Get better at Math. Get better at everything.

Come experience the Cuemath methodology and ensure your child stays ahead at math this summer.

Adaptive Platform | **Interactive Visual Simulations** | **Personalized Attention**

For Grades 1 - 10

LIVE online classes by trained and certified experts.

Get the Cuemath advantage

Book a FREE trial class

Chapter-13: Surface Area and Volumes

Exercise 13.1 (Page 244 of Grade 10 NCERT)

Q1. 2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

Difficulty Level:
Medium

Known/given:
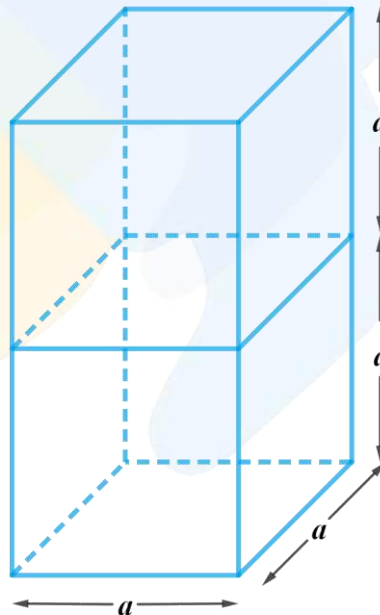Two cubes each of volume 64cm^3 are joined end to end.

Unknown:
Surface area of the resulting cuboid when two cubes are joined end to end.

Reasoning:
We will find the length of the edge of each cube by using the formula for volume of a cube $= a^3$, where length of the edge is a.

As the cubes are joined end to end, they will appear as follows

Using the formula for Surface area of a cuboid $= 2(lb + bh + lh)$, where l, b and h are length, breadth and height respectively. We'll be able to get the answer.

Solution:
Let the length of the edge of each cube is a
Therefore, volume of the cube $= a^3$

volume of the cube, $a^3 = 64cm^3$

$$a^3 = 64cm^3$$
$$a = \sqrt[3]{64cm^3}$$
$$a = \sqrt[3]{(4cm)^3}$$
$$a = 4cm$$

Therefore,

Length of the resulting cuboid, $l = a = 4cm$
Breadth of the resulting cuboid, $b = a = 4cm$
Height of the resulting cuboid, $h = 2a = 2 \times 4cm = 8cm$

Surface area of the resulting cuboid $= 2(lb + bh + lh)$

$$= 2(4cm \times 4cm + 4cm \times 8cm + 4cm \times 8cm)$$
$$= 2(16cm^2 + 32cm^2 + 32cm^2)$$
$$= 2 \times 80cm^2$$
$$= 160cm^2$$

Q2. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.
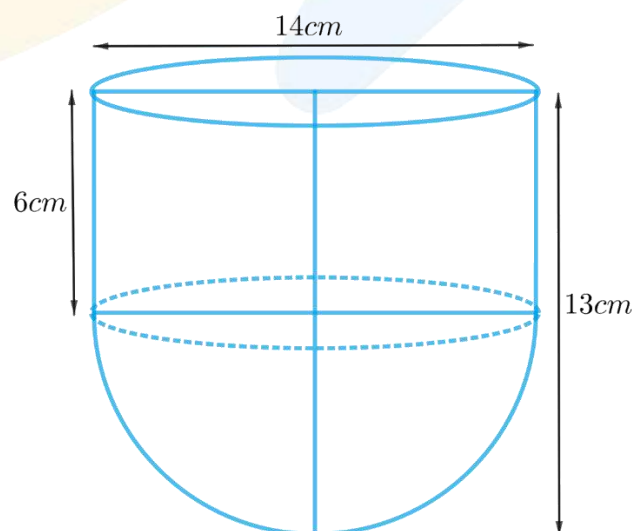
Difficulty Level:
Medium

Known/given:
The diameter of the hemisphere is 14 cm and total height of the vessel is 13 cm.
Unknown:
The inner surface area of the vessel.

Reasoning:
Create a figure of the vessel according to the given description

From the figure it's clear that the inner surface area of the vessel includes the CSA of the hemisphere and the cylinder.

Inner surface area of the vessel = CSA of the hemisphere + CSA of the cylinder

We will find the area of the vessel by using formulae;
CSA of the hemisphere $= 2\pi r^2$
where r is the radius of the hemisphere

CSA of the cylinder $= 2\pi rh$
where r and h are the radius and height of the cylinder respectively.

Height of the cylinder = Total height of the vessel − height of the hemisphere

Solution:
Diameter of the hemisphere, $d = 14cm$
Radius of the hemisphere, $r = \dfrac{14cm}{2} = 7cm$
Height of the hemisphere = radius of the hemisphere, $r = 7cm$

Radius of the cylinder, $r = 7cm$

Height of the cylinder = Total height of the vessel − height of the hemisphere
$h = 13cm - 7cm = 6cm$

Inner surface area of the vessel = CSA of the hemisphere + CSA of the cylinder

$$
\begin{aligned}
&= 2\pi r^2 + 2\pi rh \\
&= 2\pi r(r+h) \\
&= 2 \times \frac{22}{7} \times 7cm(7cm + 6cm) \\
&= 2 \times 22 \times 13cm^2 \\
&= 572cm^2
\end{aligned}
$$

Q3. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.
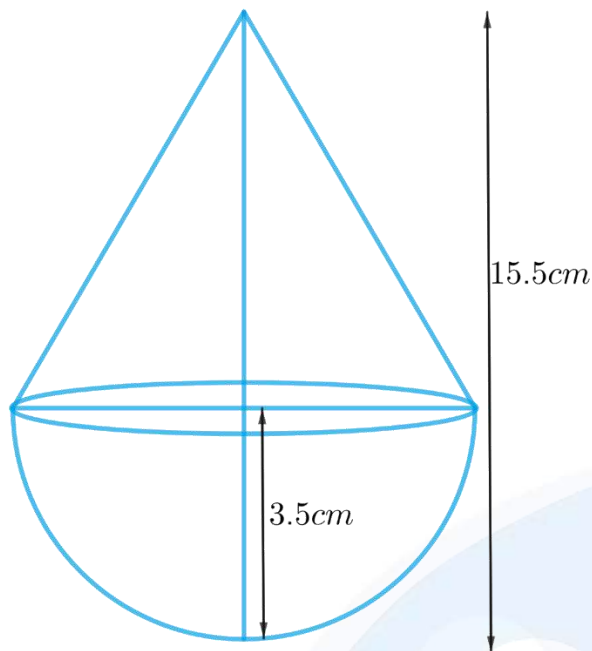
Difficulty Level:
Medium

Known/given:
The toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere with the same radius. The total height of the toy is 15.5 cm.

Unknown:
The total surface area of the toy.

Reasoning:

We can create the figure of the toy as per given information

From the figure it's clear that total surface area of the toy includes CSA of the cone and hemisphere.

Total surface area of the toy = CSA of the hemisphere + CSA of the cone

We will find the total area of the toy by using formulae;

      CSA of the hemisphere $= 2\pi r^2$

            where r is the radius of the hemisphere

      CSA of the cone $= \pi r l$

            where r and l are the radius and slant height of the cone respectively.

      Slant height of the cone, $l = \sqrt{r^2 + h^2}$

      Height of the cone, h = total height of the toy – height of the hemisphere

Solution:

Radius of the hemisphere, $r = 3.5cm$
Height of the hemisphere = radius of the hemisphere, $r = 3.5cm$

Radius of the cone, $r = 3.5cm$

Height of the cone = Total height of the toy – height of the hemisphere
                   $h = 15.5cm - 3.5cm = 12cm$

Slant height of the cone, $l = \sqrt{r^2 + h^2}$

$$l = \sqrt{r^2 + h^2}$$
$$l = \sqrt{(3.5cm)^2 + (12cm)^2}$$
$$l = \sqrt{12.25cm^2 + 144cm^2}$$
$$l = \sqrt{156.25cm^2}$$
$$l = 12.5cm$$

Total surface area of the toy = CSA of the hemisphere + CSA of the cone
$$= 2\pi r^2 + \pi r l$$
$$= \pi r(2r + l)$$
$$= \frac{22}{7} \times 3.5cm \times (2 \times 3.5cm + 12.5cm)$$
$$= \frac{22}{7} \times \frac{7}{2} cm \times (7cm + 12.5cm)$$
$$= 11cm \times 19.5cm$$
$$= 214.5cm^2$$

Q4. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

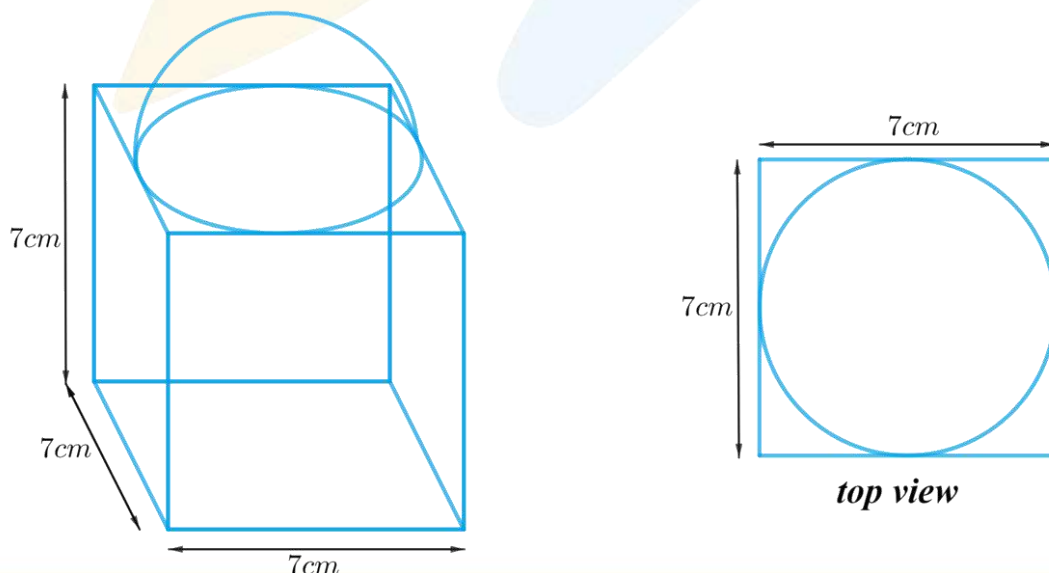**Difficulty Level:**
Hard

Known/given:
A cubical block of side 7 cm is surmounted by a hemisphere.

Unknown:
The greatest diameter the hemisphere can have and the surface area of the solid.

Reasoning:
We can create the figure of the solid as per given information

top view

From the figure it's clear that the greatest diameter the hemisphere can have is equal to the cube's edge. Otherwise a part of hemisphere's rim (circumference of its circular base) will lie outside the top part of the cube.

As it's clear from the top view of the figure that the top part of the cube is partially covered by hemispherical part.

Total surface area of the solid = Surface area of the cubical part – Area of the base of the hemispherical part + CSA of the hemispherical part

We will find the total area of the solid by using formulae;
   CSA of the hemisphere $= 2\pi r^2$
   Area of the base of the hemisphere $= \pi r^2$
      where r is the radius of the hemisphere

   Surface area of the cube $= 6l^2$
      where l is the length of the edge of the cube.

Solution:
Length of the edge of the cube, $l = 7cm$

From the figure it's clear that the greatest diameter the hemisphere can have is equal to the cube's edge
Diameter of the hemisphere, $d = l = 7cm$
Radius of the hemisphere, $r = \dfrac{d}{2} = \dfrac{7}{2}cm$

Total surface area of the solid = Surface area of the cubical part – Area of the base of the hemispherical part + CSA of the hemispherical part

$$= 6l^2 - \pi r^2 + 2\pi r^2$$
$$= 6l^2 + \pi r^2$$
$$= 6\times(7cm)^2 + \dfrac{22}{7}\times\left(\dfrac{7}{2}cm\right)^2$$
$$= 6\times 49cm^2 + \dfrac{22}{7}\times\dfrac{49}{4}cm^2$$
$$= 294cm^2 + 38.5cm^2$$
$$= 332.5cm^2$$

Q5. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Difficulty Level:
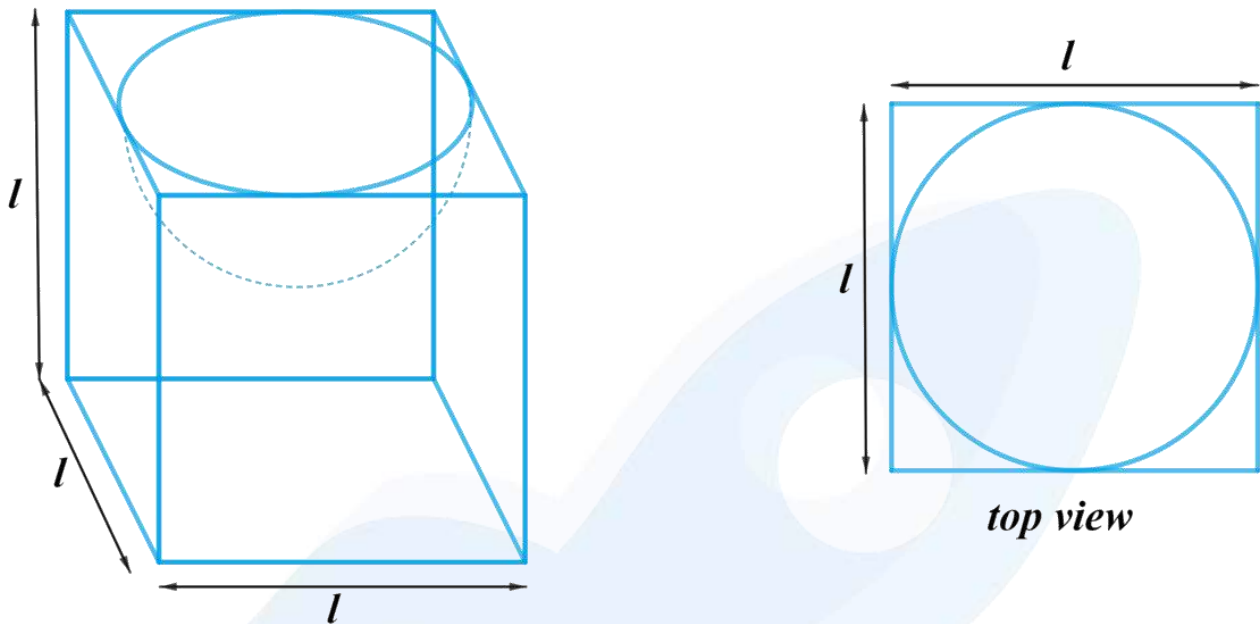Hard

Known/given:

Diameter l of the hemisphere is equal to the edge of the cube.

Unknown:

The surface area of the remaining solid.

Reasoning:

We can create the figure of the solid as per given information

top view

From the figure it's clear that the surface area of the remaining solid includes TSA of the cube, CSA of the hemisphere and excludes base of the hemisphere.

Surface area of the remaining solid = TSA of the cubical part + CSA of the hemispherical part − Area of the base of the hemispherical part

We will find the remaining area of the solid by using formulae;

TSA of the cube $= 6l^2$

where l is the length of the edge of the cube

CSA of the hemisphere $= 2\pi r^2$

Area of the base of the hemisphere $= \pi r^2$

where r is the radius of the hemisphere

Solution:

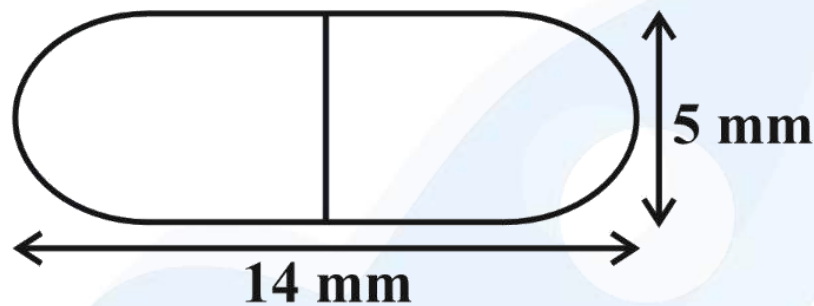Diameter of the hemisphere = Length of the edge of the cube $= l$

Radius of the hemisphere, $r = \dfrac{l}{2}$

Surface area of the remaining solid = TSA of the cubical part + CSA of the hemispherical part − Area of the base of the hemispherical part

$$= 6l^2 + 2\pi r^2 - \pi r^2$$
$$= 6l^2 + \pi r^2$$
$$= 6l^2 + \pi \left(\frac{l}{2}\right)^2$$
$$= 6l^2 + \frac{\pi l^2}{4}$$
$$= \frac{1}{4} l^2 (\pi + 24)$$

Q6. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends (see Fig. 13.10). The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.

Difficulty Level:
Medium

Known/given:
A medicine capsule is in the shape of a cylinder with two hemispheres stuck to its ends.
The length of the entire capsule is 14mm, diameter of the capsule is 5mm.

Unknown:
The surface area of the capsule.

Reasoning:
Since the capsule is in shape of a cylinder with 2 hemispheres stuck to its ends
Diameter of the capsule = diameter of its cylindrical part = diameter of its hemispherical part
From the figure, it's clear that the capsule has the curved surface of two hemispheres and the curved surface of a cylinder.

Surface area of the capsule $= 2 \times$ CSA of hemispherical part + CSA of cylindrical part

We will find the surface area of the capsule by using formulae;
CSA of the hemisphere $= 2\pi r^2$
where r is the radius of the hemisphere
CSA of the cylinder $= 2\pi rh$
where r and h are radius and height of the cylinder respectively.

