each sector}$$
$$= (14)^2 - 4 \times \frac{77}{2}$$
$$= 196 - 154$$
$$= 42 cm^2$$

Q8. Figure depicts a racing track whose left and right ends are semicircular. The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find:

(i) The distance around the track along its inner edge

(ii) The area of the track.

Difficulty Level: Hard

What is the known/given?

(i) A racing track whose right and left ends are semicircular.

(ii) The distance between the 2 inner parallel line segments is 60m and they are 106m long

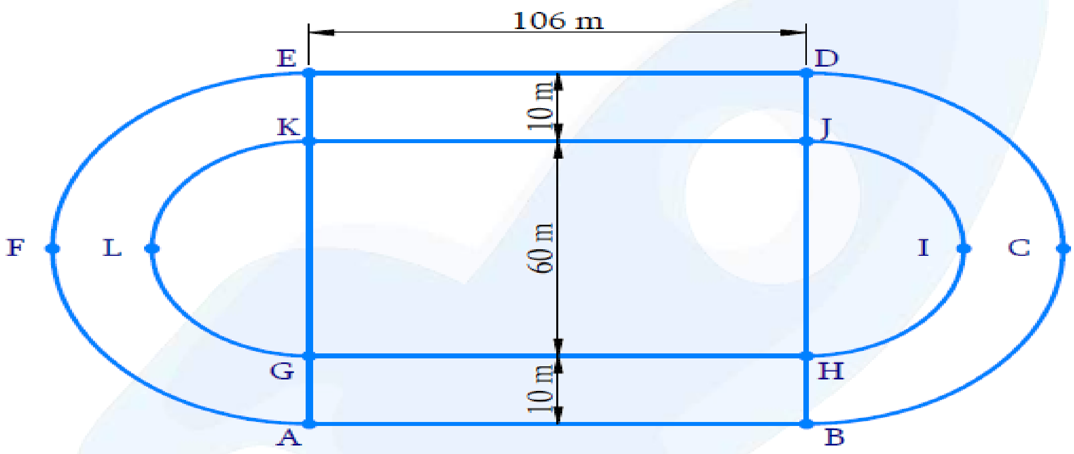
(iii) Width of track = 10m

What is the unknown?

(i) The distance around the track along its inner edge.

(ii) Area of the track.

Reasoning:



(i) Draw a figure along with dimensions to visuals the track properly.

(ii) Visually the distance around the track along its inner edge.

$$= GH + \text{arc}\, HIJ + JK + \text{arc}\, KLG$$

Where GH = JK = 106 m

And arc HIJ = arc KLG = circumference of semicircle with diameter 60m.

So, it can be easily found by substituting the required values.

i. To find area of the track

Visually it's clear that

Area of the track = Area of rectangle ABHG +Area of rectangle KJDE + (Area of semicircle BCD – Area of semicircle HIJ) + (Area of semicircle EFA – Area of semicircle KLG)

