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THE MATH EXPERT

Chapter 7: Coordinate Geometry

Exercise 7.1 (Page 161 of Grade 10 NCERT Textbook)

Q1. Find the distance between the following pairs of points:

(i) (2, 3), (4, 1)
(ii) (−5, 7), (−1, 3)
(iii) (a, b), (−a, −b)

Difficulty Level: Easy

Reasoning:

The distance between the two points can be measured using the Distance Formula which is given by:

$$\text{Distance Formula} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

Known:

The x and y co-ordinates of the points between which the distance is to be measured.

Unknown:

The distance between the given pairs of points is to be measured.

(i)

Solution:

Given,

- Let the points be A(2, 3) and B(4, 1)
- Therefore,
  - $x_1 = 2$
  - $y_1 = 3$
  - $x_2 = 4$
  - $y_2 = 1$

We know that the distance between the two points is given by the Distance Formula

$$\text{Distance Formula} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \quad \ldots (1)$$

Therefore, distance between A(2, 3) and B(4, 1) is given by

$$\begin{aligned} d &= \sqrt{(2-4)^2 + (3-1)^2} \\ &= \sqrt{(-2)^2 + (2)^2} \\ &= \sqrt{4+4} \\ &= \sqrt{8} \\ &= 2\sqrt{2} \end{aligned}$$

(ii)

Solution:

Distance between $(-5, 7)$ and $(-1, 3)$ is given by

$$\begin{aligned} d &= \sqrt{(-5-(-1))^2 + (7-3)^2} \\ &= \sqrt{(-4)^2 + (4)^2} \\ &= \sqrt{16+16} \\ &= \sqrt{32} \\ &= 4\sqrt{2} \end{aligned}$$

(iii)

Solution:

Distance between $(a, b)(-a, -b)$ is given by

$$\begin{aligned} d &= \sqrt{(a-(-a))^2 + (b-(-b))^2} \\ &= \sqrt{(2a)^2 + (2b)^2} \\ &= \sqrt{4a^2 + 4b^2} \\ &= 2\sqrt{a^2 + b^2} \end{aligned}$$

Q2. Find the distance between the points (0, 0) and (36, 15). Can you now find the distance between the two towns A and B discussed in Section 7.2?

Difficulty Level: Easy

Reasoning:

The distance between the two points can be measured using the Distance Formula which is given by:

$$\text{Distance Formula} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$.

Known:

The x and y co-ordinates of the points between which the distance is to be measured.

Unknown:

The distance between the towns A and B.

Solution:

Given:

- Let the points be A(0,0) and B(36,15)
- Hence
  - $x_1 = 0$

- $y_1 = 0$
- $x_2 = 36$
- $y_2 = 15$

We know that the distance between the two points is given by the Distance Formula,

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \quad ...(1)$$
$$= \sqrt{(0-36)^2 + (0-15)^2}$$
$$= \sqrt{(-36)^2 + (-15)^2} \quad \text{(By Substituting in the Equation (1))}$$
$$= \sqrt{1296 + 225}$$
$$= \sqrt{1521}$$
$$= 39$$

Yes, it is possible to find the distance between the given towns A and B.

The positions of the towns A & B are given by (0, 0) and (36, 15), hence, as calculated above, the distance between town A and B will be 39 km

Q3. Determine if the points (1, 5), (2, 3) and (−2, −11) are collinear.

Difficulty Level: Medium

Reasoning:

- Three or more points are said to be collinear if they lie on a single straight line.
- The distance between the two points can be measured using the Distance Formula which is given by:

$$\text{Distance Formula} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Known:

The x and y co-ordinates of the points between which the distance is to be measured.

Unknown:

To determine if the 3 points are collinear.

Solution:

- Let the points (1, 5), (2, 3), and (−2, −11) be represented by the A, B, and C of the given triangle respectively.

We know that the distance between the two points is given by the

$$\text{Distance Formula} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \quad ...(1)$$

To find AB i.e. the Distance between the Points A (1, 5) and B (2, 3)

- $x_1 = 1$
- $y_1 = 5$

- $x_2 = 2$
- $y_2 = 3$

$$\therefore AB = \sqrt{(1-2)^2 + (5-3)^2} \qquad \text{(By Substituting in (1))}$$
$$= \sqrt{5}$$

To find BC Distance between Points B (2, 3) and C (−2, −11)

- $x_1 = 2$
- $y_1 = 3$
- $x_2 = -2$
- $y_2 = -11$

$$\therefore BC = \sqrt{(2-(-2))^2 + (3-(-11))^2}$$
$$= \sqrt{4^2 + 14^2} \qquad \text{(By Substituting in the Equation (1))}$$
$$= \sqrt{16 + 196}$$
$$= \sqrt{212}$$

To find AC Distance between Points A (1, 5) and C (−2, −11)

- $x_1 = 1$
- $y_1 = 5$
- $x_2 = -2$
- $y_2 = -11$

$$\therefore CA = \sqrt{(1-(-2))^2 + (5-(-11))^2}$$
$$= \sqrt{3^2 + 16^2} \qquad \text{(By Substituting in the Equation (1))}$$
$$= \sqrt{9 + 256}$$
$$= \sqrt{265}$$

Since AB + AC ≠ BC and AB ≠ BC + AC and AC ≠ BC Therefore, the points (1, 5), (2, 3), and (−2, −11) are not collinear.

Q4. Check whether (5, −2), (6, 4) and (7, −2) are the vertices of an isosceles triangle.

Difficulty Level: Medium

Reasoning:

An isosceles triangle is a triangle that has two sides of equal length.

To check whether the given points are vertices of an isosceles triangle, the distance between any of the 2 points should be the same for two pairs of points.

Known:

The x and y co-ordinates of the points between which the distance is to be measured.

Unknown:

To check whether the given points are the vertices of an isosceles triangle.

Solution:

- Let the points (5, −2), (6, 4), and (7, −2) represent the vertices A, B, and C of the given triangle respectively.