Length of the cylindrical part = Length of the capsule $-2\times$ radius of the hemispherical part

Solution:
Diameter of the capsule, $d = 5mm$

Radius of the hemisphere, $r = \dfrac{d}{2} = \dfrac{5}{2}mm$

Radius of the cylinder, $r = \dfrac{5}{2}mm$

Length of the cylinder = Length of the capsule $-2\times$ radius of the hemisphere
$$h = 14mm - 2\times\dfrac{5}{2}mm = 9mm$$

Surface area of the capsule $= 2\times$ CSA of hemispherical part $+$ CSA of cylindrical part
$$= 2\times 2\pi r^2 + 2\pi rh$$
$$= 2\pi r\left(2r + h\right)$$
$$= 2\times\dfrac{22}{7}\times\dfrac{5}{2}mm\times\left(2\times\dfrac{5}{2}mm + 9mm\right)$$
$$= \dfrac{110}{7}mm\times 14mm$$
$$= 220mm^2$$

Q7. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of ₹ 500 per m2. (Note that the base of the tent will not be covered with canvas.).
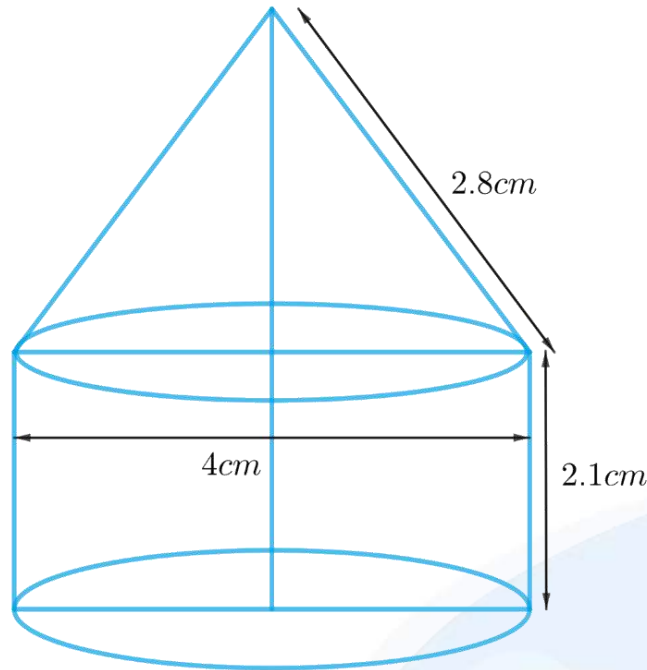
Difficulty Level:
Medium

Known/given:
The height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the conical top is 2.8 m. The rate of the canvas of the tent is ₹ 500 per m^2.

Unknown:
Area and cost of the canvas used for making the tent.

Reasoning:
We can create the figure of the tent as per given information

2.8cm

4cm

2.1cm

From the figure it's clear that, diameter of the cylindrical part is equal to the diameter of the conical part as the cylindrical part is surmounted by the conical part.

Since its given that the base of the tent will not be covered by the canvas then the base of the cylindrical part of the tent is not included to get the area of the canvas required.

Visually, the surface of the tent includes the curved surface of the cylindrical part and curved surface of the conical part only.
Area of the canvas used for making the tent is the surface area of the tent.

Area of the canvas used = CSA of the cylindrical part + CSA of the conical part

We will find the area of the canvas by using formulae;
   CSA of the cone $= \pi r l$
      where r and l are radius and slant height of the cone respectively.
   CSA of the cylinder $= 2\pi r h$
      where r and h are radius and height of the cylinder respectively.

Cost of the canvas of the tent = Area of the canvas $\times$ Rate of the canvas

Solution:
Height of the cylinder, $h = 2.1m$
Diameter of the cylinder, $d = 4m$
Radius of the cylinder, $r = \dfrac{d}{2} = \dfrac{4m}{2} = 2m$

Radius of the cone, $r = 2m$
Slant height of the cone, $l = 2.8m$

Area of the canvas used = CSA of the cylindrical part + CSA of the conical part

$$= 2\pi rh + \pi rl$$
$$= \pi r(2h + l)$$
$$= \frac{22}{7} \times 2m \times (2 \times 2.1m + 2.8m)$$
$$= \frac{44}{7}m \times 7m$$
$$= 44m^2$$

Cost of the canvas of the tent = Area of the canvas × Rate of the canvas
$$= 44m^2 \times ₹\ 500/m^2$$
$$= ₹\ 22000$$

Q8. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2.

Difficulty Level:
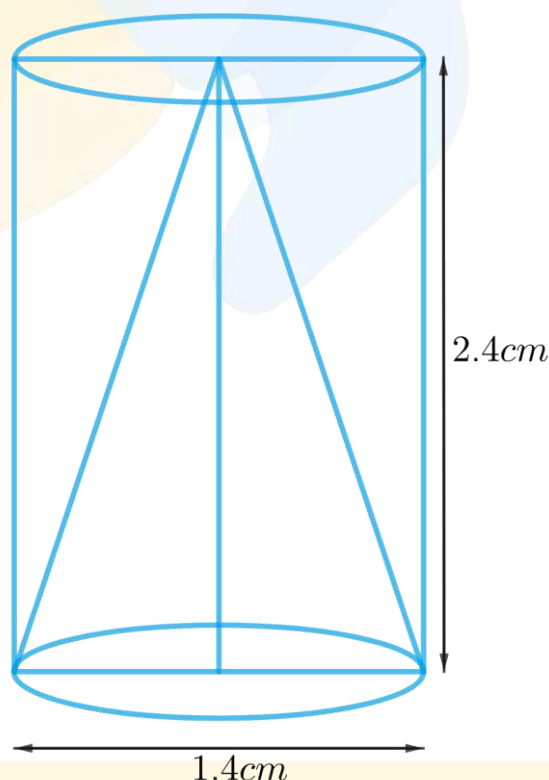Medium

Known/given:
A solid cylinder of height 2.4 cm and diameter 1.4 cm from which a conical cavity of the same height and same diameter is hollowed out.

Unknown:
Total surface area of the remaining solid.

Reasoning:
We can create the figure of the solid as per given information

2.4cm

1.4cm

Since the conical cavity of the same height and diameter has been hollowed out, it's clear that one of the bases of the cylinder is not included in total surface area of the solid.

TSA of the remaining solid = CSA of the cylindrical part + CSA of conical part +Area of the cylindrical base

We will find the area of the remaining solid by using formulae;

CSA of the cylinder $= 2\pi rh$

Area of the base of the cylinder $= \pi r^2$

where r and h are radius and height of the cylinder respectively.

CSA of the cone $= \pi rl$

Slant height of the cone, $l = \sqrt{r^2 + h^2}$

where r, h and l are radius, height and slant height of the cone respectively.

Solution:

Height of the cylinder = Height of the cone $= h = 2.4 cm$

Diameter of the cylinder = diameter of the cone $= d = 1.4 cm$

Radius of the cylinder = radius of the cone $= r = \dfrac{1.4 cm}{2} = 0.7 cm$

Slant height of the cone, $l = \sqrt{r^2 + h^2}$

$$l = \sqrt{(0.7 cm)^2 + (2.4 cm)^2}$$
$$= \sqrt{0.49 cm^2 + 5.76 cm^2}$$
$$= \sqrt{6.25 cm^2}$$
$$= 2.5 cm$$

TSA of the remaining solid = CSA of the cylindrical part + CSA of conical part +Area of the cylindrical base
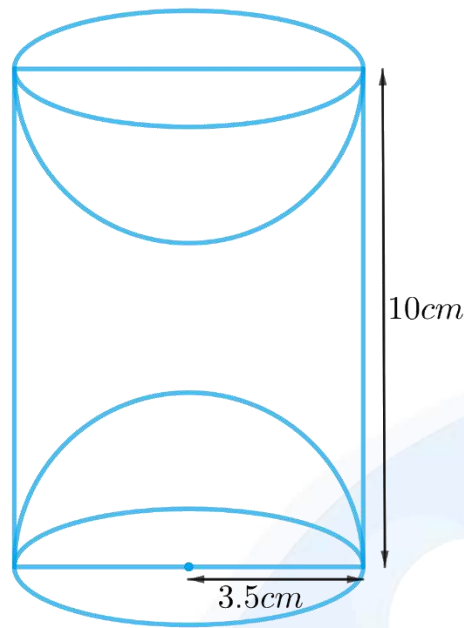
$$= 2\pi rh + \pi rl + \pi r^2$$
$$= \pi r(2h + l + r)$$
$$= \frac{22}{7} \times 0.7 cm \times (2 \times 2.4 cm + 2.5 cm + 0.7 cm)$$
$$= 2.2 cm \times 8 cm$$
$$= 17.6 cm^2$$

Hence, the total surface area of the remaining solid to the nearest cm^2 is 18 cm^2.

Q9. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in Fig. 13.11. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article.

$10cm$

$3.5cm$

Difficulty Level:
Medium

Known/given:
The height of the cylinder is 10 cm, and radius of its base is 3.5 cm

Unknown:
Total surface area of the article.

Reasoning:
From the figure it's clear that radius of the hemispheres scooped out is same as the radius of base of the cylinder since both the hemispheres are of equal radius.
So, total surface area of the article only includes the CSA of both the hemispheres and the cylinder.

TSA of the article = $2\times$CSA of the hemispherical part + CSA of the cylindrical part

We will find the TSA of the article by using formulae;

CSA of the hemisphere $= 2\pi r^2$
where r is the radius of the hemisphere

CSA of the cylinder $= 2\pi rh$
where r and h are radius and height of the cylinder respectively.

Solution:

Height of the cylinder $=h=10cm$

Radius of the cylinder = radius of the hemisphere $=r=3.5cm$

TSA of the article $=2\times$ CSA of the hemispherical part + CSA of the cylindrical part

$$=2\times2\pi r^2+2\pi rh$$

$$=2\pi r\left(2r+h\right)$$

$$=2\times\frac{22}{7}\times3.5cm\times\left(2\times3.5cm+10cm\right)$$

$$=22cm\times17cm$$

$$=374cm^2$$

Chapter-13: Surface Area and Volumes

Exercise 13.2 (Page 247)

Q1. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.
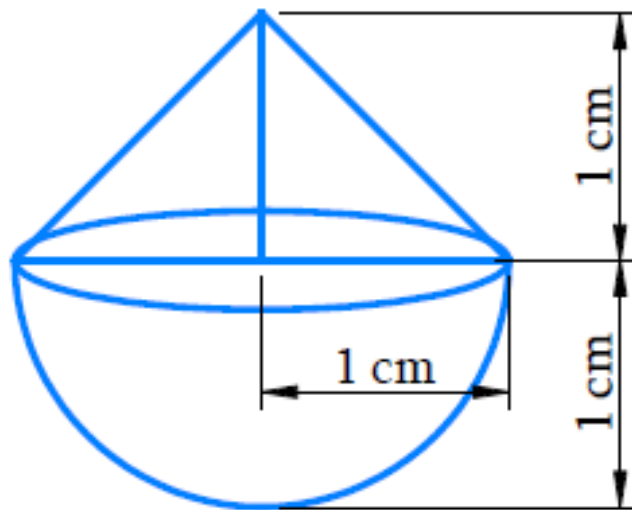
Difficulty Level:
Medium

Known/given:
A solid in the shape of a cone standing on a hemisphere. Radius and height of the cone are same as the radius of the hemisphere which is 1cm.

Unknown:
The volume of the solid.

Reasoning:
Draw a figure of the solid to visualize it

Since the solid is made up of conical part and a hemispherical part.
Volume of the solid = volume of the conical part + volume of the hemispherical part

We will find the volume of the solid by using formulae;

Volume of the hemisphere $= \dfrac{2}{3} \pi r^3$

where r is the radius of the hemisphere

Volume of the cone $= \dfrac{1}{3} \pi r^2 h$

where r and h are the radius and height of the cone respectively.

Solution:

Radius of hemispherical part = Radius of conical part $= r = 1cm$

Height of conical part $= h = r = 1cm$

Volume of the solid = volume of the conical part + volume of the hemispherical part

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi r^3 + \frac{2}{3}\pi r^3$$

$$= \pi r^3$$

$$= \pi (1cm)^3$$

$$= \pi cm^3$$

Q2. Rachel, an engineering student, was asked to make a model shaped like a Cylinder with two cones attached at its two ends by using a thin aluminum Sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.)

Difficulty Level:

Medium

Known/given:

Rachel made a model of aluminum sheet shaped like a cylinder with 2 cones attached to its 2 ends.

Diameter of the model = 3cm

Length of the model = 12cm
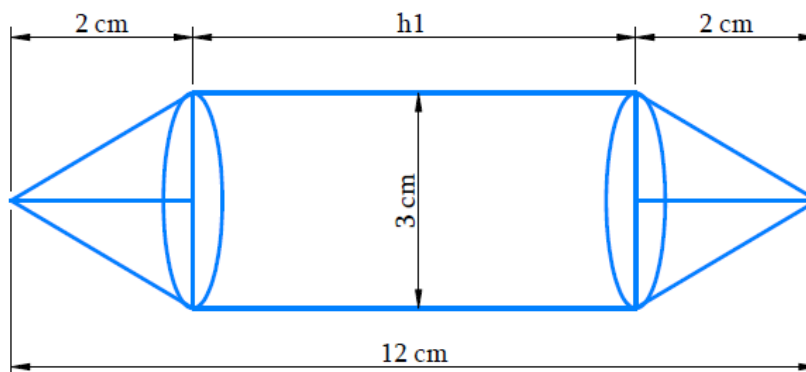
Height of each cone = 2cm

The internal and external dimensions of model are same.

Unknown:

Volume of air contained in the model made by Rachel.

Reasoning:

Draw a figure according to given data to visualize the model.

From the figure it's clear that the volume of the model includes volume of the cylindrical part and both the identical conical parts.

Volume of the model = Volume of the 2 conical parts + volume of the cylindrical part

Hence, we work on the assumption that the outer and inner dimensions of the model are nearly the same.
Therefore, Volume of air in the model = volume of the model based on outer dimensions

Since, length of the model includes height of the cylindrical part and height of both the identical conical parts and also diameter of the cylindrical part and conical parts are same as the diameter of the model.

Length of the model = Height of the cylindrical part $+ 2 \times$ Height of the conical part

We will find the volume of the model by using formulae;
    Volume of the cylinder $= \pi r^2 h_1$

        where r and h_1 are the radius and height of the cylinder respectively.
    Volume of the cone $= \dfrac{1}{3} \pi r^2 h_2$

        where r and h_2 are the radius and height of the cone respectively.

Solution:
Height of each conical part, $h_2 = 2 cm$
Height of cylindrical part = Length of the model $- 2 \times$ Height of the conical part
$$h_1 = 12 cm - 2 \times 2 cm$$
$$= 8 cm$$

Diameter of the model, $d = 3 cm$

Radius of cylindrical part = radius of conical part $= r = \dfrac{3 cm}{2} = 1.5 cm$

Volume of the model $= 2 \times$ Volume of the conical part + volume of the cylindrical part
$$= 2 \times \dfrac{1}{3} \pi r^2 h_2 + \pi r^2 h_1$$
$$= \pi r^2 \left(\dfrac{2}{3} h_2 + h_1 \right)$$
$$= \dfrac{22}{7} \times 1.5 cm \times 1.5 cm \times \left(\dfrac{2}{3} \times 2 cm + 8 cm \right)$$
$$= \dfrac{22}{7} \times 1.5 cm \times 1.5 cm \times \dfrac{28}{3} cm$$
$$= 66 cm^3$$

Volume of air in the model is $66 cm^3$.

Q3. A Gulab jamun contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 Gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm (see Fig. 13.15).

Difficulty Level:
Medium

Known/given:
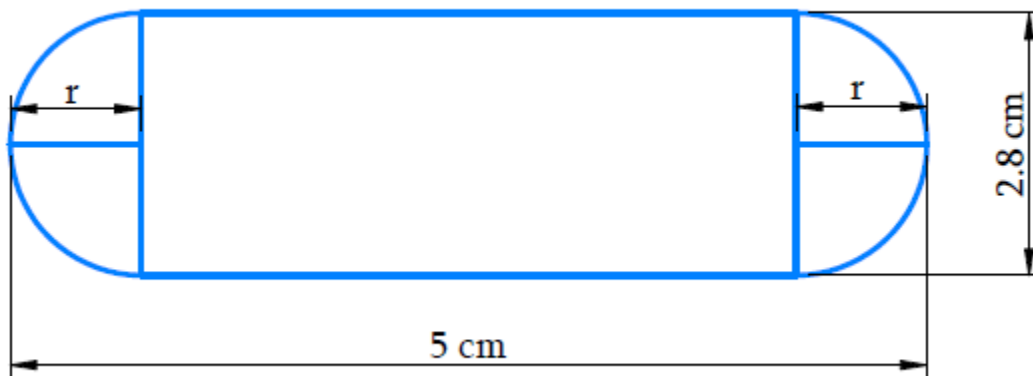Each Gulab jamun's shape is like a cylinder with 2 hemispherical ends with length 5cm and diameter 2.8cm.
Each Gulab jamun contains sugar syrup up to about 30% of its volume.

Unknown:
Volume of sugar syrup in 45 Gulab jamuns (approximately)

Reasoning:
Draw the figure to visualize the shape of Gulab jamun

From the figure it's clear that
Length of cylindrical part = length of a Gulab jamun − 2×radius of the hemispherical part

Also,
Diameter of Gulab jamun = diameter of cylindrical part

Radius of cylindrical part = radius of hemispherical part

In order to find volume of sugar syrup in 45 Gulab jamun we find volume of Gulab jamun

Using the statement

The volume of the solid formed by joining 2 basic solids will actually be sum of the volumes of its constituents.

