Chapter - 5: Arithmetic Progressions

Exercise 5.1 (Page 99 of Grade 10 NCERT)

Q1. In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?

i) The taxi fare after each km, when the fare is Rs. 15 for the first km and Rs. 8 for each of additional km.

ii) The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.

iii) The cost of digging a well after every meter of digging, when it costs ₹150 for the first meter and rises by Rs. 50 for each subsequent meter.

iv) The amount of money in the account every year, when Rs. 10000 is deposited at compound interest at 8% per annum.

Reasoning:

An arithmetic progression is a sequence of number in which each term is obtained by adding a fixed number to the preceding term except the first term.

General form of an arithmetic progression is $a,\ (a+d),\ (a+2d),\ (a+3d),$ …….. Where a is the first term and d is the common difference.

i) Known:
Charges for the first km and additional km.

Unknown:
Whether it is an arithmetic progression or not?

Solution:

Taxi fare for 1 km = Rs. $15\,(a_1)$

Taxi fare for 2 km = 15+8 = Rs. $23\,(a_2)$

Taxi fare for 3 km = 15+8+8 = Rs. 31 (a_3)

And so on.

$$(a_2)-(a_1)=Rs.(23-15)=Rs.\ 8$$

$$(a_3)-(a_2)=Rs.(31-23)=Rs.\ 8$$

Every time the difference is same.

Answer:

So, this forms an AP with first term 15 and the difference is 8.

ii) Known:

The amount of air present.

Unknown:

Whether it is an arithmetic progression or not?

Solution:

Let the amount of air in the cylinder be x.

So $a_1 = x$

After first time removal, $a_2 = x - \frac{x}{4}$

$$a_2 = \frac{3x}{4}$$

After second time removal, $a_3 = \frac{3}{4} - \frac{1}{4}\left(\frac{3x}{4}\right) = \frac{3x}{4} - \frac{3x}{16} = \frac{12x - 3x}{16} = \frac{9x}{16}$

$$a_3 = \frac{9x}{16}$$

After third time removal, $a_4 = \frac{9x}{16} - \frac{1}{4}\left(\frac{9x}{16}\right) = \frac{9x}{16} - \frac{9x}{64} = \frac{36x - 9x}{64} = \frac{27x}{64}$

$$a_4 = \frac{27x}{64}$$

$$a_2 - a_1 = \frac{3x}{4} - x = \frac{3x - 4x}{4} = \frac{-x}{4}$$

$$a_3 - a_2 = \frac{9}{16}x - \frac{3}{4}x = \frac{9x - 12x}{16} = \frac{-3x}{16}$$

$$(a_3 - a_2) \neq (a_2 - a_1)$$

Answer:

This is not forming an AP.

iii) Known:

Cost of digging well for every meter and subsequent meter.

Unknown:

Whether it is an arithmetic progression or not.

Solution:

Cost of digging the well after 1 meter = Rs. $150(a_1)$

Cost of digging the well after 2 meters = Rs. 150 + 50 = Rs. $200(a_2)$

Cost of digging the well after 3 meters = Rs. 150 +50 +50 = Rs. $250(a_3)$

$$(a_2 - a_1) = 200 - 150 = \text{Rs.}50$$
$$(a_3 - a_2) = 250 - 200 = \text{Rs.}50$$
$$(a_2 - a_1) = (a_3 - a_2)$$

Answer:

So, this list of numbers from an AP with first term as Rs. 150 and common difference is Rs. 50.

iv) Known:

Deposited amount and rating interest.

Unknown:

Whether it is an arithmetic progression or not.

Solution:

Amount present when the amount is P and the interest is r % after n years is

$$A = \left[P\left(1 + \frac{r}{100}\right)\right]$$

P = 10,000

r = 8 %

For first year $(a_1) = 10000\left(1+\frac{8}{100}\right)$

For second year $(a_2) = 10000\left(1+\frac{8}{100}\right)^2$

For third year $(a_3) = 10000\left(1+\frac{8}{100}\right)^3$

For fourth year $(a_4) = 10000\left(1+\frac{8}{100}\right)^4$ And so on.

$$(a_2 - a_1) = 10000\left(1+\frac{8}{100}\right)^2 - 10000\left(1+\frac{8}{100}\right)$$
$$= 10000\left(1+\frac{8}{100}\right)\left[1+\frac{8}{100}-1\right]$$
$$= 10000\left(1+\frac{8}{100}\right)\left(\frac{8}{100}\right)$$
$$(a_3 - a_2) = 10000\left(1+\frac{8}{100}\right)^3 - 10000\left(1+\frac{8}{100}\right)^2$$
$$= 10000\left(1+\frac{8}{100}\right)^2\left[1+\frac{8}{100}-1\right]$$
$$= 10000\left(1+\frac{8}{100}\right)^2\left(\frac{8}{100}\right)$$
$$(a_3 - a_2) \neq (a_2 - a_1)$$

Answer:

The amount will not form an AP.

Q2. Write first four terms of AP, When the first term a and the common difference d are given as follows:

i) $a = 10, d = 10$ ii) $a = -2$ $, d = 0$

iii) $a = 4, d = -3$ iv) $a = -1, d = \frac{1}{2}$

v) $a = -1.25, d = -0.25$

Reasoning:

General form of an arithmetic progression is $a,\ (a+d),\ (a+2d),\ (a+3d),\ \ldots\ldots.$
Where a is the first term and d is the common difference.

i) Known:

$a=10$ and $d=10$

Unknown:

First four terms of the AP.

Solution:

First term $a=10$
Second term $a+d=10+10=20$
Third term $a+2d=10+20=30$
Fourth term $a+3d=10+30=40$

Answer:

The first four terms of AP are 10, 20, 30, and 40.

ii) Known:

$a=-2$ and $d=0$

Unknown:

First four terms of the AP.

Solution:

First term $a=-2$
Second term $a+d=-2+0=-2$
Third term $a+2d=-2+0=-2$
Fourth term $a+3d=-2+0=-2$

Answer:

The first four terms of AP are -2, -2, -2, and -2.

iii) Known:

$a=4\ d=-3$

Unknown:

First four terms of the AP.

Solution:

First term $a = 4$
Second term $a + d = 4 + (-3) = 1$
Third term $a + 2d = 4 - 6 = -2$
Fourth term $a + 3d = 4 - 9 = -5$

Answer:

The first four terms of AP are 4, 1, -2, -5.

iv) Known:

$a = -1\ d = -\frac{1}{2}$

Unknown:

First four terms of the AP.

Solution:

$$\text{First term } = a = -1$$
$$\text{Second term } = a + d = -1 + \frac{1}{2} = -\frac{1}{2}$$
$$\text{Third term } = a + 2d = -1 + 1 = 0$$
$$\text{Fourth term } = a + 3d = -1 + \frac{3}{2} = \frac{1}{2}$$

Answer:

The first four terms of AP are $-1, -\frac{1}{2}, 0, \frac{1}{2}$.

v) Known:

$a = -1.25\ d = -\ 0.25$

Unknown:

First four terms of the AP.

Solution:

$$\begin{aligned}
\text{First term } &= a = -1.25 \\
\text{Second term } &= a + d \\
&= -1.25 + (-0.25) \\
&= -1.25 - 0.25 \\
&= -1.5 \\
\text{Third term } &= a + 2d \\
&= -1.25 + 2 \times (-0.25) \\
&= -1.25 - 0.50 \\
&= -1.75 \\
\text{Fourth term } &= a + 3d \\
&= -1.25 + 3 \times (-0.25) \\
&= -1.25 - 0.75 \\
&= -2.00
\end{aligned}$$

Answer:

The first four terms of AP are -1.25, -1.5, -1.75, and - 2.00.

Q3. For the following APs, write the first term and the common difference:

$i)$ $3, 1, -1, -3 \ldots\ldots$ $ii)$ $-5, -1, 3, 7 \ldots\ldots$

$ii)$ $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3} \ldots\ldots$ $iv)$ $0.6, 1.7, 2.8, 3.9 \ldots$

Reasoning:

General form of an arithmetic progression is a, $(a+d)$, $(a+2d)$, $(a+3d)$, …….. Where a is the first term and d is the common difference.

i) Known:

3, 1, -1, -3 are in AP

Unknown: First term and the common difference of the AP.

Solution:

The AP is 3, 1, -1, -3.

First term $a = 3$

Common difference $= a_2 - a_1$

$= 1 - 3$

$= -2$

Answer:

First term is 3 and the common difference is -2.

ii) Known:

-5, -1, 3, 7 are in AP

Unknown:

First term and the common difference of the AP.

Solution:

The AP is -5, -1, 3, 7.

First term $a = -5$

Common difference is $= a_2 - a_1$

$= -1 - (-5)$

$= -1 + 5$

$= 4$

Answer:

First term -5 and the common difference is 4.

iii) Known:

$\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}$ are in AP

Unknown:

First term and the common difference of the AP.

Solution:

The AP is $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}$

First term $a = \frac{1}{3}$

Common difference is $= a_2 - a_1 = \frac{5}{3} - \frac{1}{3} = \frac{5-1}{3} = \frac{4}{3}$

Answer:

First term is $\frac{1}{3}$ and the common difference is $\frac{4}{3}$

iv) Known:

$0.6, 1.7, 2.8, 3.9$ are in AP

Unknown:

First term and the common difference of the AP.

Solution:

The AP is $0.6, 1.7, 2.8, 3.9$

First term $a = 0.6$

Common difference is $= a_2 - a_1$
$= 1.7 - 0.6$
$= 1.1$

Answer:

First term = 0.6 and the common difference = 1.1

Q4. Which of the followings are APs? If they form an AP, Find the common difference d and write three more terms.

$i)\ 2,4,8,16................$

$ii)\ 2,\frac{5}{2},3,\frac{7}{2}................$

$iii)\ -1.2,-3.2,-5.2,-7.2................$

$iv)\ -10,-6,-2,2................$

$v)\ 3,3+\sqrt{2},3+2\sqrt{2},3+3\sqrt{2}................$

$vi)\ 0.2,0.22,0.222,0.2222................$

$vii)\ 0,-4,-8,-12................$

$viii)\ -\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},................$

$ix)\ 1,3,9,27,................$

$x)\ a,2a,3a.4a,................$

$xi)\ a,a^2,a^3,a^4................$

$xii)\ \sqrt{2},\sqrt{8},\sqrt{18},\sqrt{32}.............$

$xiii)\ \sqrt{3},\sqrt{6},\sqrt{9},\sqrt{12}.............$

$xiv)\ 1^2,3^2,5^2,7^2$

$xv) 1^2,5^2,7^2,73.............$

Reasoning:

General form of an arithmetic progression is $a,\ (a+d),\ (a+2d),\ (a+3d),$ …….. Where a is the first term and d is the common difference.

i) Known:

2, 4, 8, 16…..

Unknown:

Common difference and next three more terms of AP if it is an AP.

Solution:

The given numbers are 2, 4,8,16.
First term $a = 2$
Common difference $d = a_2 - a_1 = 4 - 2 = 2$
Common difference $d = a_3 - a_2 = 8 - 4 = 4$
$(a_3 - a_2) \neq (a_2 - a_1)$

Answer:

2, 4, 8, 16 are not in AP, because the common difference is not equal.

ii) Known:

$2, \frac{5}{2}, 3, \frac{7}{2}$

Unknown:

Common difference and next three more terms of AP if it is an AP.

Solution:

The given numbers are $2, \frac{5}{2}, 3, \frac{7}{2}$

First term $a = 2$

Common difference $d = a_2 - a_1 = \frac{5}{2} - 2 = \frac{5-4}{2} = \frac{1}{2}$

Common difference $d = a_3 - a_2 = 3 - \frac{5}{2} = \frac{6-5}{2} = \frac{1}{2}$

Since $a_3 - a_2 = a_2 - a_1$.

$2, \frac{5}{2}, 3, \frac{7}{2}$ forms an AP and common difference is $\frac{1}{2}$.

The next three terms are:

$$\begin{aligned}
\text{Fifth term } &= a + 4d \\
&= 2 + 4 \times \frac{1}{2} \\
&= 2 + 2 \\
&= 4 \\
\text{Sixth term} &= a + 5d \\
&= 2 + 5 \times \frac{1}{2} \\
&= 2 + \frac{5}{2} \\
&= \frac{4+5}{2} \\
&= \frac{9}{2} \\
\text{Seventh term} &= a + 6d \\
&= 2 + 6 \times \frac{1}{2} \\
&= 5
\end{aligned}$$

Answer:

$2, \frac{5}{2}, 3, \frac{7}{2}$ forms an AP and the common difference is $\frac{1}{2}$ .The next three terms are $4, \frac{9}{2}, 5$.

iii) Known:

$-1.2, -3.2, -5.2, -7.2 \ldots\ldots$

Unknown:

Whether it forms an AP. If it is find the common difference and the next three terms of AP.