We know that the distance between the two points is given by the Distance Formula,

$$\text{Distance Formula} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \quad \text{... (1)}$$

To find AB i.e. Distance between Points A (5, −2) and B (6, 4)

- $x_1 = 5$
- $y_1 = -2$
- $x_2 = 6$
- $y_2 = 4$

By substituting the values in the Equation (1)

$$AB = \sqrt{(5-6)^2 + (-2-4)^2}$$
$$= \sqrt{(-1)^2 + (-6)^2}$$
$$= \sqrt{1+36}$$
$$= \sqrt{37}$$

To find BC Distance between Points B (6, 4) and C (7, −2)

- $x_1 = 6$
- $y_1 = 4$
- $x_2 = 7$
- $y_2 = -2$

By substituting the values in the Equation (1)

$$BC = \sqrt{(6-7)^2 + (4-(-2))^2}$$
$$= \sqrt{(-1)^2 + (6)^2}$$
$$= \sqrt{1+36}$$
$$= \sqrt{37}$$

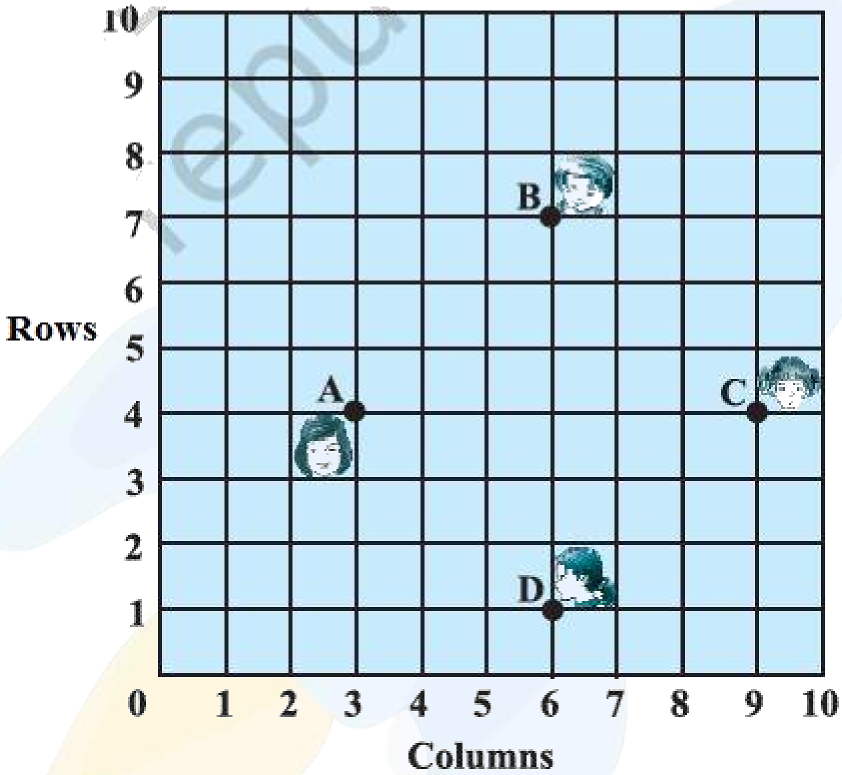
To find AC i.e. Distance between Points A (5, −2) and C (7, −2)

- $x_1 = 5$
- $y_1 = -2$
- $x_2 = 7$
- $y_2 = -2$

$$CA = \sqrt{(5-7)^2 + (-2-(-2))^2}$$
$$= \sqrt{(-2)^2 + 0^2}$$
$$= 2$$

From the above values of AB, BC and AC we can conclude that AB = BC. As, two sides are equal in length, therefore, ABC is an isosceles triangle.

Q5. In a classroom, 4 friends are seated at the points A, B, C and D as shown in the following figure. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, “Don’t you think ABCD is a square?” Chameli disagrees. Using distance formula, find which of them is correct.



Difficulty Level: Medium

Reasoning:

To prove that the points A,B,C and D from a square, the length of the four sides should be equal and the length of the two diagonals should be the same.

Known:

The x and y co-ordinates of the points between which the distance is to be measured can be deduced from the diagram.

Unknown:

To verify whether the positions of the four friends form a square or not.

Solution:

- Let A (3, 4), B (6, 7), C (9, 4), and D (6, 1) be the positions of 4 friends.

We know that the distance between the two points is given by the Distance Formula,

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \quad \text{...Equation (1)}$$

To find AB i.e. Distance between Points A (3, 4) and B (6, 7)

- $x_1 = 3$
- $y_1 = 4$
- $x_2 = 6$
- $y_2 = 7$

By substituting the values in the Equation (1), we get

$$\begin{aligned} AB &= \sqrt{(3-6)^2 + (4-7)^2} \\ &= \sqrt{(-3)^2 + (-3)^2} \\ &= \sqrt{9+9} \\ &= \sqrt{18} \\ &= 3\sqrt{2} \end{aligned}$$

To find BC i.e. Distance between Points B (6, 7) and C (9, 4)

- $x_1 = 6$
- $y_1 = 7$
- $x_2 = 9$
- $y_2 = 4$

By substituting the values in the Equation (1), we get

$$\begin{aligned} BC &= \sqrt{(6-9)^2 + (7-4)^2} \\ &= \sqrt{(-3)^2 + (3)^2} \\ &= \sqrt{9+9} \\ &= \sqrt{18} \\ &= 3\sqrt{2} \end{aligned}$$

To find CD i.e. Distance between Points C (9, 4) and D (6, 1)

- $x_1 = 9$
- $y_1 = 4$
- $x_2 = 6$
- $y_2 = 1$

By substituting the values in the Equation (1)

$$CB = \sqrt{(9-6)^2 + (4-1)^2}$$
$$= \sqrt{(3)^2 + (3)^2}$$
$$= \sqrt{9+9}$$
$$= \sqrt{18}$$
$$= 3\sqrt{2}$$

To find AD i.e. Distance between Points B (3, 4) and D (6, 1)

- $x_1 = 3$
- $y_1 = 4$
- $x_2 = 6$
- $y_2 = 1$

By substituting the values in the Equation (1)

$$AD = \sqrt{(3-6)^2 + (4-1)^2}$$
$$= \sqrt{(-3)^2 + (3)^2}$$
$$= \sqrt{9+9}$$
$$= \sqrt{18}$$
$$= 3\sqrt{2}$$