Volume of 1 Gulab jamun = volume of cylindrical part + volume of the 2 hemispherical parts

Since, it's given a Gulab jamun contains sugar syrup up to about 30% of its volume. We make an assumption to every Gulab jamun contains sugar syrup at 30% of its volume to simplify calculation

Volume of sugar syrup = 30% of volume of 45 Gulab jamun.

Solution:

Diameter of the Gulab jamun, $d = 2.8cm$

Radius of cylindrical part = radius of hemispherical part $= r = \dfrac{2.8cm}{2} = 1.4cm$

Length of cylindrical part, $h = 5cm - 2 \times 1.4cm = 2.2cm$

Volume of 1 Gulab jamun = volume of cylindrical part $+ 2 \times$ volume of the hemispherical parts

$$= \pi r^2 h + 2 \times \frac{2}{3}\pi r^3$$

$$= \pi r^2 h + \frac{4}{3}\pi r^3$$

$$= \pi r^2 \left(h + \frac{4}{3}r \right)$$

$$= \frac{22}{7} \times 1.4cm \times 1.4cm \times \left(2.2cm + \frac{4}{3} \times 1.4cm \right)$$

$$= \frac{22}{7} \times 1.4cm \times 1.4cm \times \frac{122.2}{3}cm$$

$$= \frac{75.152}{3}cm^3$$

Volume of 45 Gulab jamuns $= 45 \times$ volume of 1 Gulab jamun

$$= 45 \times \frac{75.152}{3}cm^3$$

$$= 15 \times 75.152cm^3$$

$$= 1127.28cm^3$$

Volume of sugar syrup in 45 Gulab jamuns $= 30\%$ of volume of 45 Gulab jamun

$$= \frac{30}{100} \times 1127.28cm^3$$

$$= 338.184cm^3$$

$$= 338\ cm^3 (\text{approximately})$$

Q4. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5cm and the depth is 1.4 cm. Find the volume of wood in the entire stand (see Fig. 13.16).
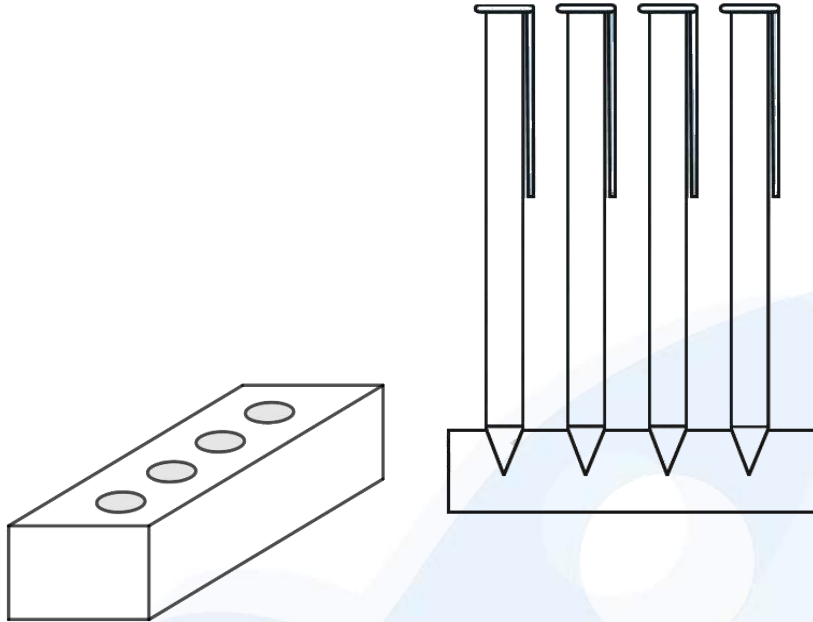
Difficulty Level:
Medium

Known/given:
A wooden pen stands in the shape of a cuboid with four conical depressions.
The dimensions of the cuboid are $15 cm \times 10 cm \times 3.5 cm$
Radius of conical depressions is 0.5 cm
Depth of conical depression is 1.4cm

Unknown:
Volume of wood in the entire pen stand.

Reasoning:
From the given figure it's clear that the conical depressions do not contain wood. Since the dimensions of all 4 conical depressions are the same, they will have identical volumes too.

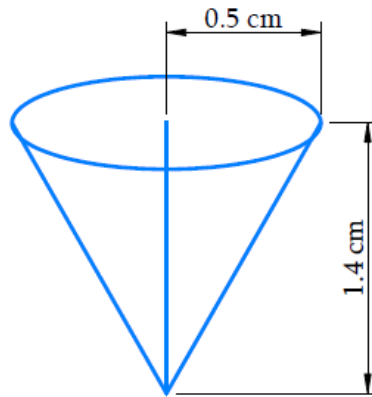Volume of wood in the entire pen stand = volume of the wooden cuboid − 4×volume of the conical depression

We will find the volume of the solid by using formulae;

Volume of the cuboid $= lbh$
where *l, b* and *h* are the length, breadth and height of the cuboid respectively.

Volume of the cone $=\dfrac{1}{3}\pi r^2 h_1$

where r and h_1 are the radius and height of the cone respectively

0.5 cm

1.4 cm

Solution:

Depth of each conical depression, $h_1 = 1.4cm$
Radius of each conical depression, $r = 0.5cm$
Dimensions of the cuboid are $15cm \times 10cm \times 3.5cm$

Volume of wood in the entire pen stand = volume of the wooden cuboid $-$ $4 \times$ volume of the conical depression

$$= lbh - 4 \times \dfrac{1}{3}\pi r^2 h_1$$

$$= \left(15cm \times 10cm \times 3.5cm\right) - \left(4 \times \dfrac{1}{3} \times \dfrac{22}{7} \times 0.5cm \times 0.5cm \times 1.4cm\right)$$

$$= 525cm^3 - 1.47cm^3$$

$$= 523.53cm^3$$

Q5. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Difficulty Level:
Medium

Known/given:
An open vessel in the form of inverted cone with height 8cm and radius of its open top is 5cm, which is filled with water up to the drum.
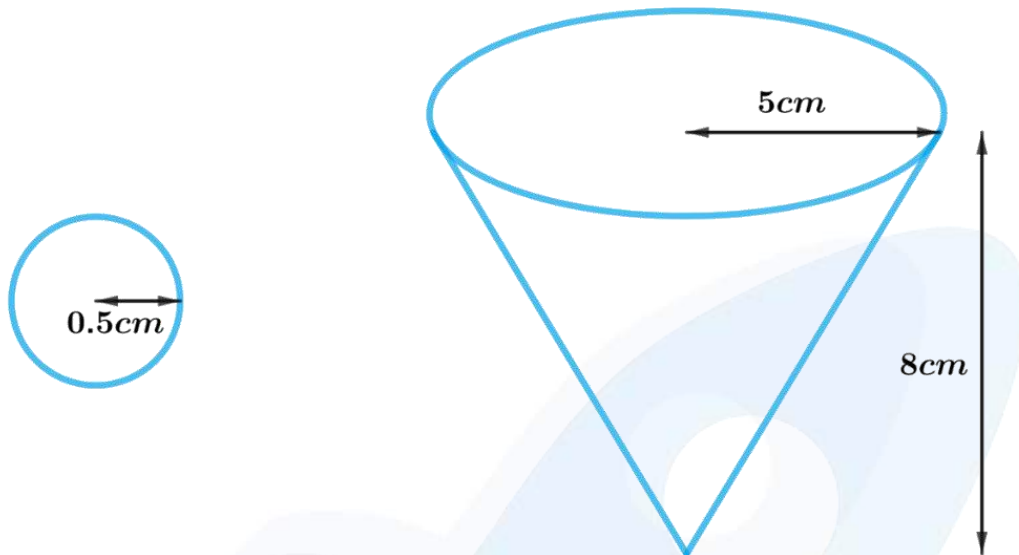
Spherical lead shots, each of radius 0.5 cm, are dropped into the vessel then one-fourth of the water in the vessel flows out.

Unknown:

Number of lead shot dropped into the vessel.

Reasoning:

Draw a figure of the vessel and lead shot to visualize it

Since the water is filled up to the brim in the vessel

Volume of water in the vessel = volume of the conical vessel.

Also, on dropping a certain number of lead shots (sphere) into the vessel one-fourth of the water flows out.

Volume of all lead shots dropped into the vessel $= \dfrac{1}{4} \times$ Volume of the water in the vessel

Hence,

Number of lead shots $= \dfrac{1}{4} \times$ volume of the water in the vessel $\div$ volume of each lead shot

We will find the volume of the water in the vessel and lead shot by using formulae;

$\qquad$ Volume of the sphere $= \dfrac{4}{3}\pi r^3$

$\qquad\qquad$ where r is the radius of the hemisphere

$\qquad$ Volume of the cone $= \dfrac{1}{3}\pi R^2 h$

$\qquad\qquad$ where R and h are the radius and height of the cone respectively

Solution:

Height of the conical vessel, $h = 8 cm$
Radius of the conical vessel, $R = 5 cm$
Radius of the spherical lead shot, $r = 0.5 cm$

Number of lead shots $= \dfrac{1}{4} \times$ volume of the water in the vessel $\div$ volume of each lead shot

$$= \left(\dfrac{1}{4} \times \dfrac{1}{3} \pi R^2 h \right) \div \left(\dfrac{4}{3} \pi r^3 \right)$$

$$= \dfrac{\pi R^2 h}{12} \times \dfrac{3}{4 \pi r^3}$$

$$= \dfrac{R^2 h}{16 r^3}$$

$$= \dfrac{5cm \times 5cm \times 8cm}{16 \times 0.5cm \times 0.5cm \times 0.5cm}$$

$$= 100$$

Q6. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm³ of iron has approximately 8g mass. (Use π = 3.14)
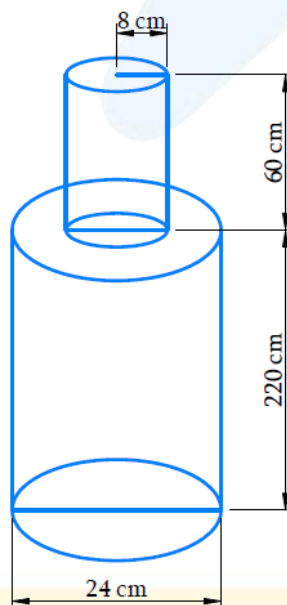
Difficulty Level:
Medium

Known/given:
A solid iron pole consisting of a cylinder of height 220cm and base diameter 24cm which is surmounted by another cylinder of height 60cm and radius 8cm
Mass of 1cm³ iron is 8g

Unknown:
The mass of the solid iron pole

Reasoning:
Draw the figure to visualize the iron pole

Visually it's clear that

Volume of the solid iron pole = volume of larger cylinder + volume of smaller cylinder

Mass of iron in the pole = 8g × volume of the solid iron pole in cm³

We will find the volume of the solid by using formula;

Volume of the cylinder $= \pi r^2 h$

where r and h are the radius and height of the cylinder respectively.

Solution:

Radius of larger cylinder, $R = \dfrac{24cm}{2} = 12cm$

Height of larger cylinder, $H = 220cm$

Radius of smaller cylinder, $r = 8cm$

Height of smaller cylinder, $h = 60cm$

Volume of the solid iron pole = volume of larger cylinder + volume of smaller cylinder

$$= \pi R^2 H + \pi r^2 h$$
$$= \pi \left(12cm \times 12cm \times 220cm + 8cm \times 8cm \times 60cm\right)$$
$$= 3.14 \times \left(31680cm^3 + 3840cm^3\right)$$
$$= 3.14 \times 35520cm^3$$
$$= 111532.8cm^3$$

Mass of 1cm³ iron is 8g

Mass of iron in the pole = 8g × volume of the solid iron pole in cm³

$$= 8g \times 111532.8$$
$$= 892262.4g$$
$$= \dfrac{892262.4}{1000}kg$$
$$= 892.2624kg$$

Mass of iron in the pole is 892.26 kg

Q7. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

Difficulty Level:
Medium

Known/given:

A solid consisting of a right circular cone of height 120cm and radius 60cm standing on a hemisphere with the same radius is placed upright in a right circular cylinder of radius 60cm and height 180cm, full of water such that it touches the bottom.

Unknown:

Volume of water left in the cylinder.

Reasoning:

Draw a figure in accordance with the given information to visualize effectively.
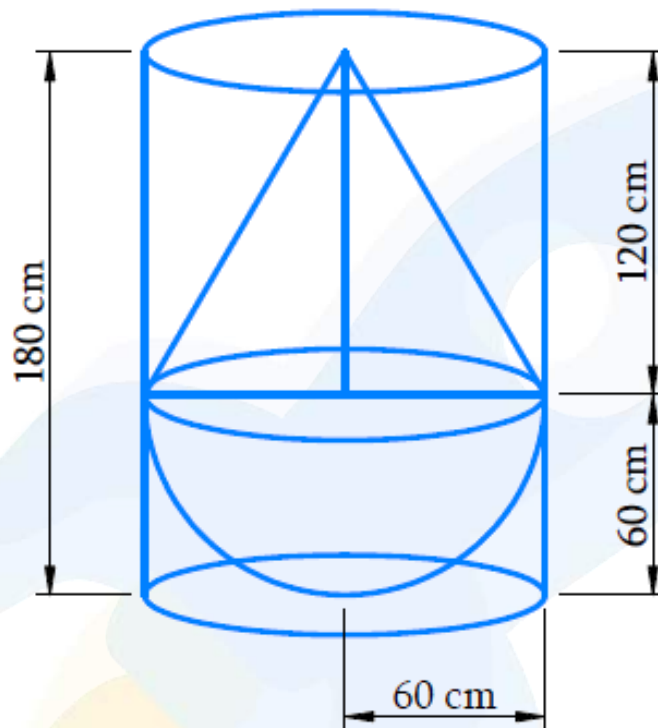
From the figure, we observe that the solid fits totally into the cylinder as the height of the solid is same as the height of the cylinder

Since the cylinder was full of water before immersing the solid into it, so the entire space in the cylinder is occupied by the solid and the left-over water.

Volume of the cylinder = volume of the solid + volume of the water left in the cylinder

Therefore,

Volume of the water left in the cylinder = volume of the cylinder – volume of the solid

Hence,

Volume of the water left in the cylinder = volume of the cylinder – volume of the hemisphere – volume of the cone

We will find the volume of the solid by using formulae;

Volume of the hemisphere $= \dfrac{2}{3}\pi r^3$

where r is the radius of the hemisphere

Volume of the cone $= \dfrac{1}{3}\pi r^2 h$

where r and h are the radius and height of the cone respectively.

Volume of the cylinder $= \pi r^2 h$

where r and h are radius and height of the cylinder respectively.

Solution:

Radius of hemispherical part = Radius of conical part = Radius of cylinder $= r = 60 cm$
Height of the conical part of solid $= h = 120 cm$
Height of the cylinder, $= H = 180 cm$

Volume of the water left in the cylinder = volume of the cylinder – volume of the hemisphere – volume of the cone

$$= \pi r^2 H - \dfrac{1}{3}\pi r^2 h - \dfrac{2}{3}\pi r^3$$

$$= \dfrac{1}{3}\pi r^2 \left(3H - h - 2r\right)$$

$$= \dfrac{1}{3} \times \dfrac{22}{7} \times 60cm \times 60cm \times \left(3 \times 180cm - 120cm - 2 \times 60cm\right)$$

$$= \dfrac{1}{3} \times \dfrac{22}{7} \times 60cm \times 60cm \times 300cm$$

$$= \dfrac{7920000}{7} cm^3$$

$$= 1131426.57 cm^3$$

$$= \dfrac{1131426.57}{1000000} m^3$$

$$= 1.13142657 m^3$$

Volume of water left in the cylinder is 1.131 m^3

Q8. A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurements, and $\pi = 3.14$.