Solution:

The given numbers are $-1.2, -3.2, -5.2, -7.2 \ldots\ldots$

First term $a = -1.2$

Common difference $d = a_2 - a_1 = -3.2 - (-1.2) = -3.2 + 1.2 = -2$

Common difference $d = a_3 - a_2 = -5.2 - (-3.2)$

$= -5.2 + 3.2 = -2$

Since $a_3 - a_2 = a_2 - a_1$.

It forms an AP.

$$\begin{aligned}
\text{The fifth term } &= a + 4d \\
&= -1.2 + 4(-2) \\
&= -1.2 - 8 = -9.2 \\
\text{The sixth term } &= a + 5\text{d} \\
&= -1.2 + 5(-2) \\
&= -1.2 - 10 \\
&= -11.2 \\
\text{The seventh term } &= a + 6\text{d} \\
&= -1.2 + 6(-2) \\
&= -1.2 - 12 \\
&= -13.2
\end{aligned}$$

Answer:

$-1.2, -3.2, -5.2, -7.2$ forms an AP with common difference -2. The next three terms of AP are -9.2, -11.2, -13.2.

iv) Known:

-10, -6, -2, 2

Unknown:

Whether it forms an AP. If it is find the common difference and the next three terms of AP.

Solution:

The given numbers are -10, -6 ,-2, 2
First term (a) = -10

$$\begin{aligned}\text{Common difference (d)is } &= a_2 - a_1 \\ &= -6-(-10) \\ &= -6+10 \\ &= 4\end{aligned}$$

$$\begin{aligned}\text{Common difference (d) is } &= a_3 - a_2 \\ &= -2-(-6) \\ &= -2+6 \\ &= 4\end{aligned}$$

Since $a_3 - a_2 = a_2 - a_1$.
Fifth Term: $a+4d = -10+16 = 6$
Sixth Term: $a+5d = -10+20 = 10$
Seventh Term: $a+6d = -10+24 = 14$

Answer:

-10, -6 ,-2, 2 forms an AP with common difference 4 and next terms are 6, 10, 14.

v)Known:

$3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2} \ldots\ldots$

Unknown:

Given list of numbers form an AP or not. If it is find the common difference and the next three terms of AP.

Solution:

The list of numbers is $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2} \ldots\ldots$

Common difference (d) is $= a_2 - a_1$

$$= 3 + \sqrt{2} - 3$$

$$= \sqrt{2}$$

Common difference (d)is $= a_3 - a_2$

$$= 3 + 2\sqrt{2} - (3 + \sqrt{2})$$

$$= 3 + 2\sqrt{2} - 3 - \sqrt{2}$$

$$= \sqrt{2}$$

since $a_3 - a_2 = a_2 - a_1$

So $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}\ldots\ldots$ forms an AP with common difference 4.

Next three terms are

Fifth term $= a + 4d$

$$= 3 + 4\sqrt{2}$$

Sixth term $= a + 5d$

$$= 3 + 5\sqrt{2}$$

Seventh term $= a + 6d$

$$= 3 + 6\sqrt{2}$$

Answer:

It is an AP with common difference $\sqrt{2}$ and Next three terms are $3+4\sqrt{2},\ 3+5\sqrt{2},\ 3+6\sqrt{2}$,.

v) Known:

$0.2, 0.22, 0.222, 0.2222\ldots\ldots$

Unknown:

Given list of numbers form an AP or not. If it is find the common difference and the next three terms of AP.

Solution:

The list of numbers are $0.2, 0.22, 0.222, 0.2222\ldots\ldots$

$$\text{Common difference } d = a_2 - a_1$$
$$= 0.22 - 0.2$$
$$= 0.20$$
$$\text{Common difference } d = a_3 - a_2$$
$$= 0.222 - 0.220$$
$$= 0.002$$
$$(a_3 - a_2) \neq (a_2 - a_1)$$

Answer: The given list of numbers does not form an AP.

vii) Known:

0,-4,-8,-12……

Unknown:

Given list of numbers form an AP or not. If it is find the common difference and the next three terms of AP.

Solution:

The list of numbers is 0,-4,-8,-12……

Common difference (d) is = $a_2 - a_1$ = - 4 – 0 = -4

Common difference (d) is = $a_3 - a_2$ = -8 – (-4) = -8 + 4 = -4

Since $a_3 - a_2 = a_2 - a_1$. It forms an AP.

$$\text{Fifth term } = a + 4d$$
$$= 0 + 4(-4)$$
$$= -16$$
$$\text{Sixth term } = a + 5d$$
$$= 0 + 5(-4)$$
$$= -20$$
$$\text{Seventh term } = a + 6d$$
$$= 0 + 6(-4)$$
$$= -24$$

Answer:

The given numbers form an AP with difference -4. The next three terms are -16, -20, -24.

viii) Known:

$$-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}.......$$

Unknown:

Given list of numbers form an AP or not. If it is find the common difference and the next three terms of AP.

Solution:

The given list of numbers is $-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}.......$

Common difference $d = a_2 - a_1$

$$= -\frac{1}{2} - \left(-\frac{1}{2}\right)$$

$$= -\frac{1}{2} + \frac{1}{2}$$

$$= 0$$

Common difference $d = a_3 - a_2$

$$= -\frac{1}{2} - \left(-\frac{1}{2}\right)$$

$$= -\frac{1}{2} + \frac{1}{2} = 0$$

Since $a_3 - a_2 = a_2 - a_1$.The list of numbers forms an AP.

$$\text{The fifth term} = a + 4d$$
$$= -\frac{1}{2} + 4(0)$$
$$= -\frac{1}{2}$$
$$\text{The sixth term} = a + 5d$$
$$= -\frac{1}{2} + 5(0)$$
$$= -\frac{1}{2}$$
$$\text{The seventh term} = a + 6d$$
$$= -\frac{1}{2} + 6(0)$$
$$= -\frac{1}{2}$$

Answer:

The given list of numbers form an AP with common difference d = 0. Next three terms are $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}$.

ix) Known:

1, 3 ,9 ,27.

Unknown:

Given list of numbers form an AP or not. If it is find the common difference and the next three terms of AP.

Solution:

The list of numbers are 1, 3 ,9 ,27

Common difference $d = a_2 - a_1 = 3 - 1 = 2$ = 3 – 1 = 2

Common difference $d = a_3 - a_2 = 9 - 3 = 6$ = 9 -3 =6

Since $a_2 - a_1 \neq a_3 - a_2$

Answer:

The given list of numbers does not form an AP.

x) Known:

$$a, 2a, 3a, 4a$$

Unknown:

Given list of numbers form an AP or not. If it is find the common difference and the next three terms of AP.

Solution:

The list of numbers is $a, 2a, 3a, 4a$.

Common difference $d = a_2 - a_1$

$= 2a - a$

$= a$

Common difference $d = a_3 - a_2$

$= 3a - 2a$

$= a$

since $a_3 - a_2 = a_2 - a_1$, $a,\ 2a,\ 3a,\ 4a$ forms an AP.

The fifth term $= a + 4d = a + 4a = 5a$

The sixth term $= a + 5d = a + 5a = 6a$

The seventh term $= a + 6d = a + 6a = 7a$

Answer:

The given list of numbers form an AP with common difference $d = a$. The next three terms are 5a, 6a, 7a.

xi) Known:

$$a, a^2, a^3, a^4$$

Unknown:

Given list of numbers form an AP or not. If it is find the common difference and the next three terms of AP.

Solution:

The list of numbers is a, a^2, a^3, a^4

Common difference $d = a_2 - a_1$

$= a^2 - a$

$= a(a - 1)$

Common difference $d = a_3 - a_2$

$$= a^3 - a^2$$
$$= a^2(a-1)$$

Since $a_2 - a_1 \neq a_3 - a_2$

Answer:

The given list of numbers does not form an AP.

xii) Known:

$$\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}$$

Unknown:

Given list of numbers form an AP or not. If it is find the common difference and the next three terms of AP.

Solution:

The list of numbers is $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}$

Common difference $d = a_2 - a_1$

$$= \sqrt{8} - \sqrt{2}$$
$$= \sqrt{4 \times 2} - \sqrt{2}$$
$$= 2\sqrt{2} - \sqrt{2}$$
$$= \sqrt{2}$$

Common difference $d = a_3 - a_2$

$$= \sqrt{18} - \sqrt{8}$$
$$= \sqrt{9 \times 2} - \sqrt{4 \times 2}$$
$$= 3\sqrt{2} - 2\sqrt{2}$$
$$= \sqrt{2}$$

since $a_2 - a_1 = a_3 - a_2$ The given numbers form an AP.

The fifth term $= a + 4d$

$$= \sqrt{2} + 4\sqrt{2}$$
$$= 5\sqrt{2}$$
$$= \sqrt{25 \times 2}$$
$$= \sqrt{50}$$

The sixth term $= a + 5d$

$$= \sqrt{2} + 5\sqrt{2}$$
$$= 6\sqrt{2}$$
$$= \sqrt{36 \times 2}$$
$$= \sqrt{72}$$

The seventh term $= \mathrm{a} + 6\mathrm{d}$

$$= \sqrt{2} + 6\sqrt{2}$$
$$= 7\sqrt{2}$$
$$= \sqrt{49 \times 2}$$
$$= \sqrt{98}$$

The list of numbers forms an AP with common difference $\sqrt{2}$. Next three terms are $\sqrt{50}, \sqrt{72}, \sqrt{98}$

xiii)Known:

$$\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}$$

Unknown:

Given list of numbers form an AP or not. If it is find the common difference and the next three terms of AP.

Solution:

The list of numbers is $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}$

Common difference $d = a_2 - a_1$

$$= \sqrt{6} - \sqrt{3}$$
$$= \sqrt{3 \times 2} - \sqrt{3}$$
$$= \sqrt{3}\left(\sqrt{2} - 1\right)$$

$$\text{Common difference } d = a_3 - a_2 = \sqrt{9} - \sqrt{6}$$
$$= \sqrt{3\times3} - \sqrt{3\times2}$$
$$= \sqrt{3}(\sqrt{3} - \sqrt{2})$$
$$\text{since } a_2 - a_1 \neq a_3 - a_2$$

Answer:

The given list of numbers does not form an AP.

xiv)Known:

$1^2, 3^2, 5^2, 7^2$

Unknown:

Given list of numbers form an AP or not. If it is find the common difference and the next three terms of AP.

Solution:

The list of numbers $1^2, 3^2, 5^2, 7^2$

Common difference $d = a_2 - a_1 = 9 - 1 = 8$

Common difference $d = a_3 - a_2 = 25 - 9 = 16$

Since $a_2 - a_1 \neq a_3 - a_2$

Answer:

The given list of numbers does not form an AP.

xv) Known:

$1^2, 5^2, 7^2, 73$

Unknown:

Given list of numbers form an AP or not. If it is find the common difference and the next three terms of AP.

Solution:

The list of numbers is $1^2, 5^2, 7^2, 73$

Common difference $d = a_2 - a_1 = 25 - 1 = 24$

Common difference $d = a_3 - a_2 = 49 - 25 = 24$

Since $a_2 - a_1 = a_3 - a_2$, they form AP

$$\begin{aligned}\text{Fifth term } &= a + 4\text{d} \\ &= 1 + 4 \times 24 \\ &= 1 + 96 \\ &= 97\end{aligned}$$

$$\begin{aligned}\text{Sixth term } &= a + 5d \\ &= 1 + 5 \times 24 \\ &= 1 + 120 \\ &= 121 \\ \text{Seventh term } &= a + 6d \\ &= 1 + 6 \times 24 \\ &= 1 + 144 \\ &= 145\end{aligned}$$

Answer:

The list of numbers forms an AP with common difference 24. The next three terms are 97, 121, and 145.