To find AC i.e. Distance between Points A (3, 4) and C (9, 4)

- $x_1 = 3$
- $y_1 = 4$
- $x_2 = 9$
- $y_2 = 4$

By substituting the values in the Equation (1), we get

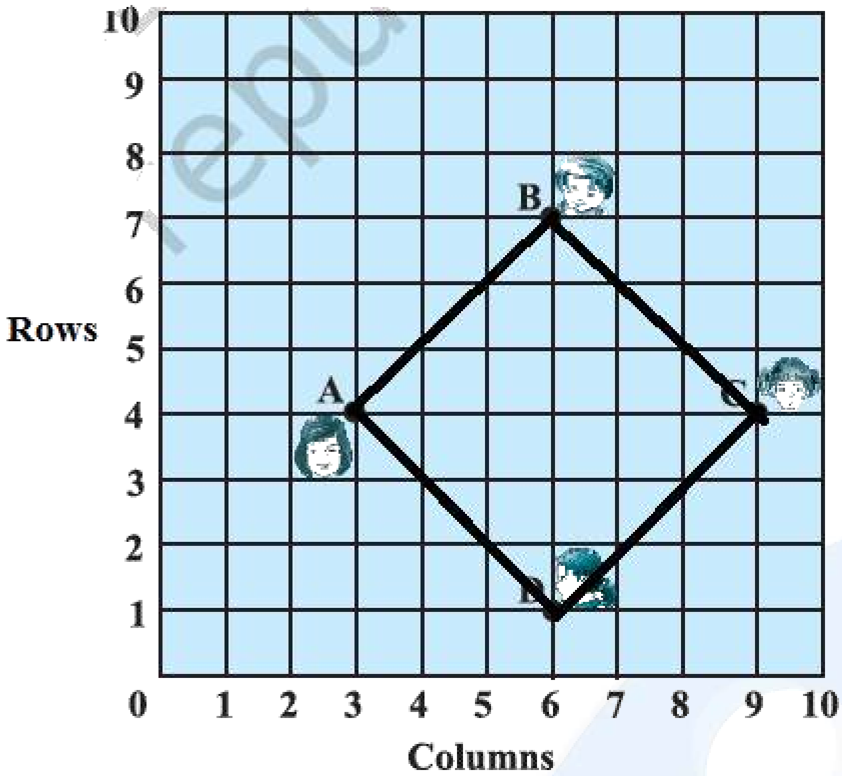
$$\text{Diagonal } AC = \sqrt{(3-9)^2 + (4-4)^2}$$
$$= \sqrt{(-6)^2 + 0^2}$$
$$= 6$$

To find BD Distance between Points B (6, 7) and D (6, 1)

- $x_1 = 6$
- $y_1 = 7$
- $x_2 = 6$
- $y_2 = 1$

By substituting the values in the Equation (1)

$$\text{Diagonal } BD = \sqrt{(6-6)^2 + (7-1)^2}$$
$$= \sqrt{0^2 + (6)^2}$$
$$= 6$$



The four sides AB, BC, CD, and AD are of same length and diagonals AC and BD are of equal length. Therefore, ABCD is a square and hence, Champa was correct

Q6. Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer:

(i) $(-1, -2), (1, 0), (-1, 2), (-3, 0)$

(ii) $(-3, 5), (3, 1), (0, 3), (-1, -4)$

(iii) $(4, 5), (7, 6), (4, 3), (1, 2)$

Difficulty Level: Medium

Reasoning:

A quadrilateral is a polygon with four edges (or sides) and four vertices or corners.

Known:

The x and y co-ordinates of the points between which the distance is to be measured.

Unknown:

To determine the type of quadrilateral formed (if any) by the given co-ordinates .

Solution:

(i) Given,

- Let the points $(-1, -2), (1, 0), (-1, 2)$, and $(-3, 0)$ represent the vertices A, B, C, and D of the given quadrilateral respectively.

We know that the distance between the two points is given by the Distance Formula,

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \qquad \ldots\text{Equation (1)}$$

To find AB i.e. Distance between Points A (−1, −2) and B (1, 0)

- $x_1 = -1$
- $y_1 = -2$
- $x_2 = 1$
- $y_2 = 0$

By substituting the values in the Equation (1)

$$\therefore AB = \sqrt{(-1-1)^2 + (-2-0)^2}$$
$$= \sqrt{(-2)^2 + (-2)^2}$$
$$= \sqrt{4+4}$$
$$= \sqrt{8}$$
$$= 2\sqrt{2}$$

To find BC i.e. Distance between Points B (1, 0) and C (−1, 2)

- $x_1 = 1$
- $y_1 = 0$
- $x_2 = -1$
- $y_2 = 2$

By substituting the values in the Equation (1)

$$BC = \sqrt{(1-(-1))^2 + (0-2)^2}$$
$$= \sqrt{(2)^2 + (-2)^2}$$
$$= \sqrt{4+4}$$
$$= \sqrt{8}$$
$$= 2\sqrt{2}$$

To find AD i.e. Distance between Points D (−3, 0) and A (−1, −2)

- $x_1 = -3$
- $y_1 = 0$
- $x_2 = -1$
- $y_2 = -2$

By substituting the values in the Equation (1)

$$AD = \sqrt{(-3-(-1))^2 + (0-(-2))^2}$$
$$= \sqrt{(-2)^2 + (2)^2}$$
$$= \sqrt{4+4}$$
$$= \sqrt{8}$$
$$= 2\sqrt{2}$$

To find CD i.e. Distance between Points C (−1, 2) and D (−3, 0)

- $x_1 = -1$
- $y_1 = 2$
- $x_2 = -3$
- $y_2 = 0$

By substituting the values in the Equation (1)

$$CD = \sqrt{(-1-(-3))^2 + (2-0)^2}$$
$$= \sqrt{(2)^2 + (2)^2}$$
$$= \sqrt{4+4}$$
$$= \sqrt{8}$$
$$= 2\sqrt{2}$$

To find AC i.e. Distance between Points A (−1, −2) and C (−1, 2)

- $x_1 = -1$
- $y_1 = -2$
- $x_2 = -1$
- $y_2 = 2$

By substituting the values in the Equation (1), we get

$$\text{Diagonal } AC = \sqrt{(-1-(-1))^2 + (-2-2)^2}$$
$$= \sqrt{0^2 + (-4)^2}$$
$$= \sqrt{16}$$
$$= 4$$

To find BD i.e. Distance between Points B (1, 0) and D (−3, 0)

- $x_1 = 1$
- $y_1 = 0$
- $x_2 = -3$
- $y_2 = 0$

By substituting the values in the Equation (1)

$$\text{Diagonal } BD = \sqrt{(1-(-3))^2 + (0-0)^2}$$
$$= \sqrt{(4)^2 + 0^2}$$
$$= \sqrt{16}$$
$$= 4$$

It can be observed that all sides of this quadrilateral are of the same length and also, the diagonals are of the same length. Therefore, the given points are the vertices of a square.