Difficulty Level:
Medium

Known/given:
A spherical glass vessel with a cylindrical neck of length 8cm and diameter 2cm
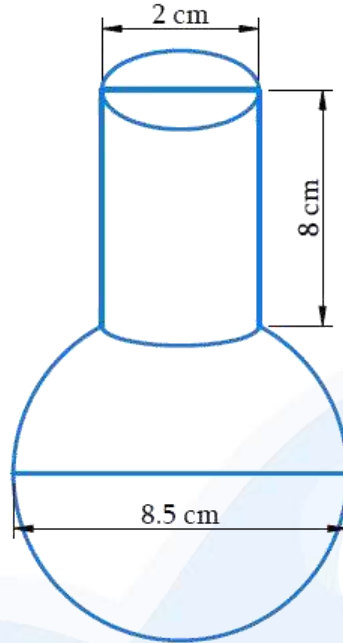The diameter of the spherical part = 8.5cm
According to the child volume of the vessel 345cm^3

Unknown:

Whether the child is correct in saying the volume of the glass vessel is 345cm³

Reasoning:

Draw a figure to visualize the glass vessel

Since the glass vessel is a combination of a sphere and a cylinder

Volume of the glass vessel = volume of the spherical part + volume of the cylindrical part

Solution:

Height of cylindrical part, $h = 8cm$

Radius of cylindrical part, $r_1 = \dfrac{2cm}{2} = 1cm$

Radius of spherical part, $r_2 = \dfrac{8.5cm}{2} = 4.25cm$

Volume of the glass vessel = volume of the spherical part + volume of the cylindrical part

$$= \frac{4}{3}\pi r_2^3 + \pi r_1^2 h$$

$$= \pi\left(\frac{4}{3}r_2^3 + r_1^2 h\right)$$

$$= 3.14 \times \left(\frac{4}{3} \times 4.25cm \times 4.25cm \times 4.25cm + 1cm \times 1cm \times 8cm\right)$$

$$= 3.14 \times \left(102.354cm^3 + 8cm^3\right)$$

$$= 3.14 \times 110.354cm^3$$

$$= 346.51cm^3$$

Therefore, the child is not correct in saying that the volume of the glass vessel is 345cm³

Chapter-13: Surface Area and Volumes

Exercise 13.3 (Page 251 of Grade 10 NCERT)

Q1. A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

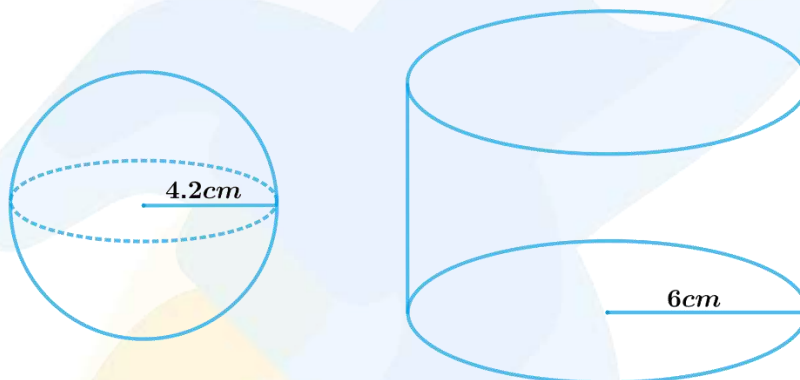**Difficulty Level:**
Medium

Known/given:
Radius of the metallic sphere is 4.2 cm and radius of the cylinder is 6 cm

Unknown:
The height of the cylinder

Reasoning:
Draw a figure to visualize the shapes better

Since, a metallic sphere is melted and recast into the shape of a cylinder then their volume must be same.
Volume of the sphere = Volume of the cylinder

We will find the volume of the sphere and cylinder by using formulae;

Volume of the sphere $= \dfrac{4}{3} \pi r^3$

where r is the radius of the sphere
Volume of the cylinder $= \pi r^2 h$
where r and h are radius and height of the cylinder respectively

Solution:

Radius of the hemisphere, $r_1 = 4.2 cm$

Radius of the cylinder, $r_2 = 6 cm$

Let the height of the cylinder be h.

Volume of sphere = Volume of cylinder

$$\frac{4}{3}\pi r_1^3 = \pi r_2^2 h$$

$$\frac{4}{3} r_1^3 = r_2^2 h$$

$$h = \frac{4r_1^3}{3r_2^2}$$

$$= \frac{4 \times 4.2cm \times 4.2cm \times 4.2cm}{3 \times 6cm \times 6cm}$$

$$= 2.74\,cm$$

Hence, the height of the cylinder so formed will be 2.74 cm.

Q2. Metallic spheres of radii 6 cm, 8 cm, and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

Difficulty Level:
Medium

Known/given:
Radii of three metallic spheres are 6 cm, 8 cm, and 10 cm.
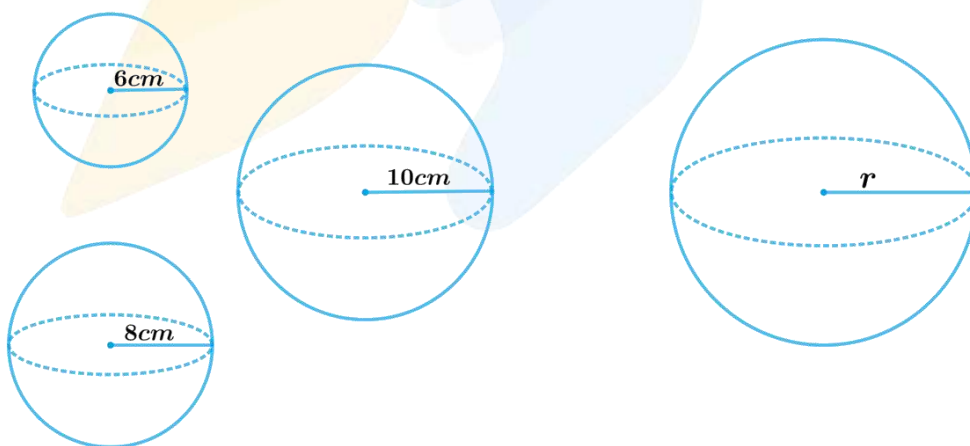
Unknown:
The radius of the resulting sphere

Reasoning:
Draw a figure to visualize the shapes better

Since, three metallic spheres are melted and recast into a single solid sphere then the sphere formed by recasting these spheres will be same in volume as the sum of the volumes of these spheres.

Volume of the resulting sphere = Sum of the volumes of three spheres

We will find the volume of the sphere by using formula;

Volume of the sphere $= \dfrac{4}{3}\pi r^3$

where r is the radius of the sphere

Solution:

Radius of 1ˢᵗ sphere, $r_1 = 6\,cm$

Radius of 2ⁿᵈ sphere, $r_2 = 8\,cm$

Radius of 3ʳᵈ sphere, $r_3 = 10\,cm$

Let the radius of the resulting sphere be r.

Volume of the resulting sphere = Sum of the volumes of three spheres

$$\dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi r_1^3 + \dfrac{4}{3}\pi r_2^3 + \dfrac{4}{3}\pi r_3^3$$

$$\dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi\left[r_1^3 + r_2^3 + r_3^3 \right]$$

$$r^3 = \left[r_1^3 + r_2^3 + r_3^3 \right]$$

$$r^3 = \left[(6cm)^3 + (8cm)^3 + (10cm)^3 \right]$$

$$r^3 = \left[216cm^3 + 512cm^3 + 1000cm^3 \right]$$

$$r^3 = 1728cm^3$$

$$r = 12cm$$

Therefore, the radius of the sphere so formed will be 12 cm.

Q3. A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform. $\left[\text{Use } \pi = \dfrac{22}{7} \right]$
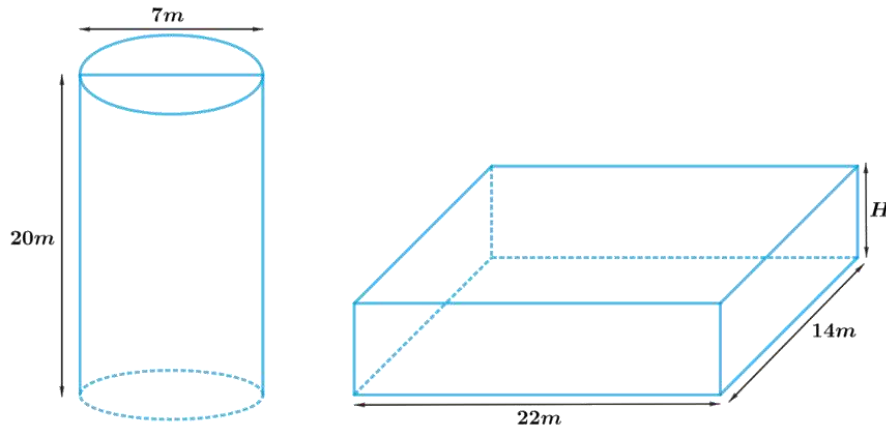
Difficulty Level:
Medium

Known/given:
Depth of the well is 20m and diameter is 7m. Length of the platform is 22m and breadth is 14m.

Unknown:
The height of the platform

Reasoning:
Draw a figure to visualize the shapes better

The shape of the well will be cylindrical, and soil evenly spread out to form a platform will be in a cuboidal shape.

Therefore, volume of the soil dug from the well will be equal to the volume of soil evenly spread out to form a platform.

Volume of soil dug out from the well = Volume of soil used to make such platform

Hence, Volume of the cylindrical well = Volume of the cuboidal platform

We will find the volume of the cylinder and cuboid by using formulae;

Volume of the cylinder $= \pi r^2 h$

where r and h are radius and height of the cylinder respectively

Volume of the cuboid $= lbH$

where l, b and H are length breadth and height of the cuboid respectively

Solution:
Depth of the cylindrical well, $h = 20m$

Radius of the cylindrical well, $r = \dfrac{7}{2}m$

Length of the cuboidal platform, $l = 22m$
Breadth of the cuboidal platform, $b = 14m$

Let the height of the cuboidal platform $= H$

Volume of the cylindrical well = Volume of the cuboidal platform

$$\pi r^2 h = lbH$$

$$H = \dfrac{\pi r^2 h}{lb}$$

$$= \dfrac{\dfrac{22}{7} \times \dfrac{7}{2}m \times \dfrac{7}{2}m \times 20m}{22m \times 14m}$$

$$= \dfrac{5}{2}m$$

$$= 2.5m$$

Therefore, the height of such platform will be 2.5 m.

Q4. A well of diameter 3m is dug 14m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4m to form an embankment. Find the height of the embankment.

Difficulty Level:
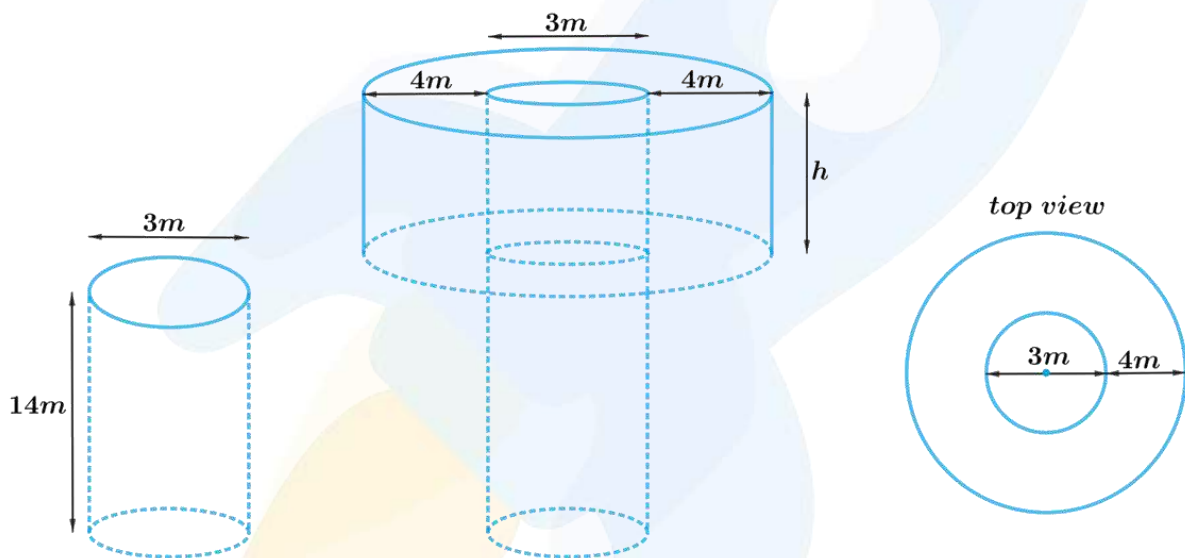Medium

Known/given:
Depth of the well is 14m and diameter is 3m. Width of the circular ring of the embankment is 4m.

Unknown:
The height of the embankment

Reasoning:
Draw a figure to visualize the shapes better

From the figure, it can be observed that the shape of the well will be cylindrical, and earth evenly spread out to form an embankment around the well in a circular ring will be in a cylindrical shape (Hollow cylinder) having outer and inner radius.

Volume of the earth taken out from well = Volume of the earth used to form embankment

Hence, Volume of the cylindrical well = Volume of the hollow cylindrical embankment

We will find the volume of the hollow cylindrical embankment by subtracting volume of inner cylinder from volume of the outer cylinder.

Volume of the cylinder $= \pi r^2 h$
where r and h are the radius and height of the cylinder respectively.

Solution:

Depth of the cylindrical well, $h_1 = 14m$

Radius of the cylindrical well, $r = \dfrac{3}{2}m = 1.5m$

Width of embankment $= 20m$

Inner radius of the embankment, $r = \dfrac{3}{2}m$

Outer radius of the embankment, R = Inner radius + Width

$$R = 1.5m + 4m$$
$$= 5.5m$$

Let the height of embankment be h

Volume of the cylindrical well = Volume of the hollow cylindrical embankment

$$\pi r^2 h_1 = \pi R^2 h - \pi r^2 h$$
$$\pi r^2 h_1 = \pi h \left(R^2 - r^2 \right)$$
$$r^2 h_1 = h \left(R - r \right) \left(R + r \right)$$
$$h = \frac{r^2 h_1}{\left(R - r \right) \left(R + r \right)}$$
$$= \frac{\left(1.5m \right)^2 \times 14m}{\left(5.5m - 1.5m \right) \left(5.5m + 1.5 \right)}$$
$$= \frac{2.25m^2 \times 14m}{4m \times 7m}$$
$$= 1.125m$$

Therefore, the height of the embankment will be 1.125 m.

Q5. A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.

Difficulty Level:
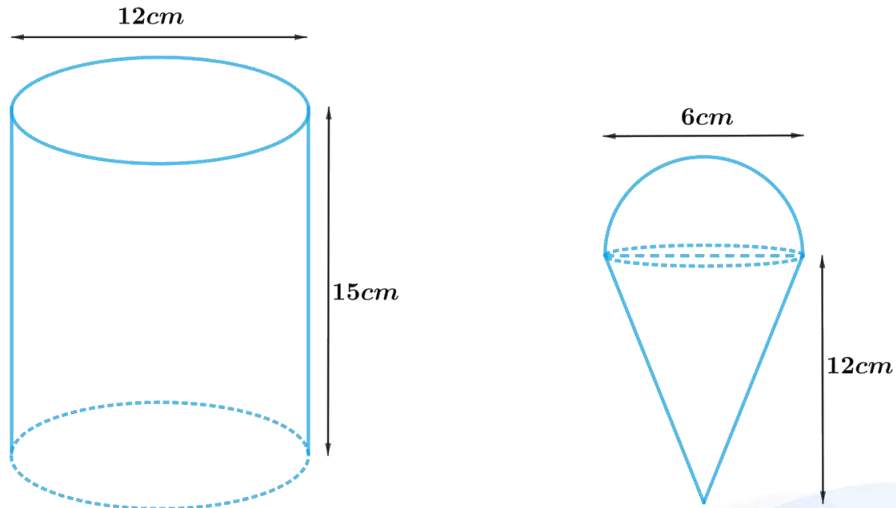Medium

Known/given:
Diameter of the right circular cylinder is 12cm and height is 15cm. Diameter of the cone is 6cm and height is 12cm.

Unknown:
The number of cones which can be filled with ice cream.

Reasoning:
Draw a figure to visualize the shapes better

From the figure it's clear that diameter of the hemisphere is same as the cone.
Since, the ice cream is to be filled into cones having a hemispherical shape on the top the volume of ice cream filled into the cone includes volume of the cone and volume of the hemisphere.

volume of the ice cream in each cone = volume of the cone + volume of the hemisphere

Since, the ice cream is to be filled from a cylindrical container, the volume of the ice cream filled into cones will be same as the volume of the ice cream in cylindrical container

volume of the ice cream in cylindrical container = number of cones filled with ice cream $\times$ volume of the ice cream in each cone

Hence, number of cones filled with ice cream = volume of the ice cream in cylindrical container $\div$ volume of the ice cream in each cone

We will find the volume of the ice cream by using formulae;

$\quad$ Volume of the hemisphere $= \dfrac{2}{3}\pi r^3$

$\qquad$ where r is the radius of the hemisphere

$\quad$ Volume of the cone $= \dfrac{1}{3}\pi r^2 h$

$\qquad$ where r and h are the radius and height of the cone respectively

$\quad$ Volume of the cylinder $= \pi r^2 h$

$\qquad$ where r and h are radius and height of the cylinder respectively.

Solution:

Height of cylindrical container, $H = 15m$

Radius of cylindrical container, $R = \dfrac{12m}{2} = 6m$

Radius of the ice-cream cone = Radius of the hemispherical top $= r = \dfrac{6m}{2} = 3m$

Height of the ice-cream cone, $h = 12m$

Let n ice-cream cones be filled with ice-cream of the container.

volume of the ice cream in container $= n \times$ volume of the ice cream in each cone

$$\pi R^2 H = n \times \left(\frac{1}{3} \pi r^2 h + \frac{2}{3} \pi r^3 \right)$$

$$\pi R^2 H = n\pi \left(\frac{1}{3} r^2 h + \frac{2}{3} r^3 \right)$$

$$R^2 H = \frac{1}{3} n r^2 \left(h + 2r \right)$$

$$n = \frac{3R^2 H}{r^2 \left(h + 2r \right)}$$

$$= \frac{3 \times \left(6cm \right)^2 \times 15cm}{\left(3cm \right)^2 \times \left(12cm + 2 \times 3cm \right)}$$

$$= \frac{3 \times 36cm^2 \times 15cm}{9cm^2 \times 18cm}$$

$$= 10$$

Therefore, 10 ice-cream cones can be filled with the ice-cream in the container.