Chapter- 5: Arithmetic Progressions

Exercise 5.2 (Page 105 of Grade 10 NCERT)

Q1. Fill in the blanks in the following table, given that a is the first term and d is the Common difference and a_n the n[th] term of the AP.

a	d	n	a_n
7	3	8	…….
- 18	……..	10	0
…….	-3	18	-5
-18. 9	2.5	…….	3.6
3.5	0	105	…….

i) Reasoning:

$$a_n = a + (n-1)d$$

Known:

$$a = 7, d = 3, n = 8$$

Unknown:

$$a_n$$

Solution:

$$\begin{aligned} a_n &= a + (n-1)d \\ a_8 &= 7 + (8-1)3 \\ &= 7 + 7 \times 3 \\ &= 7 + 21 \\ &= 28 \end{aligned}$$

Answer:

$$a_n = 28$$

ii) Known:

$$a = -18, a_n = 0, n = 10$$

Unknown:

$$d$$

Solution:

$$a_n = a + (n-1)d$$
$$0 = -18 + (10-1)d$$
$$0 = -18 + 9d$$
$$9d = 18$$
$$d = 2$$

Answer:

$$d = 2$$

iii) Known:

$$d = -3, a_n = -5, n = 18$$

Unknown:

$$a$$

Solution:

$$a_n = a + (n-1)d$$
$$-5 = a + (18-1)(-3)$$
$$-5 = a + 17 \times (-3)$$
$$-5 = a - 51$$
$$a = 51 - 5$$
$$a = 46$$

Answer:

$$a = 46$$

iv) Known:

$$a_n = 3.6, d = 2.5, a = -18.9$$

Unknown:

$$n$$

Solution:

$$a_n = a + (n-1)d$$
$$3.6 = -18.9 + (n-1)2.5$$
$$3.6 + 18.9 = 2.5(n-1)$$
$$22.5 = 2.5(n-1)$$
$$n-1 = \frac{22.5}{2.5}$$
$$n-1 = \frac{225}{25}$$
$$n-1 = 9$$
$$n = 10$$

Answer:

$$n = 10$$

v) Known:

$$a = 3.5, d = 0, n = 105$$

Unknown:

$$a_n$$

Solution:

$$a_n = a + (n-1)d$$
$$a_n = 3.5 + (105-1)(0)$$
$$a_n = 3.5 + 104 \times 0$$
$$a_n = 3.5$$

Answer:

$$a_n = 3.5$$

Q2. Choose the correct choice in the following and justify:

i) 30^{th} term of the AP: 10,7,4……. is

a) 97   b) 77   c) -77   d) -87

ii) 11^{th} term of AP: -3, $-\frac{1}{2}$,2 is:

a) 28   b) 22   c) – 38   d) $-48\frac{1}{2}$

i) Reasoning:

n^{th} term of an AP is $a_n = a + (n-1)d$

Known:

The AP

Unknown:

30^{th} term of the AP.

Solution:

The AP is 10,7,4, …..

$a = 10$

Common difference $d = a_2 - a_1$
$= 7 - 10$
$= -3$

$$
\begin{aligned}
a_n &= a + (n-1)d \\
a_{30} &= 10 + (30-1)(-3) \\
&= 10 + (29)(-3) \\
&= 10 - 87 \\
&= -77
\end{aligned}
$$

Answer:

The correct option is c = -77

30^{th} term is -77.

ii) Known:

The AP.

Unknown: 11^{th} term of the AP.

Solution:

The AP is -3, $-\frac{1}{2}$, 2

Common difference $d = a_2 - a_1$

$$= -\frac{1}{2} - (-3)$$

$$= -\frac{1}{2} + 3$$

$$= \frac{-1+6}{2}$$

$$d = \frac{5}{2}$$

$$a_n = a + (n-1)d$$

$$a_{11} = -3 + (11-1)\frac{5}{2}$$

$$= -3 + 10 \times \frac{5}{2}$$

$$= -3 + 25$$

$$a_{11} = 22$$

Answer:

The correct option is B. 11th term is 22.

Q3. In the following AP, find the missing terms in the boxes:

i) 2, $\square$, 26

ii) $\square$, 13, $\square$, 3

iii) 5, $\square$, $\square$, $9\frac{1}{2}$

iv) -4, $\square$, $\square$, $\square$, $\square$ 6

v) $\square$,38, $\square$, $\square$, $\square$, -22

i) Reasoning:

$$a_n = a + (n-1)d$$

Known:

The AP with missing term.

Unknown:

Second term.

Solution:

Let common difference be d
First term $a = 2$

Third term $= a + 2d$
$= 2 + 2d$
Third term = 26 (Given)

$$2 + 2d = 26$$
$$2d = 26 - 2$$
$$2d = 24$$
$$d = 12$$

Second term $= a + d$
$= 2 + 12$
$= 14$

Answer:

The missing term in the box is 14.

ii) Known:

The AP with missing terms.

Unknown:

First and the third term.

Solution:

$$a_n = a + (n-1)d$$
$$a_2 = 13,\ a_4 = 3$$

Second term = 13

$$a + (2-1)d = 13$$
$$a + d = 13 \qquad \ldots(1)$$

Fourth term = 3

$$a + (4-1)d = 3$$
$$a + 3d = 3 \qquad \ldots(2)$$

Solving (1) and (2) for a and d

Equation (2) – Equation (1) gives

$$3d - d = 3 - 13$$
$$2d = -10$$
$$d = -5$$

Putting d in equation (1)

$$a + (-5) = 13$$
$$a - 5 = 13$$
$$a = 18$$

- Third term =18-10
  =8

Answer:

Hence the missing terms in the boxes are 18 and 8.

iii) Known:
The AP with missing terms.

Unknown:
Missing terms in the boxes.

Solution:

First term $a = 5$

Fourth term $a_4 = \frac{19}{2}$

Fourth term $a_4 = a + (4-1)d$
$= 5 + 3d$

$$5 + 3d = \frac{19}{2}$$
$$6d = 9$$
$$d = \frac{3}{2}$$

Second term $a_2 = a + d$
$= 5 + \frac{3}{2}$
$= \frac{13}{2}$

Third term $a_3 = a + 2d$
$= 5 + 2 \times \frac{3}{2}$
$= 5 + 3$
$= 8$

Answer:

The missing terms in the boxes are $\frac{13}{2}$ and 8.

iv) Known:
The AP with missing terms.

Unknown:
Missing terms in the boxes.

Solution:

First term $a = -4$

Sixth term $a_6 = 6$

$a+5d=6$
$-4+5d=6$
$5d=10$
$d=2$

Second term $a+d=-4+2=-2$

Third term $a+2d=-4+4=0$

Fourth term $a+3d=-4+6=2$

Fifth term $a+4d=-4+8=4$

Answer:
Hence the missing terms are -2,0,2,4.

v) Known:
The AP with missing terms.

Unknown:
Missing terms in the boxes.

Solution:

Let the first term be = a

Common difference = d

Second term = 38 (Given)

$a+d = 38 \ldots(1)$

Sixth term = -22 (Given)

$a + 5d = -22 \ldots.(2)$

Solving (1) and (2) for a and d

$4d=-60$
$d=-15$

Put the d value in …(1)

$a-15=38$
$a=53$

Third term = $a+2d = 53-30=23$

Fourth term = $a + 3d$

$= 53 - 45 = 8$

Fifth term = $a + 4d$

$= 53 - 60 = -7$

Answer:

Hence the missing terms are 53,23,8,-7.

Q4. Which term of the AP: 3, 8, 13, 18……. is 78.

Reasoning:

$$a_n = a + (n-1)d$$

Known: The AP with missing terms.

Unknown: Which term will be 78.

Solution:

The AP is 3, 8, 13, 18.

First term $a = 3$

Second term $a + d = 8$

Common difference $d = 8 - 3 = 5$

$$a_n = a + (n-1)d = 78$$
$$3 + (n-1)5 = 78$$
$$5(n-1) = 78 - 3$$
$$n - 1 = 15$$
$$n = 16$$

Answer:

78 is the 16^{th} term of AP.

Q5. Find the number of terms in each of the following APs:

i) 7,13,19,……,205

ii) $18, 15\frac{1}{2}, 13, \ldots\ldots, -47$

Reasoning:

$$a_n = a + (n-1)d$$

i) Known:

The AP

Unknown:

No. of terms of the AP.

Solution:

$$a_n = a + (n-1)d$$

Where a_n is the nth term, a is the first term, d is the common difference and n is the number of terms.

$$a = 7$$
$$d = 13 - 7 = 6$$
$$a_n = 205$$
$$a_n = a + (n-1)d$$
$$a + (n-1)d = 205$$
$$7 + (n-1)6 = 205$$
$$(n-1)6 = 205 - 7$$
$$(n-1)6 = 198$$
$$(n-1) = 33$$
$$n = 33 + 1$$
$$n = 34$$

Answer:

Number of terms in the given AP is 34.

ii) Known:

The AP

Unknown:

No. of terms of the AP.

Solution:

$$\begin{aligned} d &= 15\frac{1}{2} - 18 \\ &= \frac{31}{2} - 18 \\ &= -\frac{5}{2} \\ a_n &= a + (n-1)d \\ a + (n-1)d &= -47 \\ 18 + (n-1)(-\frac{5}{2}) &= -47 \\ n - 1 &= 26 \\ n &= 26 + 1 \\ &= 27 \end{aligned}$$

Answer:

The number of terms in the given AP is 27.

Q6. Check Whether -150 is a term of the AP 11,8,5,2 …..

Reasoning:

$$a_n = a + \left(n-1\right)d$$

Where a_n is the nth term, a is the first term, d is the common difference and n is the number of terms.

Known:

The AP

Unknown:

Whether -150 is a term of AP.

Solution:

11,8,5,2 are in AP

First term $a = 11$

Common difference $d = 8 - 11 = -3$

$$a_n = a + (n-1)d$$
$$a + (n-1)d = -150$$
$$11 + (n-1)(-3) = -150$$
$$n - 1 = \frac{161}{3}$$
$$n = \frac{164}{3}$$

Answer:

$n = \frac{164}{3}$ which is a fraction. Given number is not a term of AP 11, 8, 5, 2 ……n should be positive integer and not a fraction.

Q7. Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.

Reasoning:

$a_n = a + (n-1)d$ is the general term of AP. Where a_n is the nth term, a is the first term, d is the common difference and n is the number of terms.

Known:

11th and 16th term of AP.

Unknown:

31st term of AP.

Solution:

$$a_n = a + (n-1)d$$
$$a_{11} = 38$$
$$a + (11-1)d = 38$$
$$a + 10d = 38 \ldots\ldots\ldots\ldots\ldots(1)$$
$$a_{16} = 73$$
$$a + 15d = 73 \quad \ldots.(2)$$

By solving the two equations (1) & (2) for a, d

$$5d = 35$$
$$d = 7$$

Putting d in the (1) equation

$$a = 38 - 70$$
$$= -32$$

31st terms is,

$$a_{31} = a + (31-1)d$$
$$= -32 + 30 \times 7$$
$$= -32 + 210$$
$$= 178$$

Answer:

The 31st term of AP is 178.

Q8. An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.

Reasoning:

$a_n = a + (n-1)d$ is the general term of AP. Where a_n is the nth term, a is the first term, d is the common difference and n is the number of terms.

Known:

Number of terms in the AP. 3rd term is 12 and last term is 106

Unknown:

29^{th} term.

Solution:

$$a_n = a + (n-1)d$$

Third term of AP is $a + 2d$

$$a + 2d = 12.........(1)$$

Last term = 106

50th term =106

$$a + (50-1)d = 106$$
$$a + 49d = 106..........(2)$$

Solving (1) & (2) for the values of a and d .

$$47d = 94$$
$$d = 2$$

Putting $d = 2$ in equation (1)

$$a + 2 \times 2 = 12$$
$$a + 4 = 12$$
$$a = 12 - 4$$
$$a = 8$$

29^{th} term of AP is

$$a_{29} = a + (29-1)d$$
$$a_{29} = 8 + (28)2$$
$$a_{29} = 8 + 56$$
$$a_{29} = 64$$

Answer:

29^{th} term of AP is 64.

Q9. If the 3^{rd} and the 9^{th} terms of an AP are 4 and -8, respectively, which term of this AP is zero.

Reasoning:

$a_n = a + (n-1)d$ is the general term of AP. Where a_n is the nth term, a is the first term, d is the common difference and n is the number of terms.

Known:

3rd and 9th term of AP,

Unknown:

Which term of AP is zero.