(ii) Solution:

Let the points (−3, 5), (3, 1), (0, 3), and (−1, −4) represent the vertices A, B, C, and D of the given quadrilateral respectively. We know that the distance between the two points is given by the Distance Formula,

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \qquad \text{...Equation (1)}$$

To find AB i.e. Distance between Points A (−3, 5) and B (3, 1)

- $x_1 = -3$
- $y_1 = 5$
- $x_2 = 3$
- $y_2 = 1$

By substituting the values in the Equation (1)

$$AB = \sqrt{(-3-3)^2 + (5-1)^2}$$
$$= \sqrt{(-6)^2 + (4)^2}$$
$$= \sqrt{36+16}$$
$$= \sqrt{52}$$
$$= 2\sqrt{13}$$

To find BC i.e. Distance between the Points B (3, 1) and C (0, 3)

- $x_1 = 3$
- $y_1 = 1$
- $x_2 = 0$
- $y_2 = 3$

By substituting the values in the Equation (1)

$$BC = \sqrt{(3-0)^2 + (1-3)^2}$$
$$= \sqrt{(3)^2 + (-2)^2}$$
$$= \sqrt{9+4}$$
$$= \sqrt{13}$$

To find CD i.e. Distance between Points C (0, 3) and D (−1, −4)

- $x_1 = 0$
- $y_1 = 3$
- $x_2 = -1$
- $y_2 = -4$

By substituting the values in the Equation (1)

$$CD = \sqrt{(0-(-1))^2 + (3-(-4))^2}$$
$$= \sqrt{(1)^2 + (7)^2}$$
$$= \sqrt{1+49}$$
$$= \sqrt{50}$$
$$= 5\sqrt{2}$$

To find AD i.e. Distance between Points A (−3, 5) and B (−1, −4)

- $x_1 = -3$
- $y_1 = 5$
- $x_2 = -1$

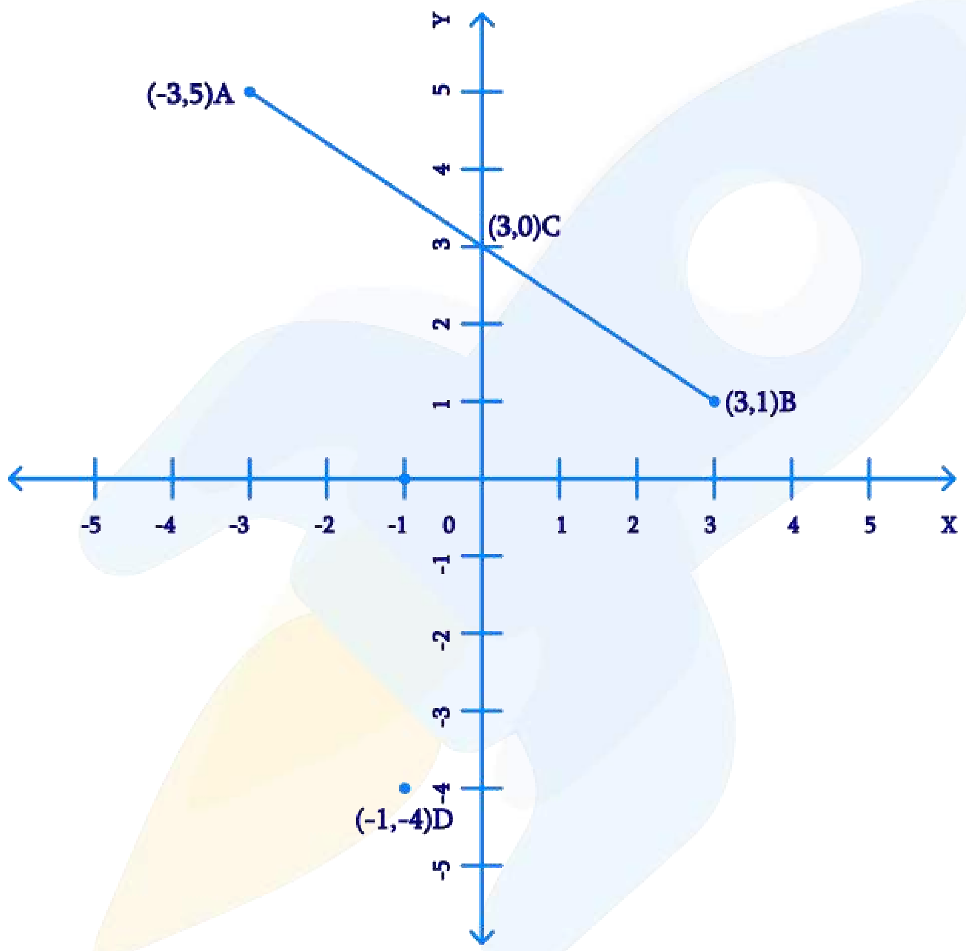
- $y_2 = -4$

By substituting the values in the Equation (1)

$$AD = \sqrt{(-3-(-1))^2 + (5-(-4))^2}$$
$$= \sqrt{(-2)^2 + (9)^2}$$
$$= \sqrt{4+81}$$
$$= \sqrt{85}$$

$AB \neq BC \neq AC \neq AD$

Also, by plotting the graph it looks like as below:



By the graph above,

A, B, C are collinear, So, no quadrilateral can be formed from these points

(iii) Solution:

- Let the points (4, 5), (7, 6), (4, 3), and (1, 2) be representing the vertices A, B, C, and D of the given quadrilateral respectively.