Q6. How many silver coins, 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions 5.5cm $\times$ 10cm $\times$ 3.5cm?

$$\left[\text{Use } \pi = \frac{22}{7} \right]$$

Difficulty Level:
Medium

Known/given:
Diameter of the coin is $1.75cm$ and thickness is $2mm$.
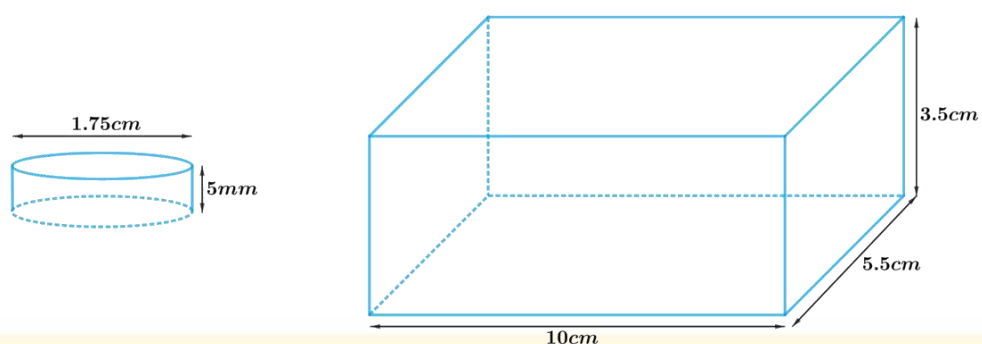Dimensions of the cuboid is $5.5cm \times 10cm \times 3.5cm$

Unknown:
The number of silver coins.

Reasoning:
Draw a figure to visualize the shapes better
From the figure it's clear that the shape of the silver coin is cylindrical.

Since, the silver coins to be melted and recast into a single solid cuboid then the cuboid formed by recasting these coins will be same in volume as the sum of the volumes of these coins.

Sum of the volumes of the silver coins = Volume of the cuboid

Therefore,

Number of the coins $\times$ volume of each coin = volume of the cuboid

We will find the volume of the solid by using formulae;

Volume of the cuboid $= lbh$

Where l, b and h are length, breadth and height of the cuboid respectively.

Volume of the cylinder $= \pi r^2 h_1$

where r and h_1 are radius and height of the cylinder respectively.

Solution:

Dimensions of the cuboid, $l \times b \times h = 5.5cm \times 10cm \times 3.5cm$

Height of the cylindrical coin, $h_1 = 2mm = \dfrac{2}{10}cm = 0.2\ cm$

Radius of the cylindrical coin, $r = \dfrac{1.75cm}{2} = 0.875cm$

Let n coins be melted to form the required cuboid.

Volume of n coins = Volume of cuboids

$$n \times \pi r^2 h_1 = l \times b \times h$$

$$n = \dfrac{l \times b \times h}{\pi r^2 h_1}$$

$$= \dfrac{5.5cm \times 10cm \times 3.5cm}{\dfrac{22}{7} \times (0.875cm)^2 \times 0.2cm}$$

$$= \dfrac{5.5cm \times 10cm \times 3.5cm}{\dfrac{22}{7} \times (0.875cm)^2 \times 0.2cm}$$

$$= \dfrac{5.5cm \times 10cm \times 3.5cm \times 7}{22 \times 0.875cm \times 0.875cm \times 0.2cm}$$

$$= 400$$

Therefore, the number of coins melted to form such a cuboid is 400.

Q7. A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm. Find the radius and slant height of the heap.

Difficulty Level:
Medium

Known/given:
Height of the cylindrical bucket is $32cm$ and radius is 18cm.
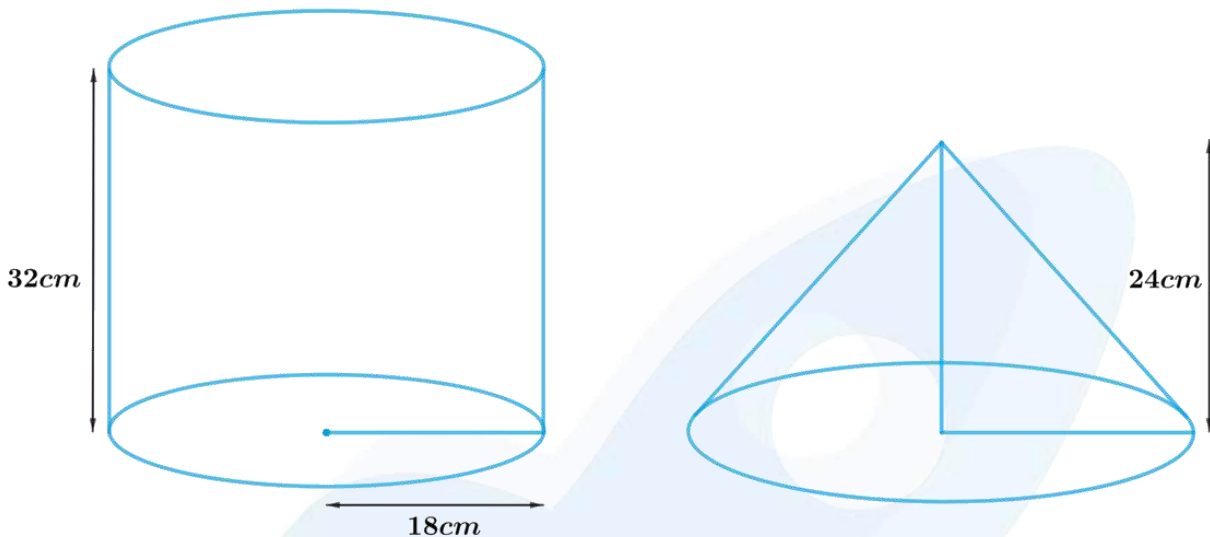Height of the conical heap is $24cm$

Unknown:
The radius and slant height of the heap.

Reasoning:
Draw a figure to visualize the shapes better

Since, a cylindrical bucket filled with sand, emptied on the ground and a conical heap of sand is formed. Then the volume of sand in the cylindrical bucket will be same as the volume the conical heap of sand.
Therefore, volume the conical heap of sand = volume of sand in the cylindrical bucket

We will find the volume of the sand by using formulae;

Volume of the cylinder $= \pi r^2 h$
  where r and h are the radius and height of the cylinder respectively

Volume of the cone $= \dfrac{1}{3}\pi r^2 h$
$$l = \sqrt{r^2 + h^2}$$
  where $r,$ h and l are radius, height and slant height of cone respectively.

Solution:
Height of the cylindrical bucket, $h_1 = 32cm$

Radius of the cylindrical bucket, $r_1 = 18cm$

Height the of conical heap, $h = 24$ cm

Let the radius of the conical heap be r and slant height be l.

volume the conical heap of sand = volume of sand in the cylindrical bucket

$$\frac{1}{3}\pi r^2 h = \pi r_1^2 h_1$$

$$\frac{1}{3}\pi r^2 h = \pi r_1^2 h_1$$

$$r^2 = \frac{3\pi r_1^2 h_1}{h}$$

$$r^2 = \frac{3\times(18cm)^2 \times 18cm \times 32cm}{24cm}$$

$$r = \sqrt{(18cm)^2 \times 4}$$

$$r = 18cm \times 2$$

$$r = 36cm$$

Slant height, $l = \sqrt{r^2 + h^2}$

$$= \sqrt{(36cm)^2 + (24cm)^2}$$

$$= \sqrt{1296cm^2 + 576cm^2}$$

$$= \sqrt{1872cm^2}$$

$$= 12\sqrt{13}cm$$

Therefore, the radius and slant height of the conical heap are 36 cm and $12\sqrt{13}$ cm respectively.

Q8. Water in canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. how much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?

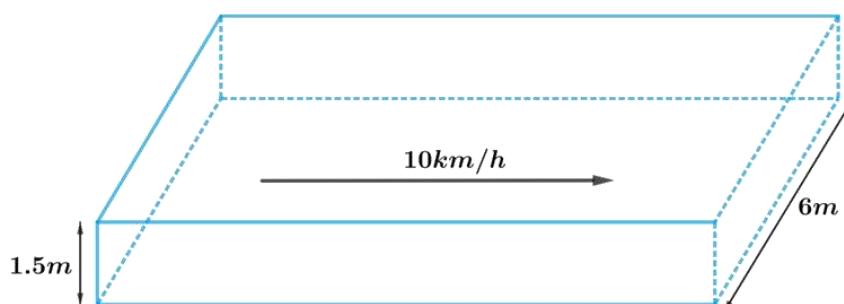**Difficulty Level:**
Hard

Known/given:
Width of the canal is $6m$ and depth is $1.5m$ and speed of flowing water is $10km/h$
Height of the standing water is $8cm$ in the area to be irrigated for 30 *minutes*.

Unknown:
The area to be irrigated in 30 *minutes* with 8cm of standing water.

Reasoning:
Draw a figure to visualize the shapes better

Canal

From the figure it's clear that shape of the cross-section of the canal is cuboidal. So, volume of the water, flowing at the speed of 10km/h for 30 minutes will be same as the volume of water to irrigate the area with 8cm of standing water.

To find the volume of the water we need to find length of the water flowing through canal in 30 minutes at the speed of 10km/h

Hence, length of the water flowing through canal in $1 hour = 10 km$
    Length of the water flowing through canal in $30 minutes = 5 km = 5000 m$

We will find the volume of the water by using formula;
    Volume of the cuboid $= lbh$
            where l, b and h are length, breadth and height of the cuboid respectively.

Volume of the water to irrigate the area = Area to be irrigated $\times$ height of the standing water

Volume of water flowing through the canal in $30 minutes$ = Area to be irrigated $\times$ height of the standing water

Solution:
Width of the cuboidal canal, $b = 6m$

Depth of the cuboidal canal, $h = 1.5m$

Speed of water flowing through the canal is $10 km/h$

Length of the water flowing through canal in $1 hour$ ($60 minutes$) $= 10 km$ Length of the water flowing through canal in $30 minutes$ $= \dfrac{30}{60} \times 10 km = 5 km$

$$l = 5 \times 1000 m = 5000 m$$

Height of the standing water, $h_1 = 8cm = \dfrac{8}{100} m = 0.08 m$

Volume of water flowing through the canal in $30 minutes$ = Area to be irrigated $\times$ height of the standing water

$$lbh = \text{Area to be irrigated} \times h_1$$

$$\text{Area to be irrigated} = \dfrac{lbh}{h_1}$$

$$= \dfrac{5000m \times 6m \times 1.5m}{0.08m}$$

$$= 562500 m^2$$

Therefore, 562500 m² area will be irrigated in 30 minutes.

Q9. A farmer connects a pipe of internal diameter 20 cm form a canal into a cylindrical tank in her field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?

Difficulty Level:
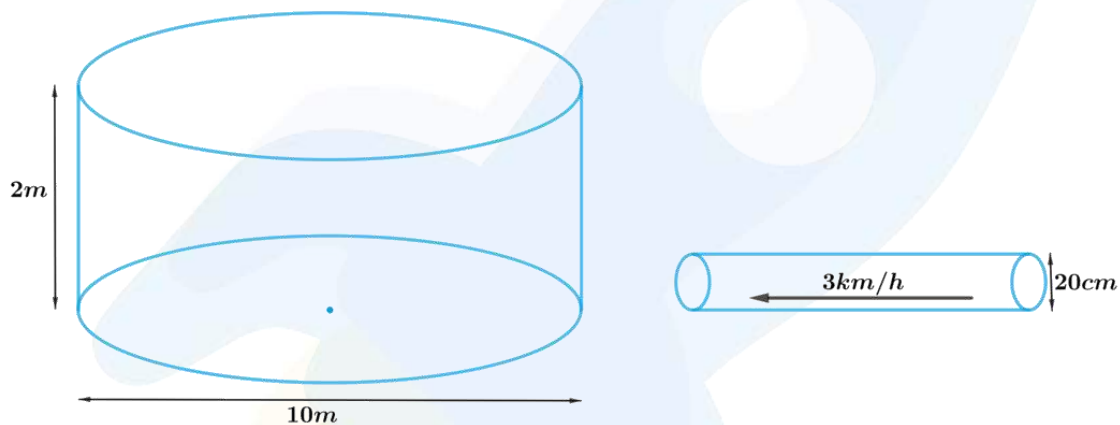Hard

Known/given:
Internal diameter of the pipe is $20cm$ and speed of water flows through the pipe is $3km/h$. Diameter of the cylindrical tank is $10m$ and depth is $2m$

Unknown:
Time taken to fill the tank.

Reasoning:
Draw a figure to visualize the shapes better

From the figure it's clear that shape of the cross-section of the pipe is cylindrical. So, volume of the water, flowing at the speed of 3km/h through the pipe to fill the tank will be same as the volume of water in the cylindrical tank.
To find the volume of the water we need to find length of the water flowing through the pipe at the speed of 3km/h
Hence, length of the water flowing through the pipe in $1 hour (60 minutes) = 3km$
        Length of the water flowing through the pipe in $1 minute = 50m$

We will find the volume of the water by using formula;
        Volume of the cylinder $= \pi r^2 h$
                where r and h are radius and height of the cylinder respectively.

Therefore,
Volume of water flowing through the pipe = volume of water in the cylindrical tank

Solution:

Radius of the cylindrical tank, $R = \dfrac{10m}{2} = 5m$
Depth of the cylindrical tank, $H = 2m$

Radius of the cylindrical pipe, $r = \dfrac{20cm}{2} = \dfrac{10}{100}m = 0.1m$

Length of the water flowing through the pipe in $1 hour$ ($60 minutes$) = $3km$

Length of the water flowing through the pipe in $1 minute$, $h = \dfrac{3km}{60} = \dfrac{3 \times 1000m}{60} = 50m$

Let the tank be filled completely in t minutes.

Volume of water flowing through pipe in t minutes = volume of water in cylindrical tank

$$t \times \pi r^2 h = \pi R^2 H$$

$$t = \frac{R^2 H}{r^2 h}$$

$$= \frac{5m \times 5m \times 2m}{0.1m \times 0.1m \times 50m}$$

$$= 100$$

Therefore, the cylindrical tank will be filled in 100 minutes.
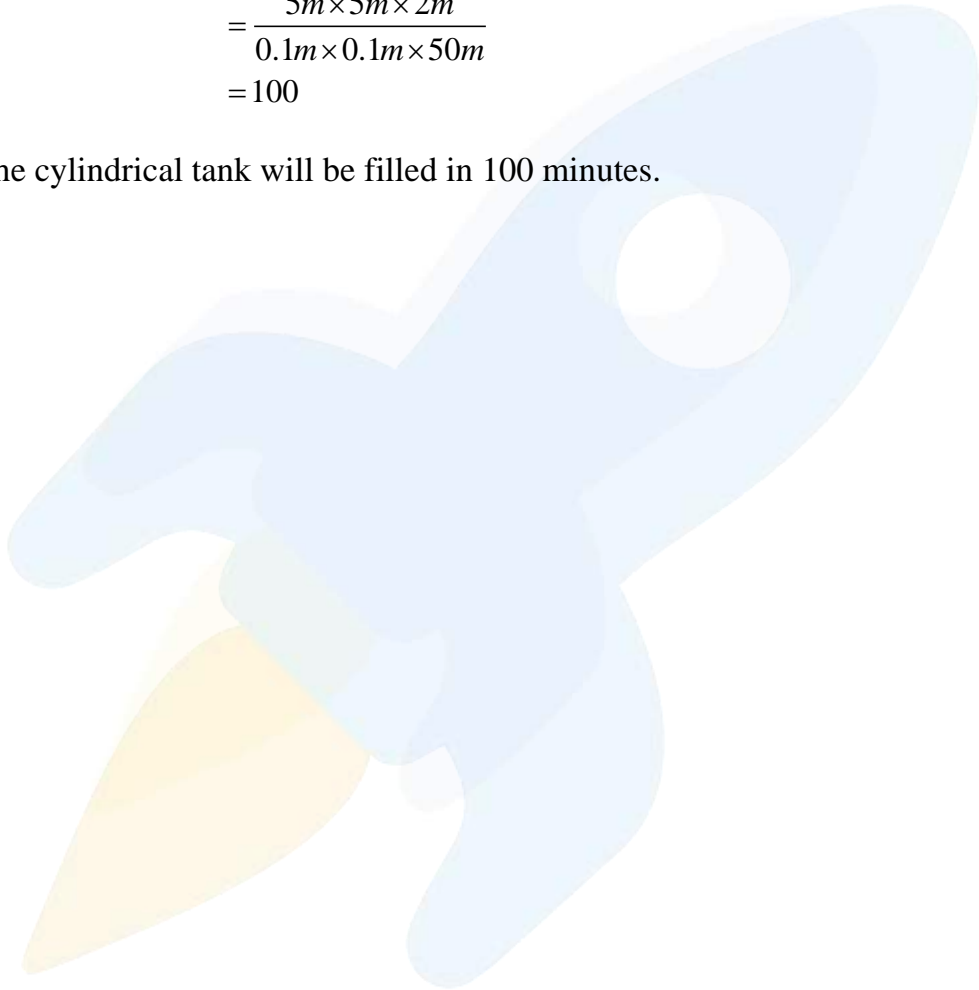
Chapter-13: Surface Area and Volumes

Exercise 13.4 (Page 257 of Grade 10 NCERT)

Q1. A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass.