Radii of semicircles HIJ and KLG $= \frac{60}{2} = 30$ m

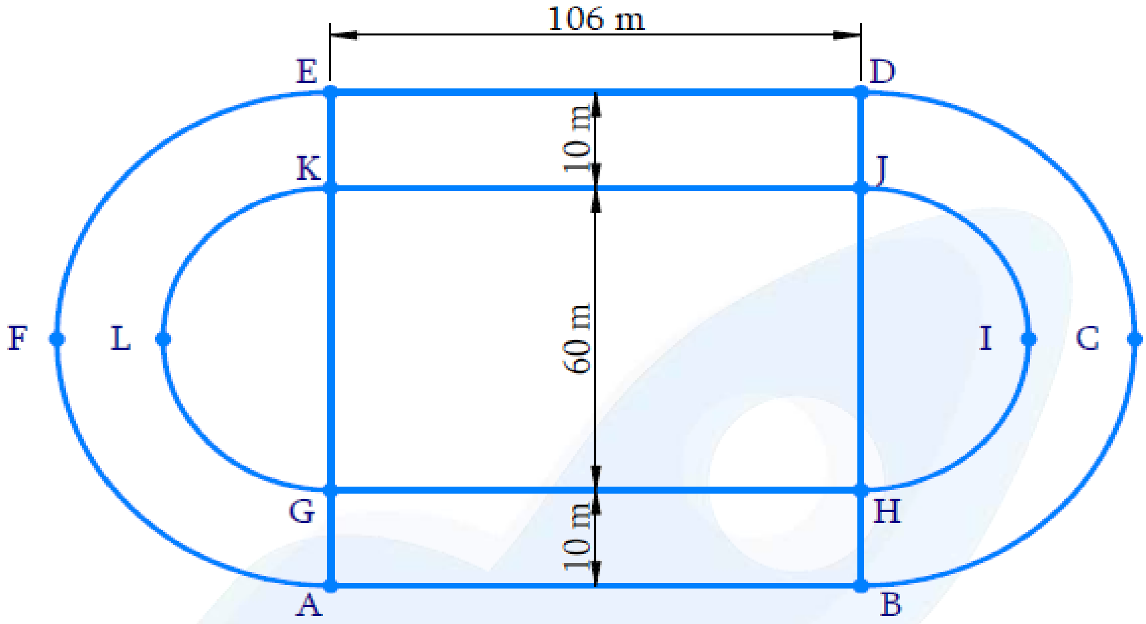
Radii of semicircles BCD and EFA = 30 m + 10 m = 40 m

And JK = GH = 106 m

And DJ = HB = 10 m

Area of the track $= GH \times HB + JK \times DJ + \left(\frac{1}{2}\pi(40)^2 - \frac{1}{2}(30)^2\right) + \left(\frac{1}{2}\pi(40)^2 - \frac{1}{2}(30)^2\right)$

Solution:



Diameter of semicircle HIJ = Diameter of semicircle KLG = 60 m

$$\therefore \text{their radius}(r_1) = \frac{60}{2} = 30\text{m}$$

i. The distance around the track along its inner edge.

$$= GH + \text{arc HIJ} + JK + \text{arc KLG}$$
$$= 106 + \frac{2\pi r_1}{2} + 106 + \frac{2\pi r_1}{2}$$
$$= 106 + \pi \times 30 + 106 + \pi \times 30$$
$$= 212 + \frac{1320}{7}$$
$$= \frac{1484 + 1320}{7}$$
$$= \frac{2804}{7} m$$

ii. Radius of semicircle BCD = Radius of semicircle EFA

$$(r_2) = 30\text{m} + 10\text{m}$$
$$= 40\text{m}$$

Area of the track = Area of rectangle ABHG + Area of rectange KJDE + (Area of semicircle BCD-Area of semicircle HIJ) + (Area of semicircle EFA – Area of semicircle KLG)

$$=(106\times10)+(106\times10)+\left[\frac{1}{2}\pi(40)^2-\frac{1}{2}\pi(30)^2\right]+\left[\frac{1}{2}\pi(40)^2-\frac{1}{2}\pi(30)^2\right]$$

$$=1060+1060+\left[\frac{1}{2}\pi(1600-900)\right]+\left[\frac{1}{2}\pi(1600-900)\right]$$

$$=1060+1060+\frac{\pi}{2}\times700+\frac{\pi}{2}\times700$$

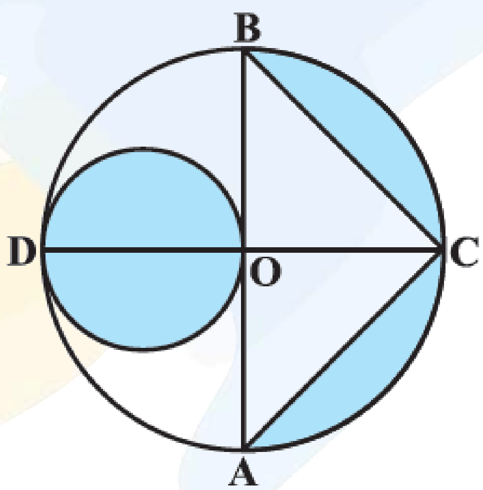
$$=2120+700\pi$$

$$=2120+700\times\frac{22}{7}$$

$$=2120+2200$$

$$=4320m^2$$

Q9. In Figure, AB and CD are two diameters of a circle (with center O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region.