We know that the distance between the two points is given by the Distance Formula,

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \quad \text{... Equation (1)}$$

To find AB i.e. Distance between Points A (4, 5) and B (7, 6)

- $x_1 = 4$
- $y_1 = 5$
- $x_2 = 7$
- $y_2 = 6$

By substituting the values in the Equation (1)

$$AB = \sqrt{(4-7)^2 + (5-6)^2}$$
$$= \sqrt{(-3)^2 + (-1)^2}$$
$$= \sqrt{9+1}$$
$$= \sqrt{10}$$

To find BC i.e. Distance between Points B (7, 6) and C (4, 3)

- $x_1 = 7$
- $y_1 = 6$
- $x_2 = 4$
- $y_2 = 3$

By substituting the values in the Equation (1)

$$BC = \sqrt{(7-4)^2 + (6-3)^2}$$
$$= \sqrt{(3)^2 + (3)^2}$$
$$= \sqrt{9+9}$$
$$= \sqrt{18}$$

To find CD i.e. Distance between Points C (4, 3) and D (1, 2)

- $x_1 = 4$
- $y_1 = 3$
- $x_2 = 1$
- $y_2 = 2$

By substituting the values in the Equation (1)

$$CD = \sqrt{(4-1)^2 + (3-2)^2}$$
$$= \sqrt{(3)^2 + (1)^2}$$
$$= \sqrt{9+1}$$
$$= \sqrt{10}$$

To find AD i.e. Distance between Points A (4, 5) and D (1, 2)

- $x_1 = 4$
- $y_1 = 5$
- $x_2 = 1$
- $y_2 = 2$

By substituting the values in the Equation (1)

$$\begin{aligned} AD &= \sqrt{(4-1)^2+(5-2)^2} \\ &= \sqrt{(3)^2+(3)^2} \\ &= \sqrt{9+9} \\ &= \sqrt{18} \end{aligned}$$

To find AC i.e. Distance between Points A (4, 5) and C (4, 3)

- $x_1 = 4$
- $y_1 = 5$
- $x_2 = 4$
- $y_2 = 3$

By substituting the values in the Equation (1)

$$\begin{aligned} \text{Diagonal AC} &= \sqrt{(4-4)^2+(5-3)^2} \\ &= \sqrt{(0)^2+(2)^2} \\ &= \sqrt{0+4} \\ &= 2 \end{aligned}$$

To find BD i.e. Distance between Points B (7, 6) and D (1, 2)

- $x_1 = 7$
- $y_1 = 6$
- $x_2 = 1$
- $y_2 = 2$

By substituting the values in the Equation (1)

$$\begin{aligned} \text{Diagonal BD} &= \sqrt{(7-1)^2+(6-2)^2} \\ &= \sqrt{(6)^2+(4)^2} \\ &= \sqrt{36+16} \\ &= \sqrt{52} \\ &= 13\sqrt{2} \end{aligned}$$

It can be observed that opposite sides of this quadrilateral are of the same length. However, the diagonals are of different lengths. Therefore, the given points are the vertices of a parallelogram.

Q7. Find the point on the x-axis which is equidistant from (2, −5) and (−2, 9).

Difficulty Level: Medium

Reasoning:

The distance between the two points can be measured using the Distance Formula which is given by:

$$\text{Distance Formula} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \quad .$$

Known:

The x and y co-ordinates of the points from which the point on the x-axis is equidistant.

Unknown:

The point on the x-axis which is equidistant from (2, −5) and (−2, 9).

Solution:

Given,

- Since the point is on x-axis the co-ordinates are (x, 0).

We have to find a point on x-axis which is equidistant from A (2, −5) and B (−2, 9).

We know that the distance between the two points is given by the Distance Formula,

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \qquad \text{... Equation (1)}$$

To find the distance between PA, substitute the values of P (x, 0) and A (2, −5) in Equation (1),

$$= \sqrt{(x-2)^2 + (0-(-5))^2}$$
$$= \sqrt{(x-2)^2 + (5)^2}$$

To find the distance between PB, substitute the values of P (x, 0) and B (−2, 9) in Equation (1),

Distance

$$= \sqrt{(x-(-2))^2 + (0-9)^2}$$
$$= \sqrt{(x+2)^2 + (-9)^2}$$

By the given condition, these distances are equal in measure.

Hence PA = PB

$$\sqrt{(x-2)^2 + (5)^2} = \sqrt{(x+2)^2 + (-9)^2}$$

Squaring on both sides

$$(x-2)^2 + 25 = (x+2)^2 + 81$$
$$x^2 + 4 - 4x + 25 = x^2 + 4 + 4x + 81$$
$$8x = 25 - 81$$
$$8x = -56$$
$$x = -7$$

Therefore, the point equidistant from the given points on the axis is (−7, 0).

Q8. Find the values of y for which the distance between the points P (2, −3) and Q (10, y) is 10 units.

Difficulty Level: Medium

Reasoning:

The distance between the two points can be measured using the Distance Formula which is given by:

$$\text{Distance Formula} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \quad .$$

Known:

The x and y co-ordinates of the points which is at a distance of 10 units.

Unknown:

The values of y for which the distance between the points P (2, −3) and Q (10, y) is 10 units

Solution:

Given,

- Distance between points A (2, −3) and B (10, y) is 10 units.

We know that the distance between the two points is given by the Distance Formula,

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \quad \text{... Equation (1)}$$

By Substituting the values of points A (2, −3) and B (10, y) in Equation (1)

Therefore,

$$\sqrt{(2-10)^2 + (-3-y)^2} = 10$$

$$\sqrt{(-8)^2 + (3+y)^2} = 10$$

Squaring on both sides

$$64 + (y+3)^2 = 100$$

$$(y+3)^2 = 36$$

$$y + 3 = \sqrt{36}$$

$$y + 3 = \pm 6$$

$$y + 3 = 6 \quad \text{or} \quad y + 3 = -6$$

Therefore, $y = 3 \text{ or } -9$ are the possible values for y?