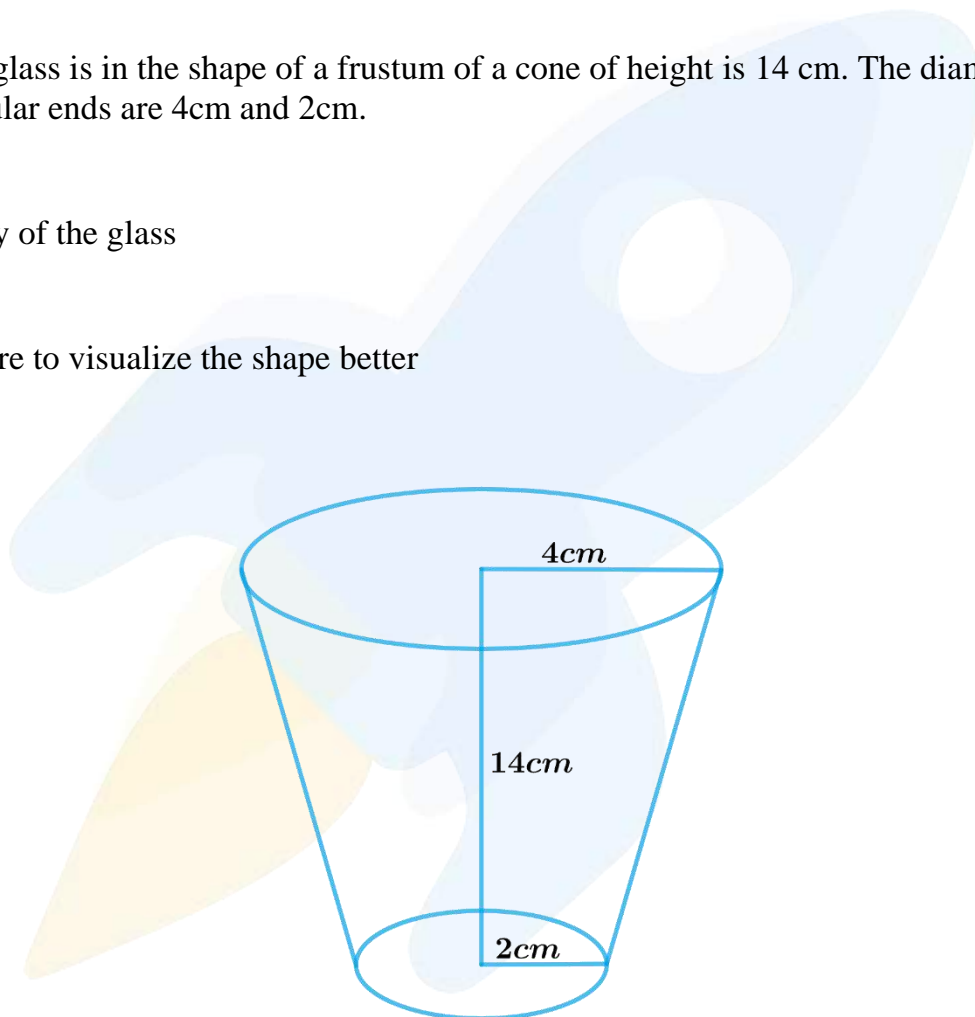**Difficulty Level:**
Medium

Known:
A drinking glass is in the shape of a frustum of a cone of height is 14 cm. The diameter of its two circular ends are 4cm and 2cm.

Unknown:
The capacity of the glass

Reasoning:
Draw a figure to visualize the shape better

Since the glass is in the shape of a frustum of a cone
Therefore, the capacity of the glass = Volume of frustum of a cone

We will find the capacity of the glass by using formulae

Volume of frustum of a cone $= \dfrac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$

where r_1, r_2 and h are the radii and height of the frustum of the cone respectively.

Solution:

Height of glass, $h = 14cm$

Radius of the larger base, $r_1 = \dfrac{4cm}{2} = 2cm$

Radius of the smaller base, $r_2 = \dfrac{2cm}{2} = 1cm$

The capacity of the glass = Volume of frustum of a cone

$$= \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 14cm \times \left((2cm)^2 + (1cm)^2 + 2cm \times 1cm\right)$$

$$= \frac{44}{3}cm \times \left(4cm^2 + 1cm^2 + 2cm^2\right)$$

$$= \frac{44}{3}cm \times 7cm^2$$

$$= \frac{308}{3}cm^2$$

$$= 102\frac{2}{3}cm^2$$

Therefore, the capacity of the glass is $102\dfrac{2}{3}cm^2$

Q2. The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

Difficulty Level:
Medium

Known:
Slant height of a frustum of a cone is 4 cm and the circumference of its circular ends are 18cm and 6 cm.

Unknown:
The curved surface area of the frustum.

Reasoning:
Draw a figure to visualize the shape better
Using the circumferences of the circular ends of the frustum to find the radii of its circular ends
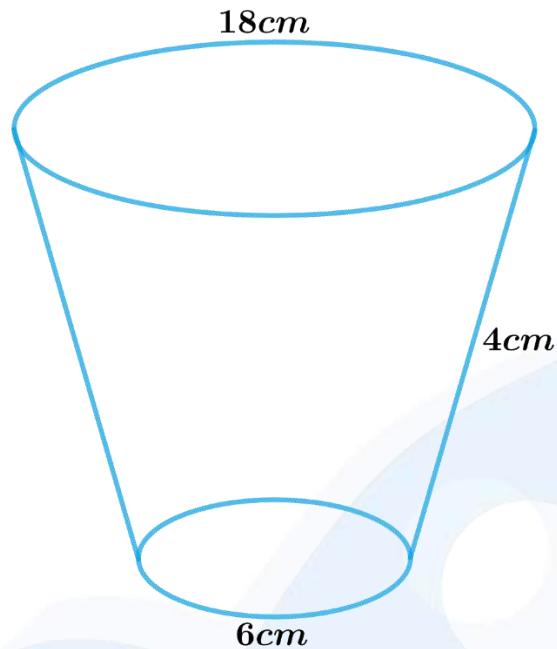
Circumference of the circle $= 2\pi r$

where r is the radius of the circle.

We will find the CSA of the frustum by using formula;

CSA of frustum of a cone $= \pi(r_1 + r_2)l$

where r_1, r_2 and l are the radii and slant height of the frustum of the cone respectively.

18cm

4cm

6cm

Solution:

Slant height of frustum of a cone, $l = 4cm$

Circumference of the larger circular end, $C_1 = 18cm$

Radius of the larger circular end, $r_1 = \dfrac{C_1}{2\pi} = \dfrac{18cm}{2\pi} = \dfrac{9}{\pi}cm$

Circumference of the smaller circular end, $C_2 = 6cm$

Radius of the smaller circular end, $r_2 = \dfrac{C_2}{2\pi} = \dfrac{6cm}{2\pi} = \dfrac{3}{\pi}cm$

CSA of frustum of a cone $= \pi(r_1 + r_2)l$

$$= \pi\left(\dfrac{9}{\pi}cm + \dfrac{3}{\pi}cm\right) \times 4cm$$

$$= \pi \times \dfrac{12}{\pi}cm \times 4cm$$

$$= 48cm^2$$

The curved surface area of the frustum is $48cm^2$

Q3. A fez, the cap used by the Turks, is shaped like the frustum of a cone (see Fig. 13.24). If its radius on the open side is 10 cm, radius at the upper base is 4 cm and its slant height is 15 cm, find the area of material used for making it.

Difficulty Level:
Medium

Known:
A fez is shaped like a frustum of a cone with radius of open side 10cm, radius at the upper base is 4cm and slant height is 15 cm

Unknown:
Area of the material used for making Fez

Reasoning:

Since the fez is in the shape of frustum of a cone and is open at the bottom.
Therefore,
Area of material used for making fez = Curved Surface Area of the frustum + Area of the upper circular end

We will find the Area of material by using formulae;

CSA of frustum of a cone $= \pi (r_1 + r_2) l$

where r_1, r_2 and l are the radii and slant height of the frustum of the cone respectively.

Area of the circle $= \pi r^2$

where r is the radius of the circle.

Solution:
Slant height, $l = 15cm$
Radius of open side, $r_1 = 10cm$
Radius of upper base, $r_2 = 4cm$

Area of material used for making fez = Curved Surface area of the frustum + area of the upper circular end

$$= \pi \left(r_1 + r_2 \right) l + \pi r^2$$
$$= \pi \left[\left(r_1 + r_2 \right) l + r_2^2 \right]$$
$$= \frac{22}{7} \left[\left(10cm + 4cm \right) 15cm + \left(4cm \right)^2 \right]$$
$$= \frac{22}{7} \left[14cm \times 15cm + 16cm^2 \right]$$
$$= \frac{22}{7} \left[210cm^2 + 16cm^2 \right]$$
$$= \frac{22}{7} \times 226cm^2$$
$$= \frac{4972}{7} cm^2$$
$$= 710\frac{2}{7} cm^2$$

$710\frac{2}{7} cm^2$ of the material used for making Fez.

Q4. A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of the milk which can completely fill the container, at the rate of ₹ 20 per litre. Also find the cost of metal sheet used to make the container, if it costs ₹ 8 per 100 cm².
(Take π = 3.14)

Difficulty Level:
Hard

Known:
A container opened from the top and made up of a metal sheet is in the shape of a frustum of a cone with of height 16 cm, radius of lower end 8 cm and radius of upper end 20 cm. Cost of 1 litre of milk is ₹ 20 and Cost of 100cm² of metal sheet is ₹ 8.

Unknown:
Cost of the milk which can completely fill the container and cost of metal sheet used to make the container

Reasoning:
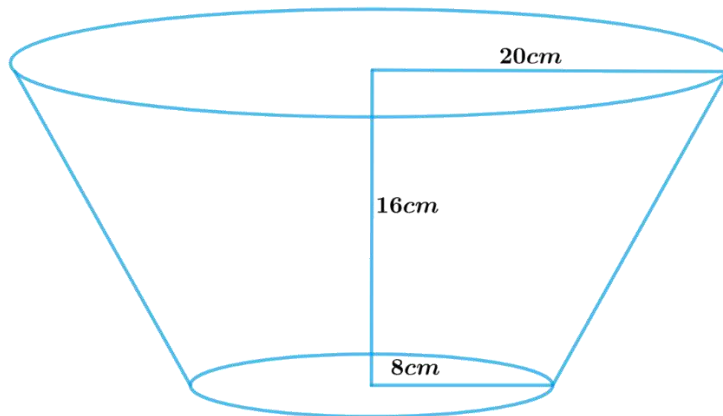
To find the cost of milk:

Since the container is in the shape of a frustum of a cone, so volume of milk which can completely fill the container will be equal to volume of frustum of cone.

Therefore, volume of milk = volume of frustum of cone

We will find the volume of the frustum by using formula;

Volume of frustum of a cone $= \frac{1}{3} \pi h \left(r_1^2 + r_2^2 + r_1 r_2 \right)$

where r_1, r_2 and h are the radii and height of the frustum of the cone respectively.

Once we know the volume of milk, we need to convert it into litre as the cost of milk is in terms of ₹ per litre.

$$1 \, litre = 1000 cm^3$$

$$1 cm^3 = \frac{1}{1000} \, litre$$

To find the cost of metal sheet:

Since the container is open from the top the metal sheet will only contain CSA of frustum of the cone and area of the lower circular end.
Therefore,
Area of metal sheet required to make the container = CSA of frustum of the cone + Area of lower circular end

We will find the CSA of the frustum by using formulae;

CSA of frustum of a cone $= \pi \left(r_1 + r_2 \right) l$

Slant height $l = \sqrt{ \left(r_1 - r_2 \right)^2 + h^2 }$ $(r_1 > r_2)$

where r_1, r_2, h and l are the radii height and slant height of the frustum of the cone respectively.

Area of the lower circular end $= \pi r_2^2$

where r_2 is the radius of the lower circular end.

Solution:

Height of the frustum of cone, $h = 16cm$

Radius of the upper end, $r_1 = 20cm$

Radius of the lower end, $r_2 = 8cm$

Slant height of the frustum, $l = \sqrt{(r_1 - r_2)^2 + h^2}$

$$l = \sqrt{(20cm - 8cm)^2 + (16cm)^2}$$
$$= \sqrt{144cm^2 + 256cm^2}$$
$$= \sqrt{400cm^2}$$
$$= 20cm$$

Volume of milk which can completely fill the container = Volume of the frustum of Cone

$$= \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$$
$$= \frac{1}{3} \times 3.14 \times 16cm \times \left((20cm)^2 + (8cm)^2 + 20cm \times 8cm\right)$$
$$= \frac{1}{3} \times 3.14 \times 16cm \times \left(400cm^2 + 64cm^2 + 160cm^2\right)$$
$$= \frac{1}{3} \times 3.14 \times 16cm \times 624cm^2$$
$$= 10449.92cm^3$$
$$= \frac{10449.92}{1000} litre$$
$$= 10.44992 litre$$
$$= 10.45 litre$$

Cost of 1 litre of milk = ₹ 20

Therefore, cost of 10.45 litre of milk = ₹ 20×10.45 = ₹ 209

Area of metal sheet required to make the container = CSA of frustum of the cone + Area of lower circular end

$$= \pi(r_1 + r_2)l + \pi r_2^2$$
$$= \pi\left[(r_1 + r_2)l + r_2^2\right]$$
$$= 3.14 \times \left[(20cm + 8cm) \times 20cm + (8cm)^2\right]$$
$$= 3.14 \times \left[560cm^2 + 64cm^2\right]$$
$$= 3.14 \times 624cm^2$$
$$= 1959.36cm^2$$

Cost of 100cm^2 of metal sheet = ₹ 8

Therefore, cost of 1959.36cm^2 of metal sheet = ₹ $\dfrac{8}{100} \times 1959.36$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ = ₹ 156.7488

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ = ₹ 156.75

Answer:

Cost of milk which can completely fill the container is ₹ 209

Cost of metal sheet required to make the container is ₹ 156.75

Q5. A metallic right circular cone 20 cm high and whose vertical angle is 60°
is cut into two parts at the middle of its height by a plane parallel to its base.
If the frustum so obtained be drawn into a wire of diameter $\dfrac{1}{16} cm$, find the length
of the wire.

Difficulty Level:

Hard

Known:

(i) A metallic right circular cone of height = 20 cm and whose vertical angle = 60° is cut
into 2 parts at the middle of its height by a plane parallel to its base

(ii) The frustum obtained is drawn into a wire of diameter = $\dfrac{1}{16}$ cm

Unknown:

Length of the wire drawn out from a frustum

Reasoning:

Draw a figure to visualize the cross section of a cone
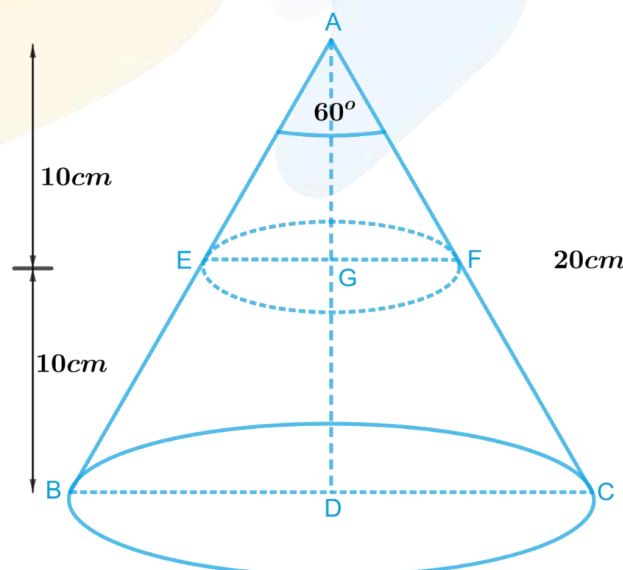
Since a metallic right circular cone is cut into two parts at the middle
of its height by a plane parallel to its base.

From the figure it's clear that
In $\triangle ABC$, EF parallel to BC and
$$AD = 20cm$$
$$AG = 10cm$$
$$\angle BAC = 60°$$

To get the values of radii of both ends of the frustum formed compare $\triangle ADB$ and $\triangle ADC$

$AD = AD$	(common)
$AB = AC$	(Slant height)
$\angle ADB = \angle ADC = 90^o$	(Right circular cone)
$\triangle ADB \cong \triangle ADC$	(RHS criterion of congruency)
$\angle BAD = \angle DAC$	(CPCT)

Then,
$$\angle BAD = \angle DAC = \frac{1}{2}\angle BAC = \frac{1}{2} \times 60° = 30°$$

In $\triangle ADB$
$$\frac{BD}{AD} = \tan 30°$$
$$BD = AD \tan 30°$$

Radius of lower end $BD = \dfrac{AD}{\sqrt{3}}$

Similarly, in $\triangle AEG$
$$\frac{EG}{AG} = \tan 30°$$
$$EG = AG \tan 30°$$

Radius of upper end $EG = \dfrac{AG}{\sqrt{3}}$

Now radii and height are known, finding volume of frustum of cone is easy

Since the frustum obtained is drawn into the wire which will be cylindrical in the shape.
Then the volume of the wire will be same as the volume of the frustum of the cone.

Therefore, Volume of the wire = Volume of frustum of the cone

We will find the volume of the frustum by using formula;

Volume of frustum of a cone $= \dfrac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$

where r_1, r_2 and h are the radii and height of the frustum of the cone
respectively.

We will find the volume of the wire by using formula;

Volume of cylinder $= \pi r^2 h$

where r and h are radius and height of the cylinder.