Difficulty Level: Medium

What is the known/given?

AB and CD are two diameters of a circle with centre O, perpendicular to each other that is $\angle BOC = 90^\circ$

OD is diameter of smaller circle, and OA = 7 cm.

What is the unknown?

Area of the shaded region.

Reasoning:

Since AB and CD are diameter of the circle.

$$\therefore OD = OC = OA = OB = R = 7cm \qquad \text{(being radii of the circle)}$$
$$\therefore AB = CD = 2R = 14cm$$

Radius of the shaded smaller circular region $(r) = \frac{7}{2}cm$

Area of the shaded smaller circular region $= \pi\left(\frac{7}{2}\right)^2$

Area of the shaded segment of larger circular region
= Area of semicircle ACB – Area of ΔABC

$$= \frac{\pi}{2}(OA)^2 - \frac{1}{2} \times AB \times OC\left(\because \angle BOC = 90^\circ\right)$$
$$= \frac{\pi}{2}(7)^2 - \frac{1}{2} \times 14 \times 7(AB = OA + OB = 2 \times OA = 14)$$

Area of shaded region = Area of the shaded smaller circular region + Area of the shaded segment of larger circular region

Solution:

OA = 7cm

AB and CD are diameter of the circle with center O

$$\therefore OD = OC = OA = OB = R = 7cm \qquad \text{(being radii of the circle)}$$
$$\therefore AB = 2R = 14cm$$

Radius of shaded circular region, $r = \frac{OD}{2} = \frac{7}{2}cm$

Area of the shaded smaller circular region $= \pi r^2$

$$= \pi\left(\frac{7}{2}cm\right)^2$$
$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}cm^2$$
$$= \frac{77}{2}cm^2$$
$$= 38.5cm^2$$

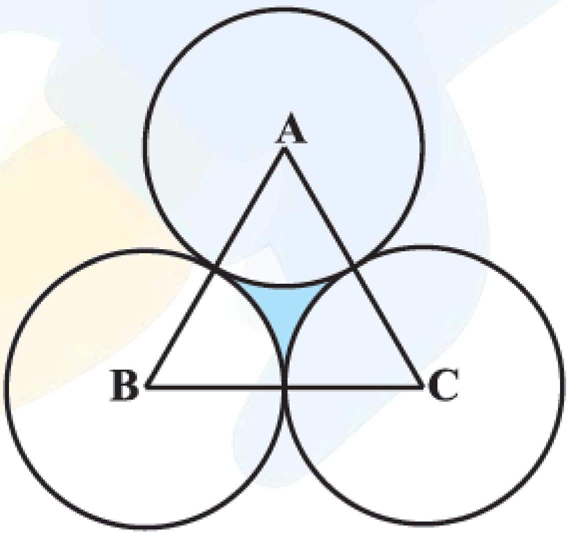
Area of the shaded segment of larger circular region

$$= \text{Area of semicircle ACB} - \text{Area of } \Delta ABC$$
$$= \frac{1}{2}\pi(OA)^2 - \frac{1}{2}\times AB \times OC \left(\because OC \perp AB\right)$$
$$= \frac{1}{2}\pi R^2 - \frac{1}{2}\times 2R \times R$$
$$= \frac{1}{2}\times\frac{22}{7}\times(7cm)^2 - \frac{1}{2}\times 14cm \times 7cm$$
$$= 77cm^2 - 49cm^2$$
$$= 28cm^2$$

Area of shaded region = Area of the shaded smaller circular region + Area of the shaded segment of larger circular region

$$= 38.5cm^2 + 28cm^2$$
$$= 66.5cm^2$$

Q10. The area of an equilateral triangle ABC is 17320.5 cm^2. With each vertex of the triangle as center, a circle is drawn with radius equal to half the length of the side of the triangle (see Figure). Find the area of the shaded region. $\left(\text{Use } \pi = 3.14 \text{ and } \sqrt{3} = 1.73205\right)$



Difficulty Level: Medium

What is the known/given?