Q9. If Q (0, 1) is equidistant from P (5, −3) and R (x, 6), find the values of x. Also find the distances QR and PR.

Difficulty Level: Easy

Reasoning:

The distance between the two points can be measured using the Distance Formula which is given by:

$$\text{Distance Formula} = \sqrt{\left(x_1 - x_2\right)^2 + \left(y_1 - y_2\right)^2}$$.

Known:

The x and y co-ordinates of the points P, Q and R between which the distance is to be measured.

Unknown:

The value of x and the distance Q R and PR.

Solution:

Given,

Since Q (0, 1) is equidistant from P (5, −3) and R (x, 6),

$$PQ = QR$$

We know that the distance between the two points is given by the Distance Formula,

$$\sqrt{\left(x_1 - x_2\right)^2 + \left(y_1 - y_2\right)^2} \quad \text{... Equation (1)}$$

Hence by applying the distance formula for the $PQ = QR$, we get

$$\sqrt{(5-0)^2 + (-3-1)^2} = \sqrt{(0-x)^2 + (1-6)^2}$$

$$\sqrt{(5)^2 + (-4)^2} = \sqrt{(-x)^2 + (-5)^2}$$

By squaring both the sides,

$$25 + 16 = x^2 + 25$$

$$16 = x^2$$

$$x = \pm 4$$

Therefore, point R is (4, 6) or (−4, 6).

Case (1),

When point R is (4, 6),

Distance between P (5, −3) and R (4, 6) can be calculated using the Distance Formula as ,

$$PR = \sqrt{(5-4)^2 + (-3-6)^2}$$

$$= \sqrt{1^2 + (-9)^2}$$

$$= \sqrt{1+81}$$

$$= \sqrt{82}$$

Distance between Q (0, 1) and R (4, 6) can be calculated using the Distance Formula as ,

$$\begin{aligned} QR &= \sqrt{(0-4)^2+(1-6)^2} \\ &= \sqrt{(-4)^2+(-5)^2} \\ &= \sqrt{16+25} \\ &= \sqrt{41} \end{aligned}$$

Case (2),
When point R is (−4, 6),

Distance between P (5, −3) and R (−4, 6) can be calculated using the Distance Formula as,

$$\begin{aligned} PR &= \sqrt{(5-(-4))^2+(-3-6)^2} \\ &= \sqrt{(9)^2+(-9)^2} \\ &= \sqrt{81+81} \\ &= 9\sqrt{2} \end{aligned}$$

Distance between Q (0, 1) and R (−4, 6) can be calculated using the Distance Formula as ,

$$\begin{aligned} QR &= \sqrt{(0-(-4))^2+(1-6)^2} \\ &= \sqrt{(4)^2+(-5)^2} \\ &= \sqrt{16+25} \\ &= \sqrt{41} \end{aligned}$$

Q10. Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (−3, 4).

Difficulty Level: Medium

Reasoning:
The distance between the two points can be measured using the Distance Formula which is given by:

$$\text{Distance Formula} = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2} \quad .$$

Known:
The x and y co-ordinates of the points between which the distance is to be measured.

Unknown:
The relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (−3, 4)

Solution:

- Let Point P (x, y) be equidistant from points A (3, 6) and B (−3, 4).

We know that the distance between the two points is given by the Distance Formula,

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \quad \text{... Equation (1)}$$

Since they are equidistant, PA = PB

Hence by applying the distance formula for PA = PB, we get

$$\therefore \sqrt{(x-3)^2 + (y-6)^2} = \sqrt{x-(-3)^2 + (y-4)^2}$$

$$\sqrt{(x-3)^2 + (y-6)^2} = \sqrt{(x+3)^2 + (y-4)^2}$$

By Squaring, $PA^2 = PB^2$

$$(x-3)^2 + (y-6)^2 = (x+3)^2 + (y-4)^2$$

$$x^2 + 9 - 6x + y^2 + 36 - 12y = x^2 + 9 + 6x + y^2 + 16 - 8y$$

$$6x + 6x + 12y - 8y = 36 - 16$$

$$12x + 4y = 20$$

$$3x + y = 5$$

$$3x + y - 5 = 0$$

Thus, the relation between x and y is given by $3x + y - 5 = 0$

Chapter 7: Coordinate Geometry

Exercise 7.2 (Page 167 of Grade 10 NCERT Textbook)

Q1. Find the coordinates of the point which divides the join of (−1, 7) and (4, −3) in the ratio 2:3.

Difficulty Level: Easy

Reasoning:
The coordinates of the point P(x, y) which divides the line segment joining the points A(x1, y1) and B(x2, y2), internally, in the ratio m1 : m2 is given by the Section Formula

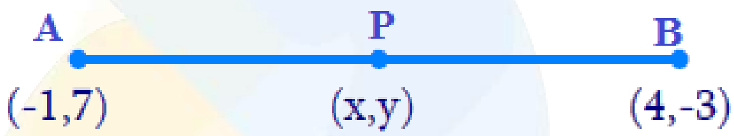
$$P(x, y) = \left[\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right]$$

Known:
The x and y co-ordinates of the points which is to be divided in the ratio 2:3

Unknown:
The coordinates of the point which divides the join of (−1, 7) and (4, −3) in the ratio 2:3

Solution:
Given,

- Let P(x, y) be the required point.



- Let A(−1, 7) and B(4, −3)
- m: n = 2:3
- Hence
  - $x_1 = -1$
  - $y_1 = 7$
  - $x_2 = 4$
  - $y_2 = -3$

By Section formula

$$P(x, y) = \left[\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right] \qquad \text{...Equation (1)}$$

By substituting the values in the Equation (1)

$$x = \frac{2\times 4 + 3x(-1)}{2+3} \qquad y = \frac{2\times(-3)+3\times 7}{2+3}$$

$$x = \frac{8-3}{5} \qquad y = \frac{-6+21}{5}$$

$$x = \frac{5}{5} = 1 \qquad y = \frac{15}{5} = 3$$

Therefore, the co-ordinates of point P are (1, 3).

Q2. Find the coordinates of the points of trisection of the line segment joining (4, −1) and (−2, −3).

Difficulty Level: Medium

Reasoning:

The coordinates of the point P(x, y) which divides the line segment joining the points A(x1, y1) and B(x2, y2), internally, in the ratio m1 : m2 is given by the Section Formula.