Solution:

In ΔABC, EF parallel to BC and

$$AD = 20cm$$

$$AG = 10cm$$

$$\angle BAC = 60°$$

To get the values of radii of both ends of the frustum formed compare ΔADB and ΔADC

$AD = AD$	(common)
$AB = AC$	(Slant height)
$\angle ADB = \angle ADC = 90^o$	$(\text{Right circular cone})$
$\Delta ADB \cong \Delta ADC$	$(\text{RHS criterion of congruency})$
$\angle BAD = \angle DAC$	(CPCT)

Then,

$$\angle BAD = \angle DAC = \frac{1}{2}\angle BAC = \frac{1}{2} \times 60° = 30°$$

In ΔADB

$$\frac{BD}{AD} = \tan 30°$$

$$BD = AD \tan 30°$$

$$BD = 20cm \times \frac{1}{\sqrt{3}}$$

$$BD = \frac{20\sqrt{3}}{3} cm$$

Similarly, in ΔAEG

$$\frac{EG}{AG} = \tan 30°$$

$$EG = AG \tan 30°$$

$$EG = 10cm \times \frac{1}{\sqrt{3}}$$

$$EG = \frac{10\sqrt{3}}{3} cm$$

Height of the frustum of the cone, $h = 10cm$

Radius of lower end, $r_1 = \frac{20\sqrt{3}}{3} cm$

Radius of upper end, $r_2 = \frac{10\sqrt{3}}{3} cm$

Diameter of the cylindrical wire, $d = \dfrac{1}{16} cm$

Radius of the cylindrical wire, $r = \dfrac{1}{2} \times \dfrac{1}{16} cm = \dfrac{1}{32} cm$

Let the length of the wire be H

Since the frustum is drawn into wire

Volume of the cylindrical wire = Volume of frustum of the cone

$$\pi r^2 H = \frac{1}{3}\pi h \left(r_1^2 + r_2^2 + r_1 r_2 \right)$$

$$H = \frac{h}{3r^2}\left(r_1^2 + r_2^2 + r_1 r_2 \right)$$

$$= \frac{10 cm}{3 \times \left(\dfrac{1}{32} cm \right)^2} \times \left(\left(\frac{20\sqrt{3}}{3} cm \right)^2 + \left(\frac{10\sqrt{3}}{3} cm \right)^2 + \frac{20\sqrt{3}}{3} cm \times \frac{10\sqrt{3}}{3} cm \right)$$

$$= \frac{10240}{3 cm} \times \left(\frac{400}{3} cm^2 + \frac{100}{3} cm^2 + \frac{200}{3} cm^2 \right)$$

$$= \frac{10240}{3 cm} \times \frac{700 cm^2}{3}$$

$$= \frac{7168000}{9 \times 100} m$$

$$= 7964.4 m$$

Length of the wire is $7964.4 m$

Chapter-13: Surface Area and Volumes

Exercise 13.5 (Page 258 of Grade 10 NCERT)

Q1. A copper wire, 3 mm in diameter, is wound about a cylinder whose length is 12 cm, and diameter 10 cm, so as to cover the curved surface of the cylinder. Find the length and mass of the wire, assuming the density of copper to be 8.88 g per cm^3.
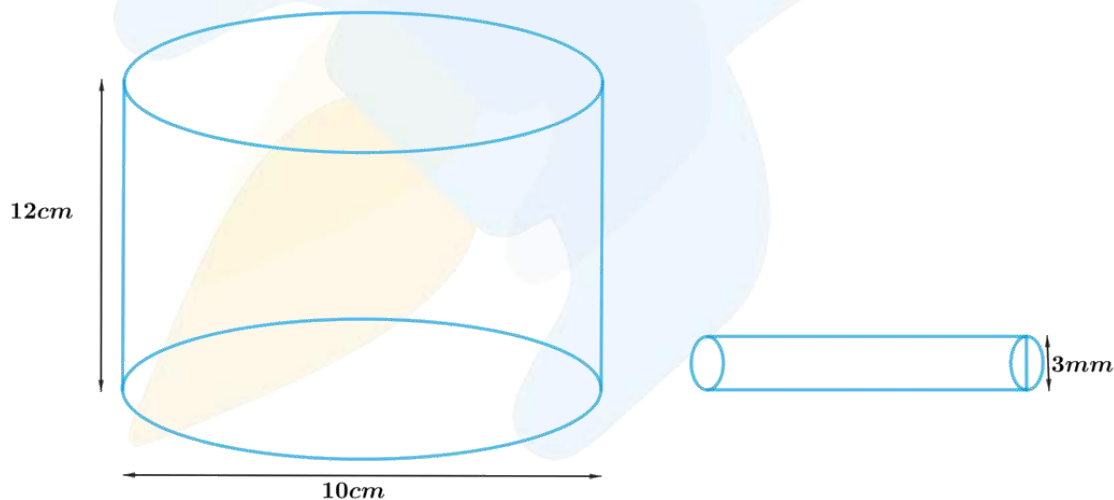
Difficulty Level:
Medium

Known:
Diameter of the copper wire is 3mm. Length of the cylinder is 12cm and diameter is 10cm. Density of copper is 8.88 g per cm^3

Unknown:
Length and mass of the wire

Reasoning:
Draw a figure to visualize the cylinder

To find length of the wire:
Since the curved surface of the cylinder to be covered with the copper wire then one round of copper wire will cover same height of cylinder as of the width (diameter) of the wire.

Therefore, Length of the wire × diameter the of wire = CSA of the cylinder

We will find the CSA of the cylinder by using formula;

CSA of the cylinder $= 2\pi rh$

where r and h are radius and height of the cylinder respectively.

To find mass of the wire:

Mass = Density $\times$ Volume

As the density of the copper is given, we need to find volume of the wire first.

Since the wire is cylindrical in shape

Therefore, Volume of the wire = volume of the cylinder

We will find the volume of the cylindrical wire by using formula;

Volume of the cylinder $= \pi r_1^2 h_1$

where r_1 and h_1 are the radius and the length of the cylindrical wire

Solution:

Height of cylinder, $h = 12cm$

Diameter of cylinder, $d = 10cm$

Radius of cylinder, $r = \dfrac{10cm}{2} = 5cm$

Diameter (width) of wire $d_1 = 3mm = \dfrac{3}{10}cm = 0.3cm$

Radius of wire $r_1 = \dfrac{0.3cm}{2} = 0.15cm$

Let the length of the wire be h_1

Since curved surface of the cylinder to be covered with the wire

Length of the wire $\times$ diameter the of wire = CSA of the cylinder

$$d_1 \times h_1 = 2\pi rh$$

$$h_1 = \dfrac{2\pi rh}{d_1}$$

$$= \dfrac{2 \times 3.14 \times 5cm \times 12cm}{0.3cm}$$

$$= 1256cm$$

Volume of the wire = volume of the cylinder

$$= \pi r_1^2 h_1$$

$$= 3.14 \times 0.15cm \times 0.15cm \times 1256cm$$

$$= 88.7364cm^3$$

Density of copper $= 8.8g/cm^3$

Mass of the copper Wire = Density $\times$ volume

$$= 8.88g/cm^3 \times 88.7364cm^3$$

$$= 787.97923g$$

Length of the wire is $1256cm$ and Mass is $788g$ (approx.)

Q2. A right triangle, whose sides are 3 cm and 4 cm (other than hypotenuse) is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed. (Choose value of π as found appropriate.)
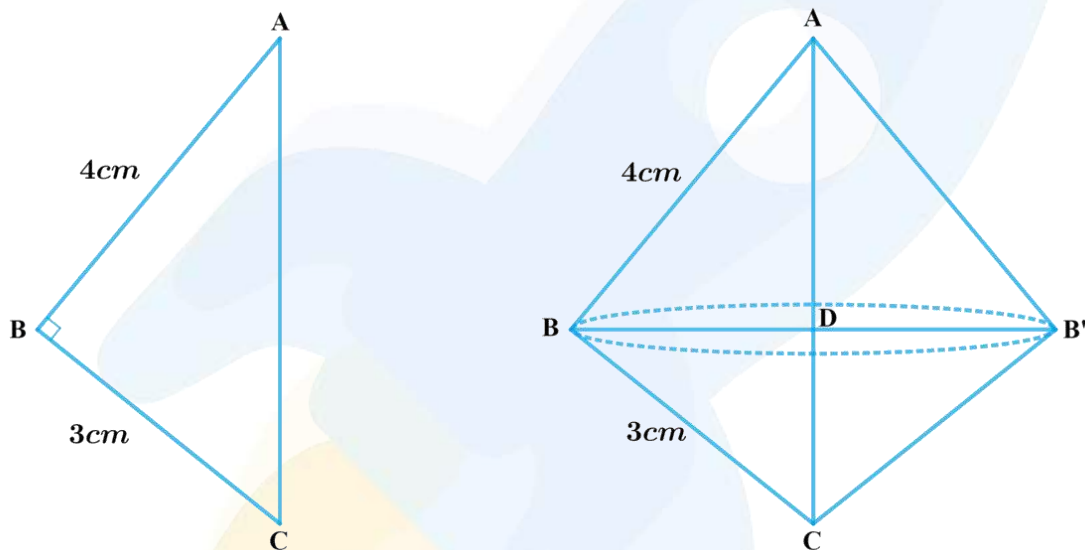
Difficulty Level:
Hard

A right triangle whose sides are 3 cm and 4 cm (other than hypotenuse) is made to revolve about its hypotenuse to form cone.

Unknown:
The volume and surface area of the double cone formed.

Reasoning:
Draw a figure to visualize the double cone formed

In order to find the volume and surface area, we need to find BD or radius of the double cone
From the figure it's clear that $BD \perp AC$

To find BD
(i)   We first find AC using Pythagoras theorem
$$AC^2 = AB^2 + BC^2$$
$$AC = \sqrt{AB^2 + BC^2}$$

(ii) Using AA criterion of similarity

Prove
$$\triangle ABC \sim \triangle BDC$$
$$\frac{AB}{BD} = \frac{AC}{BC}$$
(Corresponding Sides of similar triangles are in proportion)

Radius or $BD = \dfrac{AB}{AC} \times BC$

Since we know AB, AC and BC; BD can be found out

Since double cone is made by joining 2 cones by their bases
Therefore, Volume of double cone = Volume of Cone ABB' + Volume of Cone BCB'

We will find the volume of the cone by using formulae;

Volume of the cone $= \dfrac{1}{3}\pi r^2 h$

where r and h are the radius and height of the cone respectively.

Visually from the figure it's clear that CSA of double cone includes CSA of both the cones

Therefore, CSA of double Cone = CSA of cone ABB' + CSA of cone BCB'

We will find the CSA of the cone by using formulae;

CSA of frustum of a cone $= \pi r l$

where r and l are the radius and slant height of the cone respectively.

Solution:

In $\triangle ABC$ right-angled at B

$$AC^2 = AB^2 + BC^2$$

$$AC = \sqrt{(3cm)^2 + (4cm)^2}$$

$$= \sqrt{9cm^2 + 16cm^2}$$

$$= \sqrt{25cm^2}$$

$$= 5cm$$

Consider $\triangle ABC$ and $\triangle BDC$

$$\angle ABC = \angle CDB = 90° \quad (BD \perp AC)$$

$$\angle BCA = \angle BCD \quad (\text{common})$$

By AA criterion of similarity $\triangle ABC \sim \triangle BDC$
Therefore,

$$\dfrac{AB}{BD} = \dfrac{AC}{BC} \quad \text{(Corresponding sides of similar triangles are in proportion)}$$

$$BD = \dfrac{AB \times BC}{AC}$$

$$= \dfrac{3cm \times 4cm}{5cm}$$

$$= \dfrac{12}{5}cm$$

$$= 2.4cm$$

Volume of double cone = Volume of Cone ABB' + Volume of Cone BCB'

$$= \frac{1}{3} \times \pi (BD)^2 \times AD + \frac{1}{3} \pi (BD)^2 \times DC$$
$$= \frac{1}{3} \times \pi (BD)^2 \left[AD + DC \right]$$
$$= \frac{1}{3} \times \pi (BD)^2 \times AD$$
$$= \frac{1}{3} \times 3.14 \times 2.4 cm \times 2.4 cm \times 5 cm$$
$$= \frac{90.432}{3} cm^3$$
$$= 30.144 cm^3$$
$$= 30.14 cm^3$$

CSA of double Cone = CSA of cone ABB' + CSA of cone BCB'

$$= \pi \times BD \times AB + \pi \times BD \times BC$$
$$= \pi \times BD \left[AB + BC \right]$$
$$= 3.14 \times 2.4 cm \times \left[3cm + 4cm \right]$$
$$= 3.14 \times 2.4 cm \times 7 cm$$
$$= 52.752 cm^2$$
$$= 52.75 cm^2$$

Q3. A cistern, internally measuring 150 cm $\times$ 120 cm $\times$ 110 cm, has 129600 cm^3 of water in it. Porous bricks are placed in the water until the cistern is full to the brim. Each brick absorbs one-seventeenth of its own volume of water. How many bricks can be put in without overflowing the water, each brick being 22.5 cm $\times$ 7.5 cm $\times$ 6.5 cm?
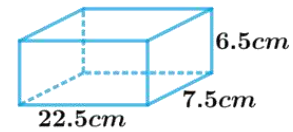
Difficulty Level:
Hard
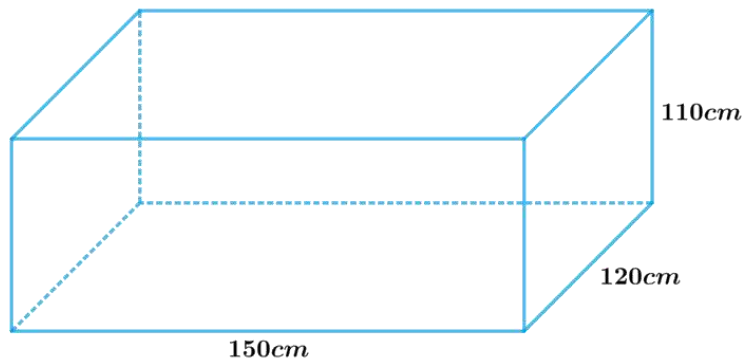
Known:
A cistern internally measuring $150cm \times 120cm \times 110cm$ has $129600 cm^3$ of water in it. Each brick is of dimensions $22.5cm \times 7.5cm \times 6.5cm$ and can absorbs one-seventeenth of its own volume of water.

Unknown:
Number of bricks which can be put in the cistern without overflowing the water.

Reasoning:

Logically, the bricks have to occupy the space left in the cistern which is not occupied by water left in the cistern, after being absorbed by the bricks.

Therefore,
Internal volume of cistern = (Volume of water in the cistern – Volume of water absorbed by bricks) + Volume of bricks

Internal volume of cistern = Volume of water in cistern $-\dfrac{1}{17}\times$ Volume of bricks +

Volume of bricks

Hence,
Internal volume of cistern = Volume of water in the cistern $+\dfrac{16}{17}$ volume of bricks

Since the dimensions of the cistern and the brick are known which are cuboidal in shape
We will find the volume by using formula;

Volume of the cuboid $= lbh$
where *l, b* and *h* are length breadth and height of the cuboid respectively.

Solution:
Internal volume of the cistern $=150cm\times120cm\times110cm=1980000cm^3$
Volume of water in the cistern $=129600cm^3$
Volume of one brick $=22.5cm\times7.5cm\times6.5cm=1069.875cm^3$

Let the required number of bricks be 'x'
Volume of x bricks $=x\times1069.875cm^3$

Internal volume of cistern = Volume of water in the cistern $+\dfrac{16}{17}$ volume of x bricks

$$1980000cm^3=129600cm^3+\frac{16}{17}\times x\times1096.875cm^3$$

$$x=\frac{\left(1980000cm^3-129600cm^3\right)\times17}{16\times1096.875cm^3}$$

$$=\frac{1850400cm^3\times17}{16\times1096.875cm^3}$$

$$=1792.41$$

Since the number of bricks should be a whole number, whole part of x should be considered as the answer. Rounding off x to the next whole number should not be considered as it will lead to overflowing of tank, if that extra fractional part is added to the cistern.

Therefore, number of bricks which can be put in the cistern without overflowing the water is 1792.

Q4. In one fortnight of a given month, there was a rainfall of 10 cm in a river valley. If the area of the valley is 7280 km^2, show that the total rainfall was approximately equivalent to the addition to the normal water of three rivers each 1072 km long, 75 m wide and 3 m deep.

Difficulty Level:
Medium

Known:
In one fortnight there was a rainfall of 10 cm in a river valley of area 7280 km^2

To show:
The total rainfall was approximately equivalent to the addition to the normal water of three rivers each 1072 km long, 75 m wide and 3 m deep.

Reasoning:
Since there was a rainfall of 10cm in a river valley of area 7280 km^2 the volume of the rainfall will be calculated by

Volume of the rainfall = Area of the river valley × height of rainfall in the river valley

Since the dimensions of three rivers are known, we can calculate the volume by
Volume of the river $= length \times breadth \times height$

Solution:
Area of the Valley, $A = 7280 km^2 = 7280 \times 1000000 m^2 = 7.28 \times 10^9 m^2$

Height of rainfall in a fortnight, $h = 10cm = \dfrac{10}{100} m = 0.1m$

Volume of the rainfall = Area of the river valley × height of rainfall in the river valley
$$= 7.28 \times 10^9 m^2 \times 0.1m$$
$$= 7.28 \times 10^8 m^3$$

Length of river, $l = 1072 km = 1072 \times 1000 m = 1.072 \times 10^6 m$

Width of river, $b = 75m$

Depth of river, $h = 3m$

Volume of 3 rivers = $3 \times$ volume of 1 river

$$= 3lbh$$

$$= 3 \times 1.072 \times 106m \times 75m \times 3m$$

$$= 723.6 \times 10^6 m^3$$

$$= 7.236 \times 10^8 m^3$$

Since $7.236 \times 10^8 m^3$ is approximately equivalent to $7.28 \times 10^8 m^3$

Therefore, we can say that total rainfall in the valley was approximately equivalent to the addition of normal water of three rivers each 1072 km long, 75 m wide and 3 m deep.