(i) Area of an equilateral triangle $\Delta ABC = 17320.5 cm^2$

(ii) with each vertex of a Δ as center a circle is drawn with radius $= \frac{1}{2}$(length of side of ΔABC)

Use $\pi = 3.14$ and $\sqrt{3} = 1.73205$

What is the unknown?

Area of the shaded region.

Reasoning:

i. Since area of triangle is given, we can find side of ΔABC using the formula of area of equilateral

$$\Delta = \frac{\sqrt{3}}{4}(\text{side})^2$$

$$\therefore (\text{side})^2 = \frac{\text{area} \times 4}{\sqrt{3}}$$

ii. Since radius $(r) = \frac{1}{2}(\text{ length of side of } \Delta)$ we can find r.

Also, all angles of an equilateral triangle are equal.

$\therefore$ Angle subtended by each sector $(\theta) = \frac{180^\circ}{3} = 60^\circ$

Using the formula

Area of the sector of angle $\theta = \frac{\theta}{360^\circ} \times \pi r^2$

We can find Area of each sector $= \frac{60^\circ}{360^\circ} \times \pi r^2 = \frac{\pi}{6} r^2$

All sectors are equal as they have same radius r and $\theta = 60^\circ$

iii. Visually from the figure it is clear that:

Area of the shaded region

$= \text{Area of } \Delta ABC - 3 \times \text{Area of each sector}$

$= 17320.5 - 3 \times \frac{\pi r^2}{6}$

$= 17320.5 - \frac{\pi r^2}{2}$

which can be easily solved using $\pi = 3.14$ (as given) and we already know r.

Solution:

1) Area of equilateral $\Delta = 17320.5 cm^2$

$$\frac{\sqrt{3}}{4}(side)^2 = 17320.5cm^2$$

$$(side)^2 = \frac{17320.5 \times 4}{\sqrt{3}} cm^2$$

$$= \frac{17320.5 \times 4}{1.73205} cm^2$$

$$side = \sqrt{10000 \times 4cm^2}$$

$$= 100 \times 2cm$$

$$= 200cm$$

Radius of each sector $(r) = \frac{1}{2} \times (side)$

$$= \frac{1}{2} \times 200cm$$

$$= 100cm$$

All interior angles of an equilateral Δ are of measure 60°

And all 3 sectors are made using these interior angles.

$\therefore$ Angles subtended at the center by each sector $(\theta) = 60^\circ$

Area of each sector $= \frac{\theta}{360^\circ} \times \pi r^2$

Area of 3 sectors $= 3 \times \frac{60^\circ}{360^\circ} \times \pi r^2$

$$= 3 \times \frac{1}{6} \times 3.14 \times (100cm)^2$$

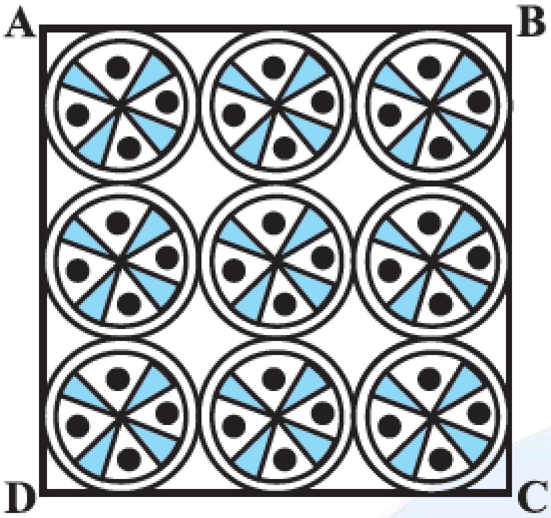
$$= 15700cm^2$$

Area of shaded region = Area of $\Delta ABC -$ Area of 3 sectors

$$= 17320.5cm^2 - 15700cm^2$$

$$= 1620.5cm^2$$

Q11. On a square handkerchief, nine circular designs each of radius 7 cm are made (see Figure). Find the area of the remaining portion of the handkerchief.