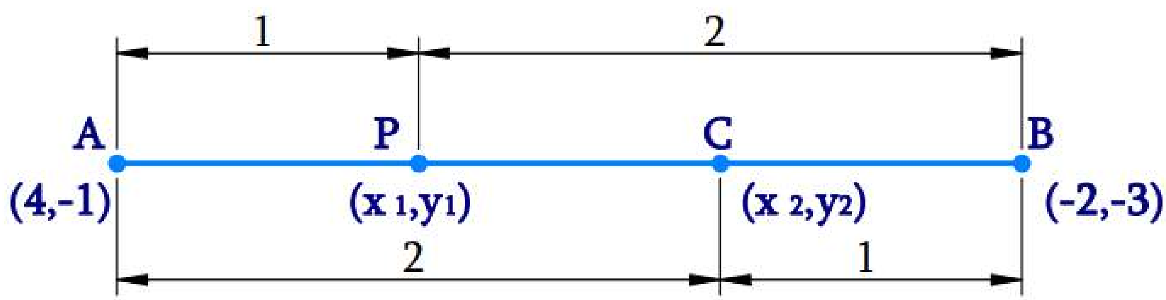
Known:

The x and y co-ordinates of the line segment joining (4, −1) and (−2, −3).

Unknown:

The coordinates of the points of trisection of the line segment joining (4, −1) and (−2, −3).

Solution:



Given,

- Let line segment joining the points be A(4, −1) and B(−2, −3).
- Let P (x_1, y_1) and Q (x_2, y_2) be the points of trisection of the line segment joining the given points i.e., AP = PQ = QB

By Section formula

$$P(x,y) = \left[\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right] \qquad \text{...Equation (1)}$$

Therefore, by observation point P divides AB internally in the ratio 1:2.

- Hence m: n = 1:2

By substituting the values in the Equation (1)

$$x_1 = \frac{1\times(-2)+2\times 4}{1+2}, \qquad y_1 = \frac{1\times(-3)+2\times(-1)}{1+2}$$

$$x_1 = \frac{-2+8}{3} = \frac{6}{3} = 2, \qquad y_1 = \frac{-3-2}{3} = \frac{-5}{3}$$

Therefore, $P(x_1, y_1) = \left(2, \frac{-5}{3}\right)$

Therefore, by observation point Q divides AB internally in the ratio 2:1.

- Hence m:n = 2:1

By substituting the values in the Equation (1)

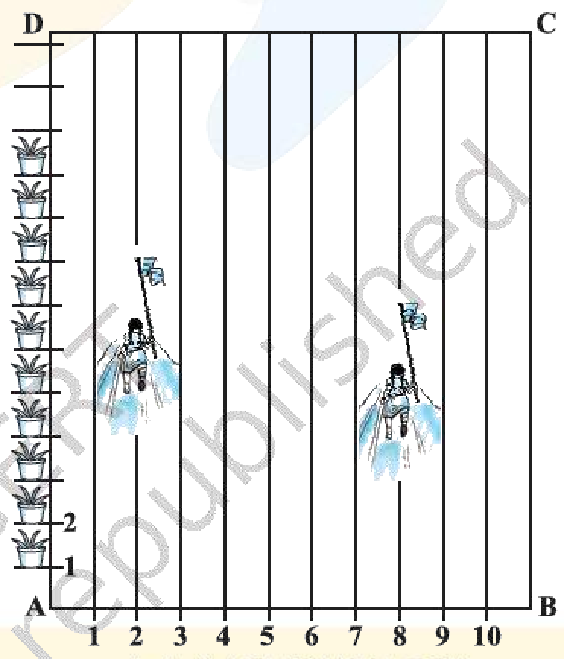
$$x_2 = \frac{2\times(-2)+1\times 4}{2+1}, \qquad y_2 = \frac{2x(-3)+1\times(-1)}{2+1}$$

$$x_2 = \frac{-4+4}{3} = 0, \qquad y_2 = \frac{-6-1}{3} = \frac{-7}{3}$$

Therefore, $Q(x_2, y_2) = \left(0, -\frac{7}{3}\right)$

Hence the points of trisection are $P(x_1, y_1) = \left(2, \frac{-5}{3}\right)$ and $Q(x_2, y_2) = \left(0, -\frac{7}{3}\right)$

Q3. To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1m each. 100 flower pots have been placed at a distance of 1m from each other along AD, as shown in the following figure. Niharika runs $\frac{1}{4}$th the distance AD on the 2nd line and posts a green flag. Preet runs $\frac{1}{5}$th the distance AD on the eighth line and posts a red flag. What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the line segments joining the two flags, where should she post her flag?



Difficulty Level: Medium

Reasoning:
The coordinates of the point P(x, y) which divides the line segment joining the points A(x1, y1) and B(x2, y2), internally, in the ratio m1 : m2 is given by the Section Formula.

Known:
- The school ground ABCD is rectangular shaped.
- Lines are drawn at a distance of 1m each and 100 flower pots have been placed at a distance of 1m each along AD.
- The distance covered by Niharika and Preet on line AD.

Unknown:
- The distance between the flags posted by Niharika and Preet.
- The position on the line segment where Rashmi has to post the flag.

Solution:
From the Figure,

Given,
- By observation, that Niharika posted the green flag at of the distance P i.e., $\left(\frac{1}{4}\times 100\right)\text{m} = 25\,\text{m}$ from the starting point of 2nd line. Therefore, the coordinates of this point P is (2, 25).
- Similarly, Preet posted red flag at $\frac{1}{5}$ of the distance Q i.e., $\left(\frac{1}{5}\times 100\right)\text{m} = 20\,\text{m}$ from the starting point of 8th line. Therefore, the coordinates of this point Q are (8, 20)

We know that the distance between the two points is given by the Distance Formula,

$$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2} \qquad \text{...Equation (1)}$$

To find the distance between these flags PQ by substituting the values in Equation (1),

$$\begin{aligned} PQ &= \sqrt{(2-8)^2+(25-20)^2} \\ &= \sqrt{36+25} \\ &= \sqrt{61\text{m}} \end{aligned}$$

- The point at which Rashmi should post her blue flag is the mid-point of the line joining these points.
- Let this point be M (x, y).