Solution:

Third term of the AP = 4

$a + 2d = 4........(1)$

9th term of AP = -8

$a + 8d = -8.......(2)$

Solving (1) and (2) for a and d

$$\begin{array}{rcl} a + 2d &=& 4 \\ a + 8d &=& -8 \\ \hline -6d &=& 12 \\ d &=& -2 \end{array}$$

Putting $d = -2$ in equation (1)

$$a - 4 = 4$$
$$a = 8$$

$$a + (n-1)d = 0$$
$$8 + (n-1)(-2) = 0$$
$$n - 1 = 4$$
$$n = 5$$

Answer:

5^{th} term will be 0.

Q10. The 17^{th} term of an AP exceeds its 10^{th} term by 7. Find the common difference.

Reasoning:

$a_n = a + (n-1)d$ is the general term of AP. Where a_n is the nth term, a is the first term, d is the common difference and n is the number of terms.

Known:

The difference between the 17^{th} and 10^{th} term.

Unknown:

Common difference d

Solution:

$$a_{17} = a + (17-1)d$$
$$a_{17} = a + 16d$$

$$a_{10} = a + (10-1)d$$
$$a_{10} = a + 9d$$

$$a_{17} - a_{10} = 7$$
$$16d - 9d = 7$$
$$d = 1$$

Answer:

The common difference is 1.

Q11. Which term of the AP 3,15,27,39….. will be 132 more than its 54^{th} term?

Reasoning:

$a_n = a + (n-1)d$ is the general term of AP. Where a_n is the nth term, a is the first term, d is the common difference and n is the number of terms.

Known:
The AP.

Unknown:

Which term will be 132 more than 54^{th} term.

Solution:

Given AP is 3, 15, 27, 39.

First term $a = 3$

Second term $a + d = 15$

$d = 15 - 3 = 12$

54^{th} term of the AP is

$$\begin{aligned} a_{54} &= a + (54-1)d \\ &= 3 + 53 \times 12 \\ &= 3 + 636 \\ &= 639 \end{aligned}$$

Let n^{th} term of AP be 132 more than 54^{th} term (Given)
We get , 132+639=771

$$a_n = 771$$

$$\begin{aligned} a_n &= a + (n-1)d \\ 771 &= 3 + (n-1)12 \\ 768 &= (n-1)12 \\ (n-1) &= 64 \\ n &= 65 \end{aligned}$$

Therefore, 65^{th} term will be 132 more than 54th term.

Alternatively,

Let n^{th} term be 132 more than 54^{th} term.

$$\begin{aligned} n &= 54 + \frac{132}{12} \\ &= 54 + 11 = 65^{th} \text{ Term} \end{aligned}$$

Answer:

65^{th} term will be 132 more than 54th term

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Q12. Two APs have the same common difference. The difference between their 100th term is 100, what is the difference between their 1000th terms?

Reasoning:

$a_n = a + (n-1)d$ is the general term of AP. Where a_n is the nth term, a is the first term, d is the common difference and n is the number of terms.

Known:

Two APs with the same common difference and difference between their 100th term.

Unknown:

Difference between their 1000th term

Solution:

Let the first term of these A.P.s be a_1 and b_1 respectively and the Common difference of these A.P's be d

For first A.P.,

$$\begin{aligned} a_{100} &= a_1 + (100-1)d \\ &= a_1 + 99\text{d} \\ a_{1000} &= a_1 + (1000-1)d \\ a_{1000} &= a_1 + 999d \end{aligned}$$

For second A.P.,

$$\begin{aligned} b_{100} &= b_1 + (100-1)d \\ &= b_1 + 99d \\ b_{1000} &= b_1 + (1000-1)d \\ &= \text{b}_1 + 999d \end{aligned}$$

Given that, difference between

100th term of these A.P.s = 100

Thus, we have

$$(a_1 + 99d) - (b_1 + 99d) = 100$$

$$a_1 - b_1 = 100 \qquad \text{..................Equation}(1)$$

Difference between 1000th terms of these A.P.s

$$(a_1 + 999d) - (b_1 + 999d) = a_1 - b_1 \qquad \text{...............Equation}(2)$$

From equation (1) & Equation (2),

This difference, $a_1 - b_1 = 100$
Hence, the difference between 1000th terms of these A.P. will be 100.

Answer:

The difference between 1000th terms of these APs will be 100.

Q13. How many three-digit numbers are divisible by 7?

Reasoning:
$a_n = a + (n-1)d$ is the general term of AP. Where a_n is the nth term, a is the first term, d is the common difference and n is the number of terms.

Known:
three-digit numbers

Unknown:
Number of all three-digit numbers which are divisible by 7

Solution:

First three-digit number that is divisible by 7 = 105
Next number = 105 + 7 = 112

Therefore, the series becomes 105, 112, 119, ...

All are three-digit numbers which are divisible by 7 and thus, all these are terms of an A.P. having first term as 105 and common difference as 7.

When we divide 999 by 7, the remainder will be 5.
Clearly, 999 − 5 = 994 is the maximum possible three-digit number that is divisible by 7.
Hence the final series is as follows:
105, 112, 119, ..., 994

Let 994 be the nth term of this A.P.

$a = 105$
$d = 7$
$a_n = 994$
$n = ?$

We know that the nth term of an A.P. Series,

$$a_n = a + (n-1)d$$
$$994 = 105 + (n-1)7$$
$$889 = (n-1)7$$
$$n-1 = \frac{889}{7}$$
$$n-1 = 127$$
$$n = 127 + 1$$
$$n = 128$$

Answer:

Therefore, 128 three-digit numbers are divisible by 7.

Q14. How many multiples of 4 lie between 10 and 250?

Reasoning:

$a_n = a + \left(n-1\right)d$ is the general term of AP. Where a_n is the nth term, a is the first term, d is the common difference and n is the number of terms.

Known:

Numbers between 10 and 250

Unknown:

Multiples of 4 between 10 and 250.

Solution:

By Observation, First multiple of 4 that is greater than 10 is 12.
Next will be 16.

Therefore, the series will be as follows: 12, 16, 20, 24, ...

All these are divisible by 4 and thus, all these are terms of an A.P. with first term as 12 and common difference as 4.

When we divide 250 by 4, the remainder will be 2. Therefore, 250 – 2 = 248 is divisible by 4 which is the largest multiple of 4 within 250.

Hence the final series is as follows:
12, 16, 20, 24, ..., 248

Let 248 be the n^{th} term of this A.P.

We know that the n^{th} term of an A.P. Series,

$$a = 12$$
$$d = 4$$
$$a_n = 248$$
$$a_n = a + (n-1)d$$
$$248 = 12 + (n-1)4$$
$$\frac{236}{4} = n - 1$$
$$n = 60$$

Answer:
Therefore, there are 60 multiples of 4 between 10 and 250.

Q15. For what value of n, are the nth terms of two APs 63, 65, 67, and 3, 10, 17, ... equal?

Reasoning:
$a_n = a + (n-1)d$ is the general term of AP. Where a_n is the nth term, a is the first term, d is the common difference and n is the number of terms.

Known:
Two different APs

Unknown:
Value of n so that two APs have equal nth term

Solution:

If nth terms of the two APs 63, 65, 67, ... and 3, 10, 17, ... are equal.
Then, $63 + (n-1)2 = 3 + (n-1)7$ ....Equation (1)
[Since In 1st AP, $a = 63$, $d = 65 - 3 = 2$ and in 2nd AP, $a = 3, d = 10 - 3 = 7$]

By Simplifying Equation (1)

$$7(n-1) - 2(n-1) = 63 - 3$$
$$7n - 7 - 2n + 2 = 60$$
$$5n - 5 = 60$$
$$n = \frac{65}{5}$$
$$n = 13$$

Answer:
The 13th terms of the two given APs are equal.

Q16. Determine the A.P. whose third term is 16 and the 7th term exceeds the 5th term by 12.

Reasoning:

$a_n = a + (n-1)d$ is the general term of AP. Where a_n is the nth term, a is the first term, d is the common difference and n is the number of terms.

Known:

Third term and relation between 5th term and 7th term.

Unknown:

The AP.

Solution:

Let a be the first term and d the common difference.

Hence from given, $a_3 = 16$ and $a_7 - a_5 = 12$

$$a + (3-1)d = 16$$
$$a + 2d = 16 \text{ ....} Equation\ (1)$$

Using $a_7 - a_5 = 12$

$$[a + (7-1)d] - [a + (5-1)d] = 12$$
$$[a + 6d] - [a + 4d] = 12$$
$$2d = 12$$
$$d = 6$$

By Substituting this in Equation (1), we obtain

$$a + 2 \times 6 = 16$$
$$a + 12 = 16$$
$$a = 4$$

Therefore, A.P. will be $4 + 6,\ 4 + 2 \times 6,\ 4 + 3 \times 6....$

Hence the series will be 4, 10, 16, 22, ...

Answer:

The series will be 4, 10, 16, 22, ...

Q17. Find the 20th term from the last term of the A.P. 3, 8, 13, ..., 253

Reasoning:

$a_n = a + (n-1)d$ is the general term of AP. Where a_n is the nth term, a is the first term, d is the common difference and n is the number of terms.

Known:

The AP

Unknown:

20^{th} term from the last term of AP.

Solution:

Given A.P. is 3, 8, 13, ..., 253

From Given,

As the 20^{th} term is considered from last $a = 253$
Common difference, $d = 3 - 8 = -5$ (Considered in reverse order)

We know that the nth term of an A.P. Series,

$a_n = a + (n-1)d$

Hence 20^{th} Term, $a_{20} = a + (20-1)d$

$$\begin{aligned} a_{20} &= 253 + (20-1)(-5) \\ &= 253 - 19 \times 5 \\ &= 253 - 95 \\ &= 158 \end{aligned}$$

Answer:

Therefore, 20th term from the last term is 158.

Q18. The sum of 4th and 8th terms of an A.P. is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the A.P.

Reasoning:

$a_n = a + (n-1)d$ is the general term of AP. Where a_n is the nth term, a is the first term, d is the common difference and n is the number of terms.

Known:

Sum of 4^{th} and 8^{th} terms is 24 and sum of 6^{th} and 10^{th} terms is 44.

Unknown:

First three terms of the AP.

Solution:

Let a be the first term and d the common difference.

Given,

$$a_4 + a_8 = 24$$
$$(a+3d)+(a+7d)=24$$
$$\Rightarrow 2a+10\,d=24$$
$$\Rightarrow a+5d \;=12 \text{ ....Equation}(1)$$

Also,

$$a_6 + a_{10} = 44$$
$$(a+5d)+(a+9d)=44$$
$$\Rightarrow 2a+14d=44$$
$$\Rightarrow a+7d=22 \text{ ....Equation}(2)$$

On subtracting Equation (1) from (2), we obtain

$$(a+7d)-(a+5d)=22-12$$
$$a+7d-a-5d=10$$
$$2d=10$$
$$d=5$$

By Substituting the value of $d=5$ in Equation (1), we obtain

$$a+5d=12$$
$$a+5\times5=12$$
$$a+25=12$$
$$a=-13$$

The first three terms are a, $(a+d)$ and $(a+2d)$

Substituting the values of a and d, we get –13, (–13+5) and (–13+2×5)

i.e., –13, –8 and –3

Answer:
Therefore, the first three terms of this A.P. are −13, −8, and −3.

Q19. Subba Rao started work in 1995 at an annual salary of Rs 5000 and received an increment of Rs 200 each year. In which year did his income reach Rs 7000?

Reasoning:

$a_n = a + (n-1)d$ is the general term of AP. Where a_n is the nth term, a is the first term, d is the common difference and n is the number of terms.

Known:

Annual salary of Rs 5000 and increment of Rs 200 each year.

Unknown:

The year in which income reached Rs 7000.

Solution:

From the Given Data, incomes received by Subba Rao in the years 1995,1996,1997... are 5000, 5200, 5400, ... 7,000

From Observation,

$$a = 5000$$
$$d = 200$$

Let after nth year, his salary be Rs 7000.
Hence $a_n = 7000$

We know that the nth term of an A.P. Series,

$$a_n = a + (n-1)d$$

By Substituting above values,

$$7000 = 5000 + (n-1)200$$
$$200(n-1) = 7000 - 5000$$
$$n - 1 = \frac{2000}{200}$$
$$n = 10 + 1$$
$$n = 11$$

Therefore, in 11th year, his income reached Rs 7000. Which means after 10 years of 1995 i.e. $1995 + 10 \Rightarrow 2005$

Answer:

In 2005 his income reached Rs 7000.

Q20. Ramkali saved Rs 5 in the first week of a year and then increased her weekly saving by Rs 1.75. If in the nth week, her weekly savings become Rs 20.75, find *n*.