Difficulty Level: Medium

What is the known/given?
On a square handkerchief, 9 circular designs each of radius (r) = 7cm are made.

What is the unknown?
Area of the remaining portion of the handkerchief.

Reasoning:
Radius of circular design = 7cm

$\therefore$ Diameter each circular design $= 2 \times 7cm = 14cm$

Visually and logically since all the 3 circular design are touching each other and cover the entire length of the square.

$\therefore$ Side of the square (s) = three times diameter of circular design
$= 3 \times 14cm = 42cm$

Also, visually from the figure it is clear that:

Area of the remaining portion of handkerchief
=Area of square $- 9 \times$ (Area of each circular design)
$= s^2 - 9\left(\pi r^2\right)$
Which can be easily solved since side (s) of square and radius (r) are known.

Solution:
Radius of each circular design, $r = 7cm$

Diameter of each circular design, $2r = 2 \times 7cm = 14cm$

From the figure, it is observed that

Side of the square, $s = 3 \times 14cm = 42cm$

Area of the remaining portion of the handkerchief.

$$= \text{Area of square} - 9 \times \text{(Area of each circular design)}$$

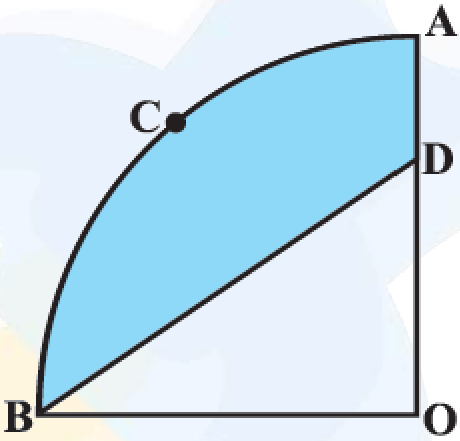
$$= s^2 - 9\pi r^2$$

$$= (42cm)^2 - 9 \times \frac{22}{7} \times (7cm)^2$$

$$= 1764cm^2 - 1386cm^2$$

$$= 378cm^2$$

Q12. In Figure, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the
(i) Quadrant OACB,
(ii) Shaded region**.**



Difficulty Level: Medium

What is the known/given?
OACB is a quadrant of a circle with centre O and radius (r) = 3.5 cm

What is the unknown?
(i) Area of the quadrant OACB
(ii) Area of the shaded region

Reasoning:

i. Since quadrant mean $\frac{1}{4}$th part.

Therefore, angle at the centre of a quadrant of a circle, $\theta = \frac{360^\circ}{4} = 90^\circ$

Area of the quadrant OACB $=\frac{1}{4}\pi r^2$

We can get area of quadrant OACB with radius r = 3.5 cm

ii. Visually from the figure it is clear that

Area of shaded region = Area of quadrant OACB – Area of ΔBDO

Since $\angle BOD = 90^\circ$

$\therefore$ For side OB of ΔBDO, OD is the

Using formula; Area of triangle $=\frac{1}{2}\times \text{base}\times \text{height}$, we can find Area of ΔBDO with base = OB = 3.5 cm (radius of quadrant) and height = OD = 2cm

Solution:

Since OACB is a quadrant, it will subtend $\theta = \frac{360^\circ}{4} = 90^\circ$ angle at O.

Radius, $r = OB = 3.5cm$

Area of quadrant OACB $=\frac{1}{4}\pi r^2$

$$= \frac{1}{4}\times\frac{22}{7}\times(3.5cm)^2$$
$$= \frac{1}{4}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}cm^2$$
$$= \frac{77}{8}cm^2$$

In $\Delta BDO,\ OB = r = 3.5cm = \frac{7}{2}cm$

$OD = 2cm$

$\angle BOD = 90^\circ$

$$\text{Area of } \Delta BDO = \frac{1}{2}\times\ base\ \times height$$
$$= \frac{1}{2}\times OB\times OD$$
$$= \frac{1}{2}\times\frac{7}{2}cm\times 2cm$$
$$= \frac{7}{2}cm^2$$

From figure, it is observed that:

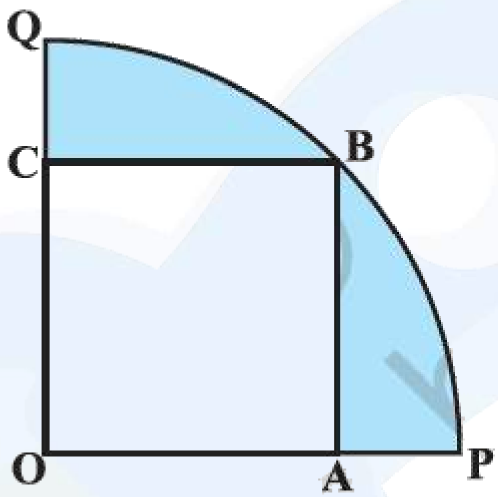
Area of shaded region = Area of Quadrant OACB – Area of ΔBDO

$$= \frac{77}{8} cm^2 - \frac{7}{2} cm^2$$

$$= \frac{77 - 28}{8} cm^2$$

$$= \frac{49}{8} cm^2$$

Q13. In Figure, a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region. (Use $\pi = 3.14$)



Difficulty Level: Medium

What is the known/given?

A square OABC is inscribed in a quadrant OPBQ, OA = 20 cm.

What is the unknown?

Area of the shaded region.

Reasoning:

Visually from the figure it is clear that;

Area of the shaded region = Area of quadrant OPBQ - Area of square OABC

Since side of the square OA = 20 cm

OB = Radius = Diagonal of the square OABC

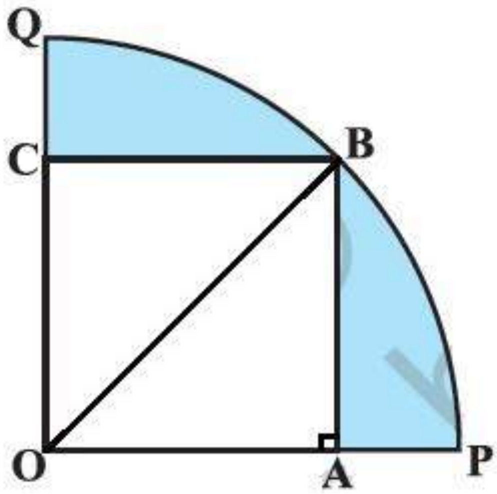
Using formula for

Area of sector of angle $\theta = \frac{\theta}{360^\circ} \times \pi r^2$

With $\theta = 90^\circ$ and $r = OB$

We can find the area of the quadrant OPBQ.

Solution:



Join OB.

We know ΔOBA is a right-angled triangle, as $\angle OAB = 90^\circ$ (angle of a square)

$\therefore$ Using Pythagoras theorem

$$OB^2 = OA^2 + AB^2$$
$$= (20cm)^2 + (20cm)^2$$
$$OB = \sqrt{2 \times (20cm)^2}$$
$$= 20\sqrt{2}cm$$

Therefore, radius of the quadrant, $r = OB = 20\sqrt{2}cm$

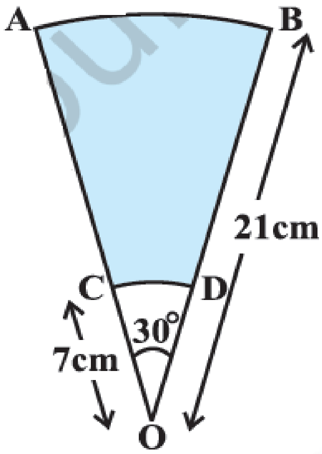
$$\text{Area of quadrant OPBQ} = \frac{90^\circ}{360^\circ} \times \pi r^2$$
$$= \frac{1}{4} \times 3.14 \times (20\sqrt{2}cm)^2$$
$$= \frac{1}{4} \times 3.14 \times 400 \times 2cm^2$$
$$= 628cm^2$$

$$\text{Area of square OABC} = (side)^2$$
$$= (OA)^2$$
$$= (20cm)^2$$
$$= 400cm^2$$

Area of the shaded region = Area of quadrant OPBQ - Area of square OABC

$$= 628cm^2 - 400cm^2$$
$$= 228cm^2$$

Q14. AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and center O (see Figure). If $\angle AOB = 30°$, find the area of the shaded region.