Q5. An oil funnel made of tin sheet consists of a 10 cm long cylindrical portion attached to a frustum of a cone. If the total height is 22 cm, diameter of the cylindrical portion is 8 cm and the diameter of the top of the funnel is 18 cm, find the area of the tin sheet required to make the funnel (see Fig. 13.25).
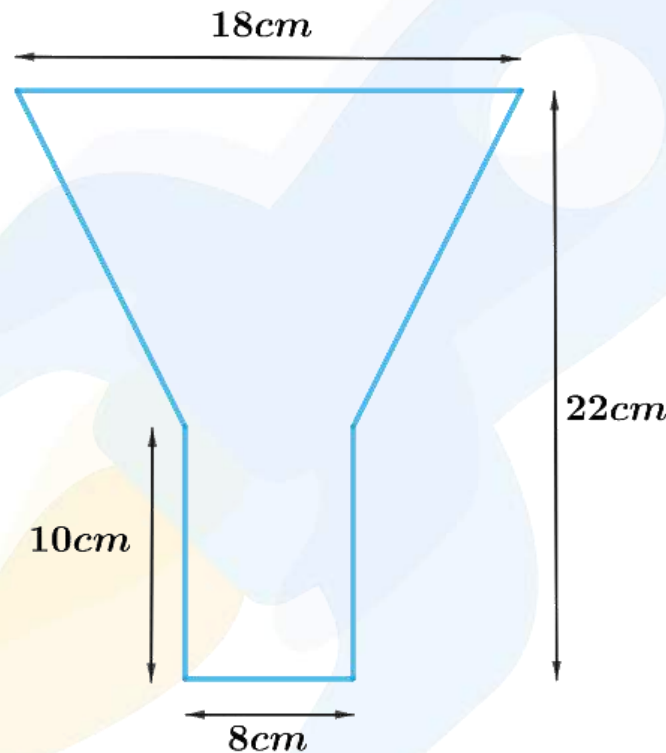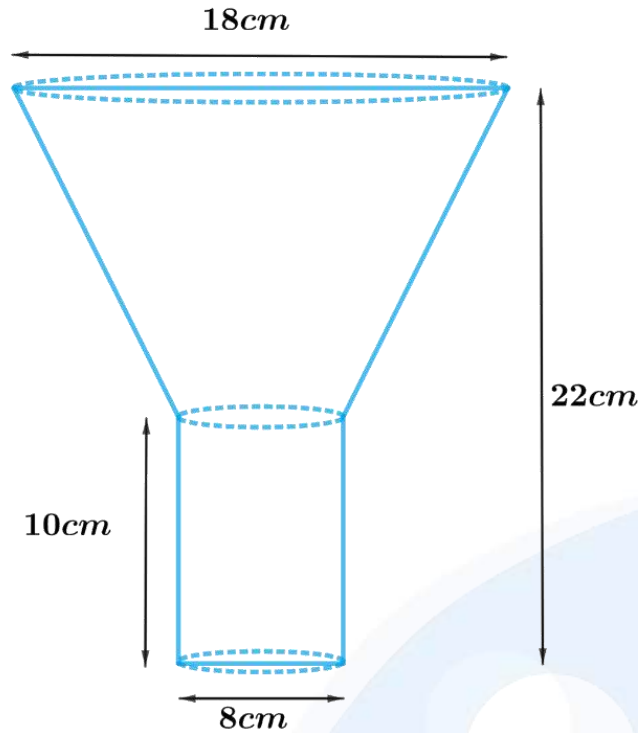

Difficulty Level:
Medium

Known:
Length of the cylindrical part is 10 cm and diameter is 8 cm. Diameter of top of the funnel is 18 cm and total height of funnel is 22 cm.

Unknown:
Area of tin sheet required to make the funnel

Reasoning:

Since the funnel is open at the top and the bottom. Hence, open at the junction of the frustum of cone and cylindrical part.

Therefore,
Area of tin sheet required to make the funnel = CSA of frustum of cone + CSA of the cylinder

We will find the CSA of the frustum by using formulae;

CSA of frustum of a cone $= \pi \left(r_1 + r_2 \right) l$

Slant height, $l = \sqrt{h^2 + \left(r_1 - r_2 \right)^2}$

where r_1, r_2, h and l are the radii height and slant height of the frustum of the cone respectively.

CSA of the cylinder $= 2\pi r h$

where r and h are radius and height of the cylinder respectively.

Solution:

Height of the funnel, $H = 22cm$

Height of cylindrical part, $h = 10cm$

Height of frustum of cone, $h_1 = 22cm - 10cm = 12cm$

Radius of top part of frustum of cone, $r_1 = \dfrac{18cm}{2} = 9cm$

Radius of lower part of frustum of cone, $r_2 = \dfrac{8cm}{2} = 4cm$

Radius of cylindrical part, $r_2 = 4cm$

Slant height of frustum, $l = \sqrt{h_1^2 + (r_1 - r_2)^2}$

$$= \sqrt{(12cm)^2 + (9cm - 4cm)^2}$$
$$= \sqrt{144cm^2 + 25cm^2}$$
$$= \sqrt{169cm^2}$$
$$= 13cm$$

Area of tin sheet required to make the funnel = CSA of frustum of cone + CSA of the cylinder

$$= \pi(r_1 + r_2)l + 2\pi r_2 h$$
$$= \pi\left[(r_1 + r_2)l + 2r_2 h\right]$$
$$= \frac{22}{7}\left[(9cm + 4cm) \times 13cm + 2 \times 4cm \times 10cm\right]$$
$$= \frac{22}{7}\left[169cm^2 + 80cm^2\right]$$
$$= \frac{22}{7} \times 249cm^2$$
$$= \frac{5478}{7}cm^2$$
$$= 782\frac{4}{7}cm^2$$

Q6. Derive the formula for the curved surface area and total surface area of the frustum of a cone, given to you in Section 13.5, using the symbols as explained.

Difficulty Level:
Hard

Known:
A frustum of a cone with h as height, l as the slant height, r_1 and r_2 radii of the ends where $r_1 > r_2$
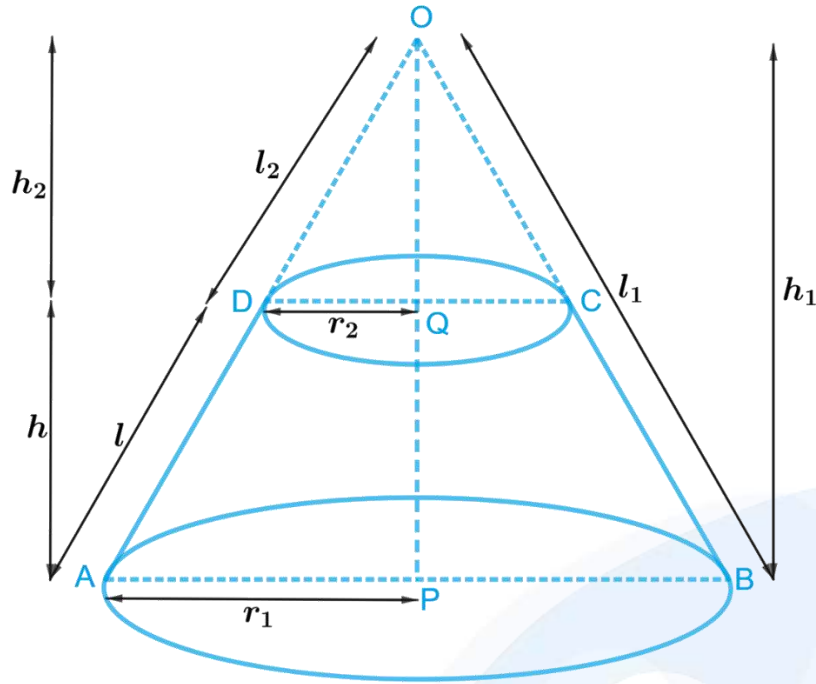
To prove:
(i) CSA of the frustum of the cone $= \pi l(r_1 + r_2)$

(ii) TSA of the frustum of the cone $= \pi l(r_1 + r_2) + \pi r_1^2 + \pi r_2^2$

where r_1, r_2, h and l are the radii height and slant height of the frustum of the cone respectively.

Construction:
Extended side BC and AD of the frustum of cone to meet at O.

Proof:

The frustum of a cone can be viewed as a difference of two right circular cones OAB and OCD. Let h_1 and l_1 be the height and slant height of cone OAB and h_2 and l_2 be the height and slant height of cone OCD respectively.

In ΔAPO and ΔDQO

$$\Delta APO = \Delta DQO = 90° \quad \text{(Since both cones are right circular cones)}$$

$$\angle AOP = \angle DOQ \quad \text{(Common)}$$

Therefore, $\Delta APO \sim \Delta DQO$ (A.A criterion of similarity)

$$\frac{AP}{DQ} = \frac{AO}{DO} = \frac{OP}{OQ} \quad \text{(Corresponding sides of similar triangles are proportional)}$$

$$\Rightarrow \frac{r_1}{r_2} = \frac{l_1}{l_2} = \frac{h_1}{h_2}$$

$$\Rightarrow \frac{r_1}{r_2} = \frac{l_1}{l_2} \qquad \qquad \Rightarrow \frac{r_2}{r_1} = \frac{l_2}{l_1}$$

$$\text{or}$$

Subtracting 1 from both sides we get

$$\frac{r_1}{r_2} - 1 = \frac{l_1}{l_2} - 1$$

$$\frac{r_1 - r_2}{r_2} = \frac{l_1 - l_2}{l_2}$$

$$\frac{r_1 - r_2}{r_2} = \frac{l}{l_2}$$

$$l_2 = \frac{l r_2}{r_1 - r_2} \qquad \qquad \text{(i)}$$

or $\qquad \qquad \Rightarrow \dfrac{r_2}{r_1} = \dfrac{l_2}{l_1}$

Subtracting 1 from both sides we get

$$\frac{r_2}{r_1} - 1 = \frac{l_2}{l_1} - 1$$

$$\frac{r_2 - r_1}{r_1} = \frac{l_2 - l_1}{l_1}$$

$$\frac{r_1 - r_2}{r_1} = \frac{l_1 - l_2}{l_1}$$

$$\frac{r_1 - r_2}{r_1} = \frac{l}{l_1}$$

$$l_1 = \frac{lr_1}{r_1 - r_2} \qquad\qquad \text{(ii)}$$

(i)   CSA of frustum of cone = CSA of cone OAB − CSA of cone OCD

$$= \pi r_1 l_1 - \pi r_2 l_2$$

$$= \pi (r_1 l_1 - r_2 l_2)$$

$$= \pi \left(r_1 \times \frac{lr_1}{r_1 - r_2} - r_2 \times \frac{lr_2}{r_1 - r_2} \right) \qquad \left[\text{using (i) and (ii)} \right]$$

$$= \pi \left(\frac{lr_1^2 - lr_2^2}{r_1 - r_2} \right)$$

$$= \pi \left(\frac{l\left(r_1^2 - r_2^2\right)}{r_1 - r_2} \right)$$

$$= \pi \left(\frac{l\left(r_1 - r_2\right)\left(r_1 + r_2\right)}{r_1 - r_2} \right) \quad \left[a^2 - b^2 = (a-b)(a+b) \right]$$

$$= \pi l \left(r_1 + r_2\right)$$

(ii)   TSA of frustum of cone = CSA of frustum + Area of lower circular end+ Area of top circular end

$$= \pi l \left(r_1 + r_2\right) + \pi r_1^2 + \pi r_2^2$$

Therefore, CSA of the frustum of the cone $= \pi l \left(r_1 + r_2\right)$
TSA of the frustum of the cone $= \pi l \left(r_1 + r_2\right) + \pi r_1^2 + \pi r_2^2$
Hence Proved.

Q7. Derive the formula for the volume of the frustum of a cone, given to you in Section 13.5, using the symbols as explained.

Difficulty Level:
Hard

Known:

A frustum of a cone with h as height, l as the slant height, r_1 and r_2 radii of the ends where $r_1 > r_2$
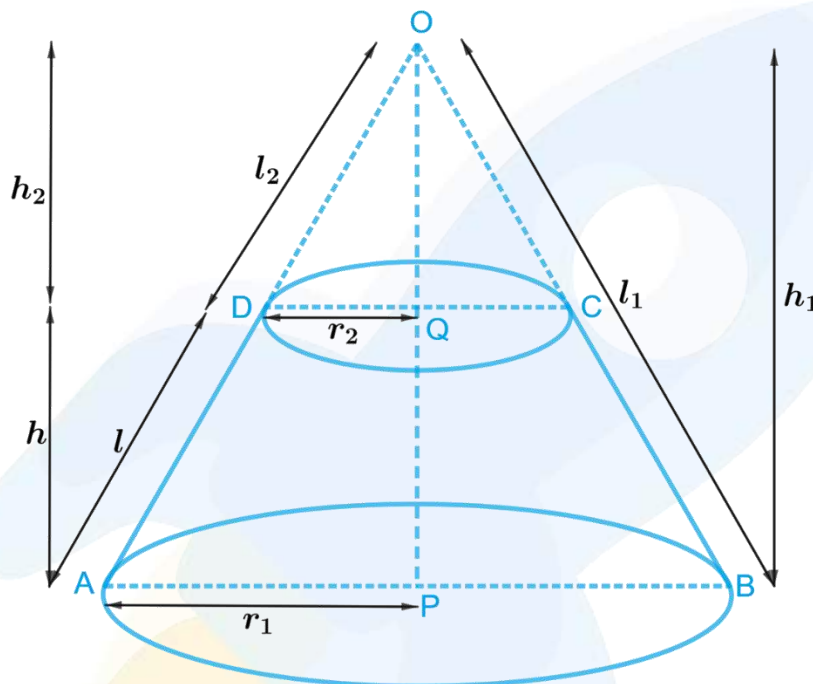
To prove:

Volume of the frustum of a cone $= \dfrac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$

where r_1, r_2 and h are the radii and height of the frustum of the cone respectively.

Construction:

Extend side BC and AD of the frustum of cone to meet at O.

Proof:

The frustum of a cone can be viewed as a difference of two right circular cones OAB and OCD.

Let h_1 and l_1 be the height and slant height of cone OAB and h_2 and l_2 be the height and slant height of cone OCD respectively.

In $\triangle APO$ and $\triangle DQO$

$\quad \triangle APO = \triangle DQO = 90°$ (Since both cones are right circular cones)

$\quad \angle AOP = \angle DOQ$ (Common)

Therefore, $\triangle APO \sim \triangle DQO$ (A.A criterion of similarity)

$\dfrac{AP}{DQ} = \dfrac{AO}{DO} = \dfrac{OP}{OQ}$ (Corresponding sides of similar triangles are proportional)

$\Rightarrow \dfrac{r_1}{r_2} = \dfrac{l_1}{l_2} = \dfrac{h_1}{h_2}$

$\Rightarrow \dfrac{r_1}{r_2} = \dfrac{h_1}{h_2}$ or $\quad \Rightarrow \dfrac{r_2}{r_1} = \dfrac{h_2}{h_1}$

Subtracting 1 from both sides

$$\frac{r_1}{r_2} - 1 = \frac{h_1}{h_2} - 1$$

$$\frac{r_1 - r_2}{r_2} = \frac{h_1 - h_2}{h_2}$$

$$\frac{r_1 - r_2}{r_2} = \frac{h}{h_2}$$

$$h_2 = \frac{hr_2}{r_1 - r_2} \qquad\qquad \text{(i)}$$

or $\qquad \Rightarrow \dfrac{r_2}{r_1} = \dfrac{h_2}{h_1}$

Subtracting 1 from both sides we get

$$\frac{r_2}{r_1} - 1 = \frac{h_2}{h_1} - 1$$

$$\frac{r_2 - r_1}{r_1} = \frac{h_2 - h_1}{h_1}$$

$$\frac{r_1 - r_2}{r_1} = \frac{h_1 - h_2}{h_1}$$

$$\frac{r_1 - r_2}{r_1} = \frac{h}{h_1}$$

$$h_1 = \frac{hr_1}{r_1 - r_2} \qquad\qquad \text{(ii)}$$

Volume of frustum of cone = Volume of cone OAB - Volume of cone OCD

$$= \frac{1}{3}\pi r_1^2 h_1 - \frac{1}{3}\pi r_2^2 h_2$$

$$= \frac{1}{3}\pi \left(r_1^2 h_1 - r_2^2 h_2 \right)$$

$$= \frac{1}{3}\pi \left(r_1^2 \times \frac{hr_1}{r_1 - r_2} - r_2^2 \times \frac{hr_2}{r_1 - r_2} \right) \qquad \left[\text{using (i) and (ii)} \right]$$

$$= \frac{1}{3}\pi \left(\frac{hr_1^3}{r_1 - r_2} - \frac{hr_2^3}{r_1 - r_2} \right)$$

$$= \frac{1}{3}\pi h \left(\frac{r_1^3 - r_2^3}{r_1 - r_2} \right)$$

$$= \frac{1}{3}\pi h \left(\frac{(r_1 - r_2)(r_1^2 + r_1^2 r_2^2 + r_2^2)}{r_1 - r_2} \right) \left[(a^3 - b^3) = (a - b)(a^2 + ab + b^2) \right]$$

$$= \frac{1}{3}\pi h \left(r_1^2 + r_2^2 + r_1^2 r_2^2 \right)$$

Volume of the frustum of a cone $= \dfrac{1}{3}\pi h \left(r_1^2 + r_2^2 + r_1 r_2 \right)$

Hence proved

When you learn math in an interesting way, you never forget.

25 Million
Math classes & counting

100K+
Students learning Math the right way

20+ Countries
Present across USA, UK, Singapore, India, UAE & more.

Why choose Cuemath?

"Cuemath is a valuable addition to our family. We love solving puzzle cards. My daughter is now visualizing maths and solving problems effectively!"

- Gary Schwartz

"Cuemath is great because my son has a one-on-one interaction with the teacher. The instructor has developed his confidence and I can see progress in his work. One-on-one interaction is perfect and a great bonus."

- Kirk Riley

"I appreciate the effort that miss Nitya puts in to help my daughter understand the best methods and to explain why she got a problem incorrect. She is extremely patient and generous with Miranda."

- Barbara Cabrera

Get the Cuemath advantage

Book a FREE trial class