Reasoning:

$a_n = a + (n-1)d$ is the general term of AP. Where a_n is the nth term, a is the first term, d is the common difference and n is the number of terms.

Known:

Savings in first week Rs 5 and increment of Rs 1.75 weekly in savings.

Unknown:

Week in which her savings become Rs 20.75

Solution:

From the given data, Ramkali's savings in the consecutive weeks are Rs 5, Rs (5+1.75), Rs (5+2×1.75), Rs (5+3×1.75) ... and so on

Hence in n^{th} weeks savings, Rs [5+(n−1)×1.75] = Rs 20.75

Now from the above we know that

$$\begin{aligned} a &= 5 \\ d &= 1.75 \\ a_n &= 20.75 \\ n &= ? \end{aligned}$$

We know that the nth term of an A.P. Series,

$$\begin{aligned} a_n &= a + (n-1)d \\ 20.75 &= 5 + (n-1)1.75 \\ 15.75 &= (n-1)1.75 \\ (n-1) &= \frac{15.75}{1.75} \\ n-1 &= \frac{1575}{175} \\ n-1 &= 9 \\ n &= 10 \end{aligned}$$

Answer:

$n = 10$

Chapter- 5: Arithmetic Progressions

Exercise 5.3 (Page 112 of Grade 10 NCERT)

Q1. Find the sum of the following APs.

(i) 2, 7, 12, ..., to 10 terms.
(ii) −37, −33, −29, ..., to 12 terms.
(iii) 0.6, 1.7, 2.8, ..., to 100 terms.
(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}$, ..., to 11 terms.

(i) 2, 7, 12, ..., to 10 terms.

Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ Where a is the first term, d is the common difference and n is the number of terms.

Known:

The AP 2,7,12, …

Unknown:

Sum up to 10 terms of the AP

Solution:

Given,

- First term, $a = 2$
- Common Difference, $d = 7 - 2 = 5$
- Number of Terms, $n = 10$

We know that Sum up to nth term of AP,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{10} = \frac{10}{2}\left[2 \times 2 + (10-1)5\right]$$

$$= 5\left[4 + 9 \times 5\right]$$

$$= 5\left[4 + 45\right]$$

$$= 5 \times 49$$

$$= 245$$

(ii) −37, −33, −29,..., to 12 terms

Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ Where a is the first term, d is the common difference and n is the number of terms.

Known:

The AP -37, -33, -29, …

Unknown:

Sum up to 12 terms

Solution:

Given,

- First term, $a = -37$
- Common Difference, $d = (-33) - (-37) = 4$
- Number of Terms, $n = 12$

We know that Sum up to nth term of AP,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{12} = \frac{12}{2}\left[2(-37) + (12-1)4\right]$$

$$= 6\left[-74 + 11 \times 4\right]$$

$$= 6\left[-74 + 44\right]$$

$$= 6 \times (-30)$$

$$= -180$$

(iii) 0.6, 1.7, 2.8, ..., to 100 terms

Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ Where a is the first term, d is the common difference and n is the number of terms.

Known:

The AP 0.6, 1.7, 2.8, ...

Unknown:

Sum up to 100 terms of the AP

Solution:

Given,

- First term, $a = 0.6$
- Common Difference, $d = 1.7 - 0.6 = 1.1$
- Number of Terms, $n = 100$

We know that Sum up to nth term of AP,

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$S_{100} = \frac{100}{2}\left[2\times 0.6+(100-1)1.1\right]$$

$$= 50\left[1.2+99\times 1.1\right]$$

$$= 50\left[1.2+108.9\right]$$

$$= 50\left[110.1\right]$$

$$= 5505$$

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots\ldots\ldots$, to 11 terms.

Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ Where a is the first term, d is the common difference and n is the number of terms.

Known:

The AP $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots\ldots\ldots$

Unknown:

Sum up to 11 terms of the AP

Solution:

Given,

- First term, $a = \frac{1}{15}$
- Common Difference, $d = \frac{1}{12} - \frac{1}{15} = \frac{1}{60}$
- Number of Terms, $n = 11$

We know that Sum up to nth term of AP,

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$S_{11} = \frac{11}{2}\left[2\times\frac{1}{15}+(11-1)\frac{1}{60}\right]$$

$$= \frac{11}{2}\left[\frac{2}{15}+\frac{1}{6}\right]$$

$$= \frac{11}{2}\left[\frac{4+5}{30}\right]$$

$$= \frac{11}{2}\times\frac{3}{10}$$

$$= \frac{33}{20}$$

Q2. Find the sums given below

(i) $7 + 10\frac{1}{2} + 14 + \ldots\ldots\ldots\ldots + 84$

(ii) $34 + 32 + 30 + \ldots\ldots\ldots\ldots + 10$

(iii) $-5 + (-8) + (-11) + \ldots\ldots\ldots\ldots + (-230)$

(i) $7 + 10\frac{1}{2} + 14 + \ldots\ldots\ldots\ldots + 84$

Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ or $S_n = \frac{n}{2}[a + l]$, and

nth term of an AP is $a_n = a + (n-1)d$

Where a is the first term, d is the common difference and n is the number of terms and l is the last term.

Known:

The AP $7 + 10\frac{1}{2} + 14 + \ldots\ldots\ldots\ldots + 84$

Unknown:

Sum of the AP

Solution:

Given,

- First term, $a = 7$
- Common Difference, $d = 10\frac{1}{2} - 7 = \frac{21}{2} - 7 = \frac{7}{2}$
- Last term, $l = 84$

$$l = a_n = a + (n-1)d$$

$$84 = 7 + (n-1)\frac{7}{2}$$

$$77 = (n-1)\frac{7}{2}$$

$$22 = n - 1$$

$$n = 23$$

Hence, $n = 23$

$$S_n = \frac{n}{2}(a + l)$$

$$S_{23} = \frac{23}{2}[7 + 84]$$

$$= \frac{23}{2} \times 91$$

$$= \frac{2093}{2}$$

$$= 1046\frac{1}{2}$$

(ii) 34 + 32 + 30 + ... + 10

Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ or $S_n = \frac{n}{2}[a+l]$, and

nth term of an AP is $a_n = a+(n-1)d$

Where a is the first term, d is the common difference and n is the number of terms and l is the last term.

Known:

The AP 34 + 32 + 30 + ... + 10

Unknown:

Sum of the AP

Solution:

Given,

- First term, $a = 34$
- Common Difference, $d = 32-34 = -2$
- Last term, $l = 10$

$$\begin{aligned} l &= a+(n-1)d \\ 10 &= 34+(n-1)(-2) \\ n-1 &= 12 \\ n &= 13 \end{aligned}$$

Hence, $n = 13$

$$\begin{aligned} S_n &= \frac{n}{2}(a+l) \\ S_{23} &= \frac{13}{2}[34+10] \\ &= \frac{13}{2}\times 44 \\ &= 13\times 22 \\ &= 286 \end{aligned}$$

(iii) (−5) + (−8) + (−11) + ... + (−230)

Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ or $S_n = \frac{n}{2}[a+l]$, and

nth term of an AP is $a_n = a+(n-1)d$

Where a is the first term, d is the common difference and n is the number of terms and l is the last term.

Known: The AP (−5) + (−8) + (−11) + ... + (−230)

Unknown:
Sum of the AP

Solution:
Given,

- First term, $a = -5$
- Common Difference, $d = (-8) - (-5) = -8 + 5 = -3$
- Last term, $l = -230$

$$l = a + (n-1)d$$
$$-230 = (-5) + (n-1)(-3)$$
$$n - 1 = \frac{225}{3}$$
$$n = 75 + 1$$
$$n = 76$$

Hence, $n = 76$

$$S_n = \frac{n}{2}[a + l]$$
$$= \frac{76}{2}[(-5) + (-230)]$$
$$= 38[-235]$$
$$= -8930$$

Q3. In an AP

(i) Given $a = 5,\ d = 3,\ a_n = 50$, find n and S_n.

(ii) Given $a = 7,\ a_{13} = 35$, find d and S_{13}.

(iii) Given $a_{12} = 37,\ d = 3$, find a and S_{12}.

(iv) Given $a_3 = 15,\ S_{10} = 125$, find d and a_{10}.

(v) Given $d = 5,\ S_9 = 75$, find a and a_9.

(vi) Given $a = 2,\ d = 8,\ S_n = 90$, find n and a_n.

(vii) Given $a = 8,\ a_n = 62,\ S_n = 210$, find n and d.

(viii) Given $a_n = 4,\ d = 2,\ S_n = -14$, find n and a.

(ix) Given $a = 3,\ n = 8,\ S = 192$, find d.

(x) Given $l = 28,\ S = 144$ and there are total 9 terms. Find a.

(i) Given $a = 5,\ d = 3,\ a_n = 50$, find n and S_n.

Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ or $S_n = \frac{n}{2}[a + l]$, and

nth term of an AP is $a_n = a + (n-1)d$

Where a is the first term, d is the common difference and n is the number of terms and l is the last term.

Known:

$a = 5,\ d = 3,\ a_n = 50$

Unknown:

n and S_n.

Solution:

Given,

- First term, $a = -5$
- Common difference, $d = 3$
- nth term, $l = a_n = 50$

$$\begin{aligned} \text{As } a_n &= a + (n-1)d \\ 50 &= 5 + (n-1)3 \\ 45 &= (n-1)3 \\ 15 &= n - 1 \\ n &= 16 \end{aligned}$$

$$\begin{aligned} S_n &= \frac{n}{2}[a + l] \\ S_{16} &= \frac{16}{2}[5 + 50] \\ &= 8 \times 55 \\ &= 440 \end{aligned}$$

(ii) Given $a = 7,\ a_{13} = 35$, find d and S_{13}.

Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ or $S_n = \frac{n}{2}[a + l]$, and

nth term of an AP is $a_n = a + (n-1)d$

Where a is the first term, d is the common difference and n is the number of terms and l is the last term.

Known:

$a = 7,\ a_{13} = 35$,

Unknown:

d and S_{13}.

Solution:

Given,

- First term, $a = 7$
- 13th term, $l = a_{13} = 35$

As $a_n = a + (n-1)d$

$$a_{13} = a + (13-1)d$$
$$35 = 7 + 12d$$
$$35 - 7 = 12d$$
$$d = \frac{28}{12}$$
$$d = \frac{7}{3}$$

$$S_n = \frac{n}{2}[a + l]$$
$$S_{13} = \frac{13}{2}[7 + 35]$$
$$= \frac{13}{2} \times 42$$
$$= 13 \times 21$$
$$= 273$$

(iii) Given $a_{12} = 37,\ d = 3$, find a and S_{12}.

Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ or $S_n = \frac{n}{2}[a + l]$, and

nth term of an AP is $a_n = a + (n-1)d$

Where a is the first term, d is the common difference and n is the number of terms and l is the last term.

Known:

$a_{12} = 37,\ d = 3$

Unknown:

a and S_{12}.

Solution:

Given,

- 12th term, $a_{12} = 37$
- Common Difference, $d = 3$

$$\text{As } a_n = a + (n-1)d$$
$$a_{12} = a + (12-1)3$$
$$37 = a + 33$$
$$a = 4$$

$$S_n = \frac{n}{2}[a + l]$$
$$S_{12} = \frac{12}{2}[4 + 37]$$
$$S_{12} = 6 \times 41$$
$$S_{12} = 246$$

(iv) Given $a_3 = 15$, $S_{10} = 125$, find d and a_{10}.

Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}[2a + (n-1)d]$, and nth term of an AP is $a_n = a + (n-1)d$

Where a is the first term, d is the common difference and n is the number of terms.

Known:

$a_3 = 15$, $S_{10} = 125$

Unknown:

d and a_{10}.

Solution:

Given,

- 3rd term, $a_3 = 15$
- Sum up to ten terms, $S_{10} = 125$

$$\text{As } a_n = a + (n-1)d$$
$$a_3 = a + (3-1)d$$
$$15 = a + 2d \text{ ---------------------- } Equation\ (i)$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$S_{10} = \frac{10}{2}\left[2a + (10-1)d\right]$$
$$125 = 5\left[2a + 9d\right]$$
$$25 = 2a + 9d \text{ --------------- } Equation\ (ii)$$

On multiplying equation (i) by 2, we obtain

$$30 = 2a + 4d \text{ ----------- } Equation\ (iii)$$

On subtracting equation (iii) from Equation (ii), we obtain

$$-5 = 5d$$
$$d = -1$$

From equation (i),

$$15 = a + 2(-1)$$
$$15 = a - 2$$
$$a = 17$$

$$a_{10} = a + (10-1)d$$
$$a_{10} = 17 + 9(-1)$$
$$a_{10} = 17 - 9$$
$$a_{10} = 8$$

(v) Given $d = 5,\ S_9 = 75$, find a and a_9.

Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$, and nth term of an AP is $a_n = a + (n-1)d$

Where a is the first term, d is the common difference and n is the number of terms.

Known:

$d = 5,\ S_9 = 75$

Unknown:

a and a_9.

Solution:

Given,

- Common difference, $d = 5$

- Sum up to nine terms, $S_9 = 75$

$$\text{As } S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$
$$S_9 = \frac{9}{2}\left[2a+(9-1)5\right]$$
$$75 = \frac{9}{2}(2a+40)$$
$$25 = 3(a+20)$$
$$25 = 3a+60$$
$$a = \frac{-35}{3}$$

We know that nth term of the AP series, $a_n = a+(n-1)d$

$$a_9 = a+(9-1)\times 5$$
$$= \frac{-35}{3}+8\times 5$$
$$= \frac{-35}{3}+40$$
$$= \frac{-35+120}{3}$$
$$= \frac{85}{3}$$

(vi) Given $a=2,\ d=8,\ S_n=90$, find n and a_n.

Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ or $S_n = \frac{n}{2}[a+l]$, and

nth term of an AP is $a_n = a+(n-1)d$

Where a is the first term, d is the common difference and n is the number of terms and l is the last term.

Known:

$a=2,\ d=8,\ S_n=90$

Unknown:

n and a_n.

Solution:

Given,

- First term, $a = 2$
- Common difference, $d = 5$
- Sum up to nth terms, $S_n = 90$

As $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$

$90 = \frac{n}{2}\left[4+(n-1)8\right]$

$90 = n\ [2+(n-1)4]$

$90 = n\left[2+4n-4\right]$

$90 = n\left[4n-2\right]$

$90 = 4n^2 - 2n$

$4n^2 - 2n - 90 = 0$

$4n^2 - 20n + 18n - 90 = 0$

$4n(n-5) + 18(n-5) = 0$

$(n-5)(4n+18) = 0$

Either $(n-5) = 0$ or $(4n+18) = 0$

$n = 5$ or $n = \frac{-9}{2}$

However, n can neither be negative nor fractional.
Therefore, $n = 5$

$a_n = a + (n-1)d$

$a_5 = 2 + (5-1)8$

$a_5 = 2 + 4 \times 8$

$a_5 = 2 + 32$

$a_5 = 34$

(vii) Given $a = 8,\ a_n = 62,\ S_n = 210$, find n and d.

Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ or $S_n = \frac{n}{2}\left[a+l\right]$, and

nth term of an AP is $a_n = a + (n-1)d$

Where a is the first term, d is the common difference and n is the number of terms and l is the last term.

Known:

$a = 8,\ a_n = 62,\ S_n = 210$

Unknown:

n and d.

Solution:

Given,

- First term, $a = 8$

- nth term, $l = a_n = 62$
- Sum up to nth terms, $S_n = 210$

$$\text{As } S_n = \frac{n}{2}[a+l]$$
$$210 = \frac{n}{2}[8+62]$$
$$210 = \frac{n}{2} \times 70$$
$$n = 6$$

We know that n[th] term of the AP series, $a_n = a + (n-1)d$

$$62 = 8 + (6-1)d$$
$$62 - 8 = 5d$$
$$54 = 5d$$
$$d = \frac{54}{5}$$

(viii) Given $a_n = 4,\ d = 2,\ S_n = -14$, find n and a.

Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}[2a+(n-1)d]$ or $S_n = \frac{n}{2}[a+l]$, and nth term of an AP is $a_n = a + (n-1)d$

Where a is the first term, d is the common difference and n is the number of terms and l is the last term.

Known:

$a_n = 4,\ d = 2,\ S_n = -14$

Unknown:

n and a

Solution:

Given,

- Common difference, $d = 2$
- nth term, $l = a_n = 4$
- Sum up to nth terms, $S_n = -14$

We know that nth term of AP series, $a_n = a + (n-1)d$

$$4 = a + (n-1)2$$
$$4 = a + 2n - 2$$
$$a = 6 - 2n..................(1)$$

$$S_n = \frac{n}{2}[a + l]$$

$$-14 = \frac{n}{2}[6 - 2n + 4] \text{.............} [from(1)]$$

$$-14 = n(5 - n)$$

$$-14 = 5n - n^2$$

$$n^2 - 5n - 14 = 0$$

$$n^2 - 7n + 2n - 14 = 0$$

$$n(n-7) + 2(n-7) = 0$$

$$(n-7)(n+2) = 0$$

Either $n - 7 = 0$ or $n + 2 = 0$
$n = 7$ or $n = -2$

However, n can neither be negative nor fractional.
Therefore, $n = 7$

From equation (1), we obtain

$$a = 6 - 2n$$

$$a = 6 - 2 \times 7$$

$$a = 6 - 14$$

$$a = -8$$

(ix) Given $a = 3,\ n = 8,\ S = 192$, find d.

Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}[2a + (n-1)d]$

Where a is the first term, d is the common difference and n is the number of terms.

Known:

$a = 3,\ n = 8,\ S = 192$

Unknown:

d

Solution:

Given,

- First term, $a = 3$
- Number of terms, $n = 8$
- Sum up to nth terms, $S_n = 192$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$192 = \frac{8}{2}\left[2\times 3 + (8-1)d\right]$$
$$192 = 4\left[6 + 7d\right]$$
$$48 = 6 + 7d$$
$$42 = 7d$$
$$d = 6$$

(x) Given $l = 28,\ S = 144$ and there are total 9 terms. Find a.

Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ or $S_n = \frac{n}{2}\left[a + l\right]$

Where a is the first term, d is the common difference and n is the number of terms and l is the last term.

Known:

$l = 28,\ S = 144,\ n = 9$

Unknown:

a

Solution:

Given,

- Last term, $l = a_n = 28$
- Number of terms, $n = 9$
- Sum up to nth terms, $S_n = 192$

$$S_n = \frac{n}{2}(a + l)$$
$$144 = \frac{9}{2}(a + 28)$$
$$32 = a + 28$$
$$a = 4$$

Q4: How many terms of the AP. 9, 17, 25 ... must be taken to give a sum of 636?

Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

Where a is the first term, d is the common difference and n is the number of terms.

Known:

The AP and sum.

Unknown:
Number of terms.

Solution:
Given,

- First term, $a = 9$
- Common difference, $d = 17 - 9 = 8$
- Sum up to nth terms, $S_n = 636$

We know that sum of n terms of AP

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$636 = \frac{n}{2}\left[2 \times 9 + (n-1)8\right]$$

$$636 = \frac{n}{2}[18 + 8n - 8]$$

$$636 = \frac{n}{2}[10 + 8n]$$

$$636 = n[5 + 4n]$$

$$636 = 5n + 4n^2$$

$$4n^2 + 5n - 636 = 0$$

$$4n^2 + 53n - 48n - 636 = 0$$

$$n(4n + 53) - 12(4n + 53) = 0$$

$$(4n + 53)(n - 12) = 0$$

Either $4n + 53 = 0$ or $n - 12 = 0$

$$n = -\frac{53}{4} \text{ or } n = 12$$

n cannot be $\frac{-53}{4}$. As the number of terms can neither be negative nor fractional, therefore, $n = 12$

Q5. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Reasoning:
Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ or $S_n = \frac{n}{2}[a + l]$, and nth term of an AP is $a_n = a + (n-1)d$

Where a is the first term, d is the common difference and n is the number of terms and l is the last term.

Known:

a, l, and S_n

Unknown:

n and l

Solution:

Given,

- First term, $a = 5$
- Last term, $l = 45$
- Sum up to nth terms, $S_n = 400$

We know that sum of n terms of AP

$$S_n = \frac{n}{2}(a+l)$$
$$400 = \frac{n}{2}(5+45)$$
$$400 = \frac{n}{2} \times 50$$
$$n = 16$$

$$l = a_n = a + (n-1)d$$
$$45 = 5 + (16-1)d$$
$$40 = 15d$$
$$d = \frac{40}{15}$$
$$d = \frac{8}{3}$$

Q6. The first and the last term of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}[2a + (n-1)d]$ or $S_n = \frac{n}{2}[a+l]$, and

nth term of an AP is $a_n = a + (n-1)d$

Where a is the first term, d is the common difference and n is the number of terms and l is the last term.

Known:

a, l, and d

Unknown:

n and S_n

Solution:

Given,

- First term, $a = 17$
- Last term, $l = 350$
- Common difference, $d = 9$

We know that nth term of AP, $l = a_n = a + (n-1)d$

$$350 = 17 + (n-1)9$$
$$333 = (n-1)9$$
$$(n-1) = 37$$
$$n = 38$$

Sum of n terms of AP,

$$S_n = \frac{n}{2}(a+l)$$
$$S_{38} = \frac{38}{2}(17+350)$$
$$= 19 \times 367$$
$$= 6973$$

Thus, this A.P. contains 38 terms and the sum of the terms of this A.P. is 6973.

Q7. Find the sum of first 22 terms of an AP in which $d = 7$ and 22nd term is 149.

Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}[2a + (n-1)d]$ or $S_n = \frac{n}{2}[a+l]$, and nth term of an AP is $a_n = a + (n-1)d$

Where a is the first term, d is the common difference and n is the number of terms and l is the last term.

Known:

d and a_{22}

Unknown:

S_{22}

Solution:

Given,

- 22nd term, $l = a_{22} = 149$
- Common difference, $d = 7$

We know that nth term of AP, $a_n = a + (n-1)d$

$$a_{22} = a + (22-1)d$$
$$149 = a + 21 \times 7$$
$$149 = a + 147$$
$$a = 2$$

$$S_n = \frac{n}{2}(a+l)$$
$$= \frac{22}{2}(2+149)$$
$$= 11 \times 151$$
$$= 1661$$

Q8. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}[2a + (n-1)d]$, and nth term of an AP is $a_n = a + (n-1)d$

Where a is the first term, d is the common difference and n is the number of terms and l is the last term.

Known:

a_2 and a_3

Unknown:

S_{51}

Solution:

Given,

- 2nd term, $a_2 = 14$
- 3rd term, $a_3 = 18$
- Common difference, $d = a_3 - a_2 = 18 - 14 = 4$

We know that nth term of AP, $a_n = a + (n-1)d$

$$a_2 = a + d$$
$$14 = a + 4$$
$$a = 10$$

Sum of n terms of AP series,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{51} = \frac{51}{2}\left[2 \times 10 + (51-1)4\right]$$

$$= \frac{51}{2}\left[20 + 50 \times 4\right]$$

$$= \frac{51}{2} \times 220$$

$$= 51 \times 110$$

$$= 5610$$

Q9. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$, and nth term of an AP is $a_n = a + (n-1)d$

Where a is the first term, d is the common difference and n is the number of terms and l is the last term.

Known:

S_7 and S_{17}

Unknown:

S_n

Solution:

Given,

- Sum of first 7 terms, $S_7 = 49$
- Sum of first 17 terms, $S_{17} = 289$

We know that sum of n term of AP is,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_7 = \frac{7}{2}\left[2a + (7-1)d\right]$$

$$49 = \frac{7}{2}\left[2a + 6d\right]$$

$$a + 3d = 7 \text{....}(i)$$

$$S_{17} = \frac{17}{2}\left[2a + (17-1)d\right]$$

$$289 = \frac{17}{2}\left[2a + 16d\right]$$

$$a + 8d = 17 \text{......}(ii)$$

Subtracting equation (i) from equation (ii),

$$5d = 10$$
$$d = 2$$

From equation (i),

$$7 = a + 3 \times 2$$
$$7 = a + 6$$
$$a = 1$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$= \frac{n}{2}\left[2 \times 1 + (n-1)2\right]$$

$$= \frac{n}{2}\left[2 + 2n - 2\right]$$

$$= \frac{n}{2} \times 2n$$

$$= n^2$$

Q10. Show that $a_1, a_2, \dots, a_n, \dots$ form an AP where a_n is defined as below

(i) $a_n = 3 + 4n$

(ii) $a_n = 9 - 5n$

Also find the sum of the first 15 terms in each case.

i)Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$, and the general term of an AP is $a_n = a + (n-1)d$

Where a is the first term, d is the common difference and n is the number of terms

Known:

$a_n = 3 + 4n$

Unknown:

Whether $a_1, a_2, \ldots, a_n, \ldots$ form an AP

Solution:

Given,

- nth term, $a_n = 3 + 4n$

$$\begin{aligned} a_1 &= 3 + 4 \times 1 = 7 \\ a_2 &= 3 + 4 \times 2 = 3 + 8 = 11 \\ a_3 &= 3 + 4 \times 3 = 3 + 12 = 15 \\ a_4 &= 3 + 4 \times 4 = 3 + 16 = 19 \end{aligned}$$

It can be observed that

$$\begin{aligned} a_2 - a_1 &= 11 - 7 = 4 \\ a_3 - a_2 &= 15 - 11 = 4 \\ a_4 - a_3 &= 19 - 15 = 4 \end{aligned}$$

i.e., the difference of a_n and a_{n-1} is constant.