Difficulty Level: Medium

What is the known/given?

AB and CD are arcs of two concentric circle of radii 21 cm and 7 cm respectively and centre O.

$\angle AOB = 30°$

What is the unknown?

Area of the shaded region.

Reasoning:

Area of the shaded region = Area of sector ABO - Area of sector CDO

Areas of sectors ABO and CDO can be found by using the formula of

Area of sector of angle $\theta = \frac{\theta}{360^o} \times \pi r^2$

where r is radius of the circle and angle with degree measure θ

For both the sectors ABO and CDO angle, $\theta = 30^o$ and radii 21cm and 7cm respectively

Solution:

Radius of the sector ABO, $R = OB = 21cm$

Radius of the sector CDO, $r = OD = 7cm$

For both the sectors ABO and CDO angle, $\theta = 90^o$

Area of shaded region = Area of sector ABO – Area of sector CDO

$$= \frac{\theta}{360^o} \times \pi R^2 - \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{\theta}{360^o} \times \pi \left(R^2 - r^2\right)$$

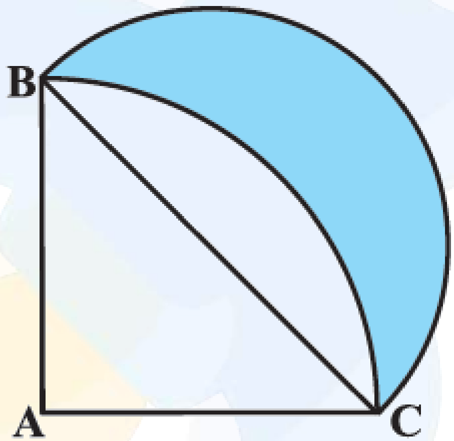
$$= \frac{30^o}{360^o} \times \frac{22}{7}\left((21cm)^2 - (7cm)^2\right)$$

$$= \frac{1}{12} \times \frac{22}{7} \times \left(441cm^2 - 49cm^2\right)$$

$$= \frac{11}{42} \times 392cm^2$$

$$= \frac{308}{3} cm^2$$

Q15. In Figure, ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region.



Difficulty Level: Hard

What is the known/given?

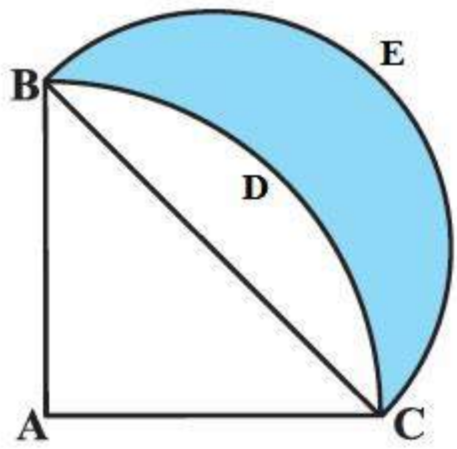
ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter.

What is the unknown?

Area of the shaded region.

Reasoning:

To visualize the shaded region better, mark point D on arc BC of quadrant ABC and E on semicircle drawn with BC as diameter.



From the figure it is clear that

Area of the shaded region = Area of semicircle BEC - Area of segment BDC

- To find area of semicircle BEC, we need to find radius or diameter (BC) of the semicircle.

  ΔABC is a right angled Δ, right angled at A (ABC being a quadrant)

  Using Pythagoras theorem, we can find the hypotenuse (BC)

- To find the area of the segment BDC

  Area of segment BDC = Area of quadrant ABDC - Area of ΔABC

  Area of quadrant ABDC can be found by using the formula

  Area of sector of angle $\theta = \frac{\theta}{360^\circ} \times \pi r^2$

  $$\text{Area of } \Delta ABC = \frac{1}{2} \times \text{ base } \times \text{ height}$$
  $$= \frac{1}{2} \times AC \times AB \left(\because \angle A = 90^\circ\right)$$

Solution:

ΔABC is a right angled Δ, right angled at A

$$BC^2 = AB^2 + AC^2$$
$$= (14cm)^2 + (14cm)^2$$
$$BC = \sqrt{2 \times (14cm)^2}$$
$$= 14\sqrt{2}cm$$

$\therefore$ Radius of semicircle BEC, $r = \frac{BC}{2} = \frac{14\sqrt{2}}{2}cm = 7\sqrt{2}cm$

Area of the shaded region = Area of semicircle BEC - Area of segment BDC
= Area of semicircle BEC – (Area of quadrant ABDC – Area ΔABC)

$$= \frac{\pi r^2}{2} - \left(\frac{90}{360} \times \pi (14)^2 - \frac{1}{2} \times AC \times AB \right)$$

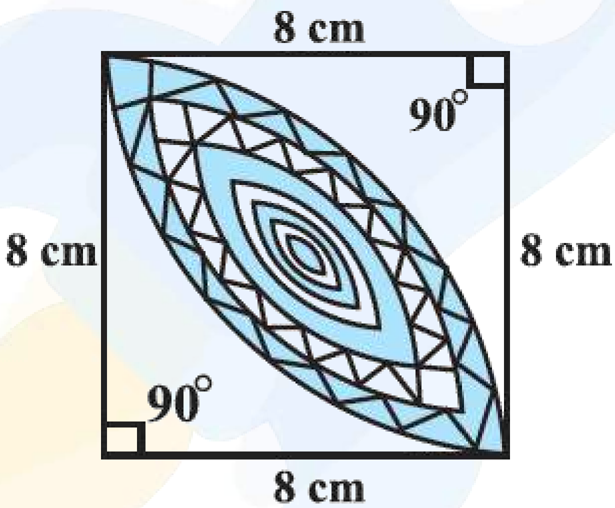
$$= \frac{\pi \left(7\sqrt{2}\right)^2}{2} - \left(\frac{(14)^2 \pi}{4} - \frac{1}{2} \times 14 \times 14 \right)$$

$$= \frac{22 \times 7 \times 7 \times 2}{7 \times 2} - \left(\frac{22 \times 14 \times 14}{7 \times 4} - 7 \times 14 \right)$$

$$= 154 - (154 - 98)$$

$$= 98 cm^2$$

Q16. Calculate the area of the designed region in Figure common between the two quadrants of circles of radius 8 cm each.



Difficulty Level: Hard

What is the known/given?
Designed region is the common area between the 2 quadrants of circles of radius 8 cm.

What is the unknown?
Area of the designed region.

Reasoning:
In a circle with radius *r* and angle at the centre with degree measure θ;

(i) Area of the sector $= \frac{\theta}{360^\circ} \times \pi r^2$

(ii) Area of the segment = Area of the sector – Area of the corresponding triangle

From the figure, it is observed that

Area of the designed region $= 2\times$ Area of the segment of the quadrant of radius 8cm

$= 2\times$[Area of the quadrant – Area of the right triangle]

$$\text{Area of the quadrant} = \frac{90^\circ}{360^\circ}\times \pi r^2 = \frac{1}{4}\pi r^2$$

$$\text{Area of the right-triangle} = \frac{1}{2}\times base \times height$$

Solution:

From the figure, it is observed that

Area of the designed region $= 2\times$ Area of the segment of the quadrant of radius 8cm

$= 2\times$[Area of the quadrant – Area of the right triangle]

$$= 2\times\left[\frac{1}{4}\pi r^2 - \frac{1}{2}\times base\times height\right]$$

$$= 2\times\left[\frac{1}{4}\times\frac{22}{7}\times 8cm\times 8cm - \frac{1}{2}\times 8cm\times 8cm\right]$$

$$= 2\times\left[\frac{352}{7}cm^2 - 32cm^2\right]$$

$$= 2\times\left[\frac{352-224}{7}cm^2\right]$$

$$= 2\times\frac{128}{7}cm^2$$

$$= \frac{256}{7}cm^2$$