Therefore, this is an AP with common difference as 4 and first term as 7.

Sum of n terms,

$$\begin{aligned} S_n &= \frac{n}{2}\left[2a + (n-1)d\right] \\ S_{15} &= \frac{15}{2}\left[2 \times 7 + (15-1)4\right] \\ &= \frac{15}{2}\left[14 + 14 \times 4\right] \\ &= \frac{15}{2} \times 70 \\ &= 15 \times 35 \\ &= 525 \end{aligned}$$

ii)Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$, and the general term of an AP is $a_n = a + (n-1)d$

Where a is the first term, d is the common difference and n is the number of terms

Known:

$a_n = 9 - 5n$

Unknown:

Whether $a_1,\ a_2, \ldots,\ a_n, \ldots$ form an AP

Given,

- nth term, $a_n = 9 - 5n$

$$a_1 = 9 - 5\times 1 = 9 - 5 = 4$$
$$a_2 = 9 - 5\times 2 = 9 - 10 = -1$$
$$a_3 = 9 - 5\times 3 = 9 - 15 = -6$$
$$a_4 = 9 - 5\times 4 = 9 - 20 = -11$$

It can be observed that

$$a_2 - a_1 = (-1) - 4 = -5$$
$$a_3 - a_2 = (-6) - (-1) = -5$$
$$a_4 - a_3 = (-11) - (-6) = -5$$

i.e., the difference of a_n and a_{n-1} is constant.

Therefore, this is an A.P. with common difference as (−5) and first term as 4.

Sum of n terms of AP,

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$
$$S_{15} = \frac{15}{2}\left[2\times 4+(15-1)(-5)\right]$$
$$= \frac{15}{2}\left[8+14(-5)\right]$$
$$= \frac{15}{2}[8-70]$$
$$= \frac{15}{2}[-62]$$
$$= -465$$

Q11. If the sum of the first n terms of an AP is $4n-n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly find the 3rd, the10th and the nth terms.

Reasoning:

Sum of the first n terms of an AP is given by $S_n=\frac{n}{2}\left[2a+(n-1)d\right]$ or $S_n=\frac{n}{2}[a+l]$, and

nth term of an AP is $a_n=a+(n-1)d$

Where a is the first term, d is the common difference and n is the number of terms and l is the last term.

Known:

$S_n=4n^2-n^2$

Unknown:

$S_1,\ S_2,\ a_2,\ a_3,\ a_{10}$ and a_n

Solution:

Given,

- Sum of first n terms, $S_n=4n-n^2$

Therefore,

Sum of first term, $a=S_1=4\times1-1^2=4-1=3$

Sum of first two terms, $S_2=4\times2-2^2=8-4=4$

Sum of first three terms, $S_3=4\times3-3^2=12-9=3$

Second term, $a_2=S_2-S_1=4-3=1$

Third term, $a_3=S_3-S_2=3-4=-1$

Tenth term, $a_{10}=S_{10}-S_9$

$$\begin{aligned}&=\left(4\times10-10^2\right)-\left(4\times9-9^2\right)\\&=(40-100)-(36-81)\\&=-60+45\\&=-15\end{aligned}$$

nth term, $a_n=S_n-S_{n-1}$

$$\begin{aligned}&=\left[4n-n^2\right]-\left[4(n-1)-(n-1)^2\right]\\&=4n-n^2-4n+4+(n-1)^2\\&=4-n^2+n^2-2n+1\\&=5-2n\end{aligned}$$

Hence, the sum of first two terms is 4.

The second term is 1.

3rd, 10th, and nth terms are −1, −15, and $(5-2n)$ respectively.

Q12. Find the sum of first 40 positive integers divisible by 6.

Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$

Where a is the first term, d is the common difference and n is the number of terms.

Known:

Positive integers divisible by 6

Unknown:

Sum of first 40 positive integers divisible by 6, S_{40}

Solution:

The positive integers that are divisible by 6 are 6, 12, 18, 24, ...
It can be observed that these are making an AP
Hence,

- First term, $a = 6$
- Common difference, $d = 6$
- Number of terms, $n = 40$

As we know that Sum of n terms,

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$S_{40} = \frac{40}{2}\left[2\times 6+(40-1)6\right]$$

$$= 20\left[12+39\times 6\right]$$

$$= 20\left[12+234\right]$$

$$= 20\times 246$$

$$= 4920$$

Q13: Find the sum of first 15 multiples of 8.

Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$

Where a is the first term, d is the common difference and n is the number of terms.

Known:

Multiples of 8

Unknown:

Sum of first 15 multiples of 8, S_{15}

Solution:

The multiples of 8 are 8, 16, 24, 32, ...
These are in an A.P.,
Hence,

- First term, $a = 8$
- Common difference, $d = 8$
- Number of terms, $n = 15$

As we know that Sum of n terms,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{15} = \frac{15}{2}\left[2\times 8 + (15-1)8\right]$$

$$= \frac{15}{2}[16 + 14\times 8]$$

$$= \frac{15}{2}[16 + 112]$$

$$= \frac{15}{2}\times 128$$

$$= 15\times 64$$

$$= 960$$

Q14: Find the sum of the odd numbers between 0 and 50.

Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}[a + l]$, and nth term of an AP is

$a_n = a + (n-1)d$

Where a is the first term, d is the common difference and n is the number of terms and l is the last term.

Known:

Odd numbers between 0 and 50

Unknown:

Sum of the odd numbers between 0 and 50

Solution:

The odd numbers between 0 and 50 are 1, 3, 5, 7, 9 ... 49
Therefore, it can be observed that these odd numbers are in an A.P.
Hence,

- First term, $a = 1$
- Common difference, $d = 2$
- Last term, $l = 1$

We know that nth term of AP, $a_n = l = a + (n-1)d$

$$49 = 1 + (n-1)2$$
$$48 = 2(n-1)$$
$$n - 1 = 24$$
$$n = 25$$

We know that sum of n terms of AP,

$$S_n = \frac{n}{2}(a + l)$$
$$S_{25} = \frac{25}{2}(1 + 49)$$
$$= \frac{25}{2} \times 50$$
$$= 25 \times 25$$
$$= 625$$

Q15. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs. 200 for the first day, Rs. 250 for the second day, Rs. 300 for the third day, etc., the penalty for each succeeding day being Rs. 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days.

Solution:

Reasoning:

General form of an arithmetic progression is $a, (a+d), (a+2d), (a+3d)$.

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}[2a + (n-1)d]$

Where a is the first term, d is the common difference and n is the number of terms.

Known:

The penalty for delay of completion by a day, Rs 200 and Rs 50 more each succeeding day.

Unknown:

Amount has to pay as a penalty.

Solution:

Penalty for 1st day Rs 200.
Penalty for 2nd day Rs 250
Penalty for 3rd day Rs 300

By observation that these penalties are in an A.P. having first term as 200 and common difference as 50 and number of terms as 30.

$$a = 200$$
$$d = 50$$
$$n = 30$$

Penalty that has to be paid if he has delayed the work by 30 days

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$S_{30} = \frac{30}{2}\left[2 \times 200 + (30-1)50\right]$$
$$= 15\left[400 + 1450\right]$$
$$= 15 \times 1850$$
$$= 27750$$

Therefore, the contractor has to pay Rs 27750 as penalty.

Q16. A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs 20 less than its preceding prize, find the value of each of the prizes.

Reasoning:

General form of an arithmetic progression is $a,\ (a+d),\ (a+2d),\ (a+3d),.........$

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

Where a is the first term, d is the common difference and n is the number of terms.

Known:

7 cash prizes are given, and each prize is Rs 20 less than its preceding prize.

Unknown:

Value of each of the prizes

Solution:

Let the cost of 1st prize be x.
Then the cost of 2nd prize $= x - 20$
And the cost of 3rd prize $= x - 40$

Prizes are $x,\ (x-20),\ (x-40),.........$

By observation that the costs of these prizes are in an A.P., having common difference as -20 and first term as x.

$$a = x$$
$$d = -20$$

Given that, $S_7 = 700$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$\frac{7}{2}\left[2x + (7-1)d\right] = 700$$
$$\left[2x + (6)\times(-20)\right] = 200$$
$$x + 3\times(-20) = 100$$
$$x - 60 = 100$$
$$x = 160$$

Therefore, the value of each of the prizes was Rs 160, Rs 140, Rs 120, Rs 100, Rs 80, Rs 60, and Rs 40.

Q17.In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of class I will plant 1 tree, a section of class II will plant 2 trees and so on till class XII. There are three sections of each class. How many trees will be planted by the students?

Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

Where a is the first term, d is the common difference and n is the number of terms.

Known:

The number of trees planted by 3 sections of each class (I to XII).

Unknown:

Number of trees planted by the students.

Solution:

	Class I	Class II	Class III	………………………	Class XII
Section A	1	2	3	………………………	12
Section B	1	2	3	………………………	12
Section C	1	2	3	………………………	12
Total	**3**	**6**	**9**		**36**

It can be observed that the number of trees planted by the students are in an AP.
3, 6, 9, 12, 15, ...36

- First term, $a = 3$
- Common difference, $d = 6 - 3 = 3$
- Number of terms, $n = 12$

We know that sum of n terms of AP,

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$S_{12} = \frac{12}{2}\left[2\times 3+(12-1)\times 3\right]$$
$$= 6\left[6+11\times 3\right]$$
$$= 6\left[6+33\right]$$
$$= 6\times 39$$
$$= 234$$

Therefore, 234 trees will be planted by the students.

Q18. A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A of radii 0.5, 1.0 cm, 1.5 cm, 2.0 cm, ... as shown in figure. What is the total length of such a spiral made up of thirteen consecutive semicircles? $\left[\text{Take } \pi = \frac{22}{7}\right]$

Images need to be inserted

Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$

Where a is the first term, d is the common difference and n is the number of terms.

Known:

Radii of the 13 semicircles.

Unknown:

Length of the spiral made.

Solution:

Semi-perimeter of circle, $l = \pi r$

$$l_1 = \pi\times(0.5 \text{ cm}) = 0.5\pi \text{ cm}$$
$$l_2 = \pi\times(1 \text{ cm}) = \pi \text{ cm}$$
$$l_3 = \pi\times(1.5 \text{ cm}) = 1.5\pi \text{ cm}$$

Therefore, l_1, l_2, l_3, i.e. the lengths of the semi-circles are in an A.P.,

$$0.5\pi, \pi, 1.5\pi, 2\pi ....$$
$$a = 0.5\pi$$
$$d = \pi - 0.5\pi = 0.5\pi$$

We know that the sum of n terms of an A.P. is given by

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{13} = \frac{13}{2}\left[2\times(0.5\pi) + (13-1)(0.5\pi)\right]$$

$$= \frac{13}{2}\left[\pi + 6\pi\right]$$

$$= \frac{13}{2}\times 7\pi$$

$$= \frac{13}{2}\times 7\times\frac{22}{7}$$

$$= 143$$

Therefore, the length of such spiral of thirteen consecutive semi-circles will be 143 cm.

Q19. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how many rows are the 200 logs placed and how many logs are in the top row?

Images need to be inserted

Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ and nth term of an AP is $a_n = a + (n-1)d$

Where a is the first term, d is the common difference and n is the number of terms.

Known:

Stack of 200 logs, 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on.

Unknown:

Number of rows and number of logs in the top row

Solution:

It can be observed that the numbers of logs in rows are in an A.P.

20, 19, 18, ...

For this A.P.,

- First terms, $a = 20$
- Common difference, $d = 19 - 20 = -1$
- Sum of the n terms, $S_n = 200$

We know that sum of n terms of AP,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$200 = \frac{n}{2}[2 \times 20 + (n-1)(-1)]$$

$$400 = n[40 - n + 1]$$

$$400 = n[41 - n]$$

$$400 = 41n - n^2$$

$$n^2 - 41n + 400 = 0$$

$$n^2 - 16n - 25n + 400 = 0$$

$$n(n-16) - 25(n-16) = 0$$

$$(n-16)(n-25) = 0$$

Either $(n-16) = 0$ or $(n-25) = 0$

$\therefore n = 16$ or $n = 25$

$$a_n = a + (n-1)d$$

$$a_{16} = 20 + (16-1) \times (-1)$$

$$a_{16} = 20 - 15$$

$$a_{16} = 5$$

Similarly,

$$a_{25} = 20 + (25-1) \times (-1)$$

$$a_{25} = 20 - 24$$

$$a_{25} = -4$$

Clearly, the number of logs in 16th row is 5. However, the number of logs in 25th row is negative 4, which is not possible.

Therefore, 200 logs can be placed in 16 rows.
The number of logs in the top (16^{th}) row is 5.

Q20. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato and other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line.

Images need to be inserted

A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?
[Hint: to pick up the first potato and the second potato, the total distance (in metres) run by a competitor is 2 × 5 + 2 × (5 + 3)]

Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$

Where a is the first term, d is the common difference and n is the number of terms.

Known:

Distance of 10 potatoes from the bucket, first potato is $5m$ away from bucket and other potatoes are placed $3m$ apart in a straight line.

Unknown:

Total distance the competitor has to run.

Solution:

The distances of potatoes are as follows.
5, 8, 11, 14, ...

It can be observed that these distances are in A.P.

- First term, $a = 5$
- Common difference, $d = 8 - 5 = 3$
- Number of terms, $n = 10$

We know that the sum of n terms,

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$S_{10} = \frac{10}{2}[2\times 5+(10-1)\times 3]$$

$$S_{10} = 5[10+27]$$

$$S_{10} = 5\times 37$$

$$S_{10} = 185$$

As every time she has to run back to the bucket, therefore, the total distance that the competitor has to run will be two times of it.
Therefore, total distance that the competitor will run $= 2\times 185m = 370\ m$

Alternatively,

The distances of potatoes from the bucket are 5, 8, 11, 14...
Distance run by the competitor for collecting these potatoes are two times of the distance at which the potatoes have been kept.
Therefore, distances to be run are

First potato, $a = 2\times 5m = 10m$
Second potato, $a_2 = 2\times(5+3)m = 16m$
Third potato, $a_3 = 2\times(5+2\times 3)m = 22m$
Number of potatoes, $n = 10$

$$10, 16, 22, 28, 34, \ldots\ldots\ldots$$
$$a = 10$$
$$d = 16 - 10 = 6$$
$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$S_{10} = \frac{10}{2}[2 \times 10 + (10-1)6]$$
$$= 5[20 + 54]$$
$$= 5 \times 74$$
$$= 370$$

Therefore, the competitor will run a total distance of 370 m.

Chapter- 5: Arithmetic Progressions

Exercise 5.4

Q1. Which term of the A.P. 121, 117, 113 ... is its first negative term?
[Hint: Find n for $a_n < 0$]

Reasoning:

nth term of an AP is $a_n = a + (n-1)d$

Where a is the first term, d is the common difference and n is the number of terms.

Known:

A.P. Series: 121, 117, 113 ...

Unknown:

First negative term of the AP

Solution:

Given:

- First Term, $a = 121$
- Common difference, $d = 117 - 121 = -4$

We know that n^{th} term of AP, $a_n = a + (n-1)d$

$$\begin{aligned} a_n &= 121 + (n-1) \times (-4) \\ &= 121 - 4n + 4 \\ &= 125 - 4n \end{aligned}$$

For the first negative term of this A.P,

$$\begin{aligned} & a_n < 0 \\ & 125 - 4n < 0 \\ & 125 < 4n \\ & n > \frac{125}{4} \\ & n > 31.25 \end{aligned}$$

Therefore, 32nd term will be the first negative term of this A.P.

Q2. The sum of the third and the seventh terms of an A.P is 6 and their product is 8. Find the sum of first sixteen terms of the A.P.

Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ and nth term of an AP is $a_n = a+(n-1)d$

Where a is the first term, d is the common difference and n is the number of terms.

Known:

$a_3 + a_7 = 6$ and $a_3 \times a_7 = 8$

Unknown:

Sum of first 16 terms, S_{16}

Solution:

Given:

- $a_3 + a_7 = 6$ ---------------- *Equation* (1)
- $a_3 \times a_7 = 8$ ---------------- *Equation* (2)

We know that n^{th} term of AP Series,

$$a_n = a+(n-1)d$$

Third term, $a_3 = a+(3-1)d$

$$a_3 = a+2d \text{ ------------------ } Equation\ (3)$$

Seventh term, $a_7 = a+(7-1)d$

$$a_7 = a+6d \text{ ------------------ } Equation\ (4)$$

Using Equation (3) and Equation (4) in Equation (1)

$$(a+2d)+(a+6d) = 6$$
$$2a+8d = 6$$
$$a+4d = 3$$
$$a = 3-4d \text{ ---------------- } Equation(5)$$

Using Equation (3) and Equation (4) in Equation (2)

$$(a+2d)\times(a+6d) = 8$$

Substituting the value of Equation (5) in above,

$$(3-4d+2d)\times(3-4d+6d)=8$$
$$(3-2d)\times(3+2d)=8$$
$$(3)^2-(2d)^2=8$$
$$9-4d^2=8$$
$$4d^2=1$$
$$d^2=\frac{1}{4}$$
$$d=\pm\frac{1}{2}$$
$$d=\frac{1}{2} \text{ or } \left(-\frac{1}{2}\right)$$

Hence by substituting both the values of d,

When $d=\frac{1}{2}$

$$a=3-4d$$
$$=3-4\times\frac{1}{2}$$
$$=3-2$$
$$=1$$

When $d=-\frac{1}{2}$

$$a=3-4d$$
$$=3-4\times\left(-\frac{1}{2}\right)$$
$$=3+2$$
$$=5$$

We know that Sum of nth term of AP Series, $S_n=\frac{n}{2}[2a+(n-1)d]$

When $a=1$ and $d=\frac{1}{2}$

$$S_{16}=\frac{16}{2}\left[2\times1+(16-1)\times\frac{1}{2}\right]$$
$$=8\left[2+\frac{15}{2}\right]$$
$$=8\times\frac{19}{2}$$
$$=76$$

When $a=5$ and $d=-\frac{1}{2}$

$$\begin{aligned}S_{16} &= \frac{16}{2}\left[2\times 5+(16-1)\times\left(-\frac{1}{2}\right)\right]\\ &= 8\left[10-\frac{15}{2}\right]\\ &= 8\times\frac{5}{2}\\ &= 20\end{aligned}$$

Therefore, $S_{16} = 20, 76$

Q3. A ladder has rungs 25 cm apart. (See figure). The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and bottom rungs are $2\frac{1}{2}$ m apart, what is the length of the wood required for the rungs? [Hint: number of rungs = $\frac{250}{25}$]

Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}[a+l]$

Where a is the first term, l is the last term and n is the number of terms.

Known:

The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top and the top and bottom rungs are $2\frac{1}{2}$ m apart.

Unknown:

The length of the wood required for the rungs

Solution:

Given:

- Distance between the rungs = 25cm
- Distance between the top and bottom rungs = $2\frac{1}{2}$ m = $2\frac{1}{2}\times 100$ cm

$\therefore$ Total number of rungs $= \dfrac{2\frac{1}{2}\times 100}{25}+1 = \dfrac{250}{25}+1 = 11$

From the given Figure, we can observe that the lengths of the rungs decrease uniformly, hence we can conclude that they will be in an AP

The length of the wood required for the rungs equals the sum of all the terms of this A.P.

- First term, $a = 45$
- Last term, $l = 25$
- Number of terms, $n = 11$

Hence Sum of n terms of the AP Series,

$$S_n = \frac{n}{2}[a+l]$$

$$S_{11} = \frac{11}{2}[45+25]$$

$$= \frac{11}{2}\times 70$$

$$= 11\times 35$$

$$= 385$$

Therefore, the length of the wood required for the rungs is 385 cm.

Q4. The houses of a row are number consecutively from 1 to 49. Show that there is a value of x such that the sum of numbers of the houses preceding the house numbered x is equal to the sum of the number of houses following it. Find this value of x. [Hint $S_{x-1} = S_{49} - S_x$]

Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}[2a+(n-1)d]$

Where a is the first term, d is the common difference and n is the number of terms.

Known:

The houses of a row are number consecutively from 1 to 49

Unknown:

Value of x

Solution:

Given:

Number of houses are 1,2,3, ...49

By Observation, the numbers of houses are in an A.P.

Hence

- First term, $a = 1$
- Common difference, $d = 1$

Let us assume that the number of x^{th} house can be expressed as below:

We know that Sum of n terms in an A.P. $S_n = \frac{n}{2}[2a+(n-1)d]$

Sum of numbers of houses preceding x^{th} house = S_{x-1}

$$S_{x-1} = \frac{(x-1)}{2}\left[2a+((x-1)-1)d\right]$$
$$= \frac{(x-1)}{2}\left[2\times1+(x-2)\times1\right]$$
$$= \frac{(x-1)}{2}\left[2+x-2\right]$$
$$= \frac{x(x-1)}{2}$$

By the given we know that, Sum of number of houses following x^{th} house = $S_{49} - S_x$

$$S_{49} - S_x = \frac{49}{2}\left[2\times1+(49-1)\times1\right] - \frac{x}{2}\left[2\times1+(x-1)\times1\right]$$
$$= \frac{49}{2}\left[2+48\right] - \frac{x}{2}\left[2+x-1\right]$$
$$= \frac{49}{2}\left[2+48\right] - \frac{x}{2}\left[2+x-1\right]$$
$$= \frac{49}{2}\times50 - \frac{x}{2}\left[x+1\right]$$
$$= 1225 - \frac{x(x+1)}{2}$$

It is given that these sums are equal.

$$\frac{x(x-1)}{2} = 1225 - \frac{x(x+1)}{2}$$
$$\frac{x^2}{2} - \frac{x}{2} = 1225 - \frac{x^2}{2} - \frac{x}{2}$$
$$x^2 = 1225$$
$$x = \pm 35$$

As the number of houses cannot be a negative number, we consider number of houses, $x = 35$

Therefore, house number 35 is such that the sum of the numbers of houses preceding the house numbered 35 is equal to the sum of the numbers of the houses following it.

Q5: A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete. Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m (See figure) calculate the total volume of concrete required to build the terrace.

Reasoning:

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$

Where a is the first term, d is the common difference and n is the number of terms.

Known:

15 steps each of which is $50m$ long and each step has a rise of $\frac{1}{4}m$ and a tread of $\frac{1}{2}m$

Unknown:

Total volume of concrete required to build the terrace.

Solution:

- From the figure, it can be observed that

  Height of 1st step is $\frac{1}{4}m$

  Height of 2nd step is $\left(\frac{1}{4}+\frac{1}{4}\right)m=\frac{1}{2}m$

  Height of 3rd step is $\left(\frac{1}{2}+\frac{1}{4}\right)m=\frac{3}{4}m$

  Therefore, height of the each step is increasing by $\frac{1}{4}m$

- length $50m$ and width (tread) $\frac{1}{2}m$ remain the same for each of the steps.

Volume of Step can be considered as Volume of Cuboid $= length \times breadth \times height$

Volume of concrete in 1st step $= 50m \times \frac{1}{2}m \times \frac{1}{4}m = 6.25m^3$

Volume of concrete in 2nd step $= 50m \times \frac{1}{2}m \times \frac{1}{2}m = 12.50m^3$

Volume of concrete in 3rd step $= 50m \times \frac{1}{2}m \times \frac{3}{4}m = 18.75m^3$

It can be observed that the volumes of concrete in these steps are in an A.P.

$6.25m^3, 12.50m^3, 18.75m^3, ...$

First term, $a = 6.25$
Common difference, $d = 6.25$
Number of steps, $n = 15$

Sum of n terms, $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$

$$S_{15} = \frac{15}{2}\left[2\times 6.25+(15-1)\times 6.25\right]$$
$$= \frac{15}{2}[12.50+14\times 6.25]$$
$$= \frac{15}{2}[12.50+87.50]$$
$$= \frac{15}{2}\times 100$$
$$= 750$$

Therefore, Volume of concrete required to build the terrace is $750m^3$.