By Section formula

$$P(x, y) = \left[\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right] \qquad \text{... Equation (2)}$$

$$x = \frac{2+8}{2}, \qquad y = \frac{25+20}{2}$$

$$x = \frac{10}{2} \qquad y = \frac{45}{2}$$

$$x = 5, \qquad y = 22.5$$

Therefore, Rashmi should post her blue flag at 22.5 m on 5^{th} line

Q4. Find the ratio in which the line segment joining the points (−3, 10) and (6, −8) is divided by (−1, 6).

Difficulty Level: Medium

Reasoning:

The coordinates of the point P(x, y) which divides the line segment joining the points A(x1, y1) and B(x2, y2), internally, in the ratio m1 : m2 is given by the Section Formula.

$$P(x, y) = \left[\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right]$$

Known:

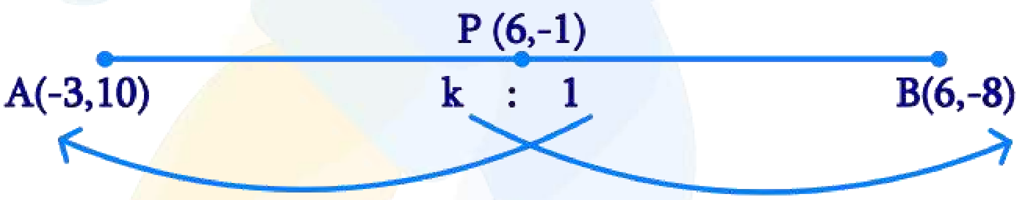
The x and y co-ordinates of the line segment which is divided by the point (−1, 6).

Unknown:

The ratio in which the line segment joining the points (−3, 10) and (6, −8) is divided by (−1, 6).

Solution:

From the figure,



Given,

- Let the ratio in which the line segment joining A(−3, 10) and B(6, −8) is divided by point P(−1, 6) be k:1.

By Section formula

$$P(x, y) = \left[\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right] \qquad \ldots \text{Equation (2)}$$

Therefore,

$$-1 = \frac{6k-3}{k+1}$$

$$-k-1 = 6k-3$$

$$7k = 2 \qquad \text{(By Cross Multiplying \& Transposing)}$$

$$k = \frac{2}{7}$$

Hence the point P divides AB in the ratio 2:7

Q5. Find the ratio in which the line segment joining A (1, −5) and B (−4, 5) is divided by the x-axis. Also find the coordinates of the point of division.

Difficulty Level: Medium

Reasoning:

The coordinates of the point P(x, y) which divides the line segment joining the points A(x1, y1) and B(x2, y2), internally, in the ratio m1 : m2 is given by the Section Formula.

Known:

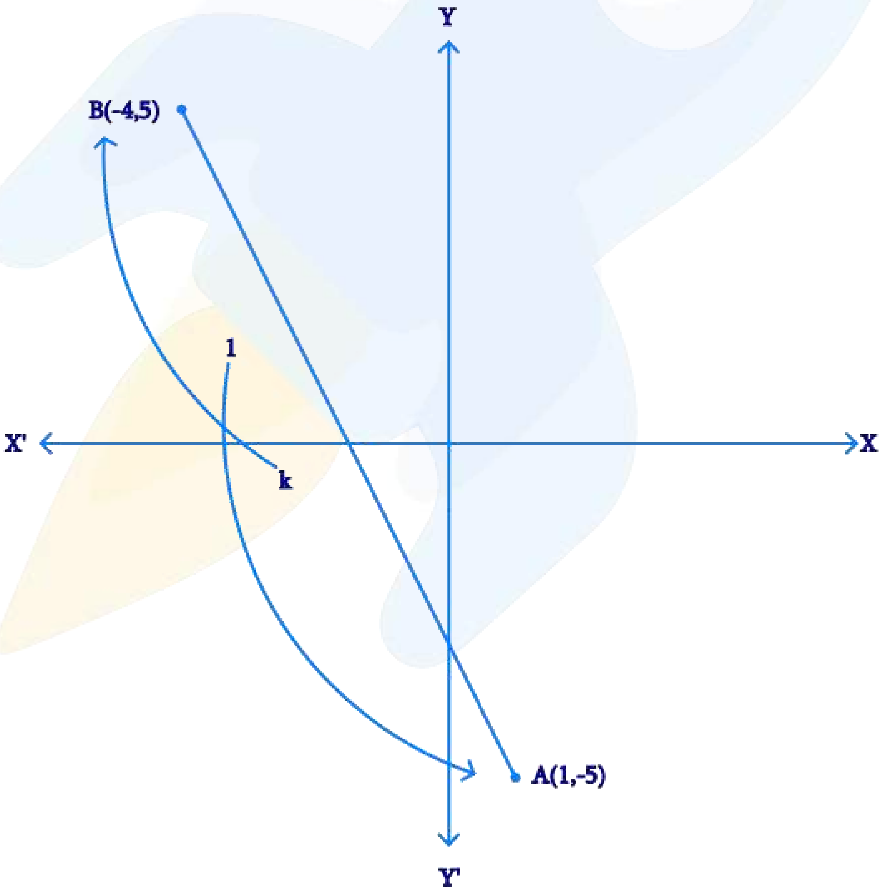
The x and y co-ordinates of the line segment which is divided by the x-axis.

Unknown:

The ratio in which the line segment joining A (1, −5) and B (−4, 5) is divided by the x-axis and the coordinates of the point of division

Solution:

From the Figure,



Given,

- Let the ratio be k : 1.
- Let the line segment joining A (1, −5) and B (−4, 5)

By Section formula

$$P(x, y) = \left[\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right] \quad \text{...Equation (1)}$$

By substituting the values in Equation (1)

Therefore, the coordinates of the point of division is $\left(\frac{-4k+1}{k+1}, \frac{5k-5}{k+1}\right)$

We know that y-coordinate of any point on x-axis is 0.

$$\therefore \frac{5k-5}{k+1} = 0$$
$$5k - 5 = 0$$
$$\rightarrow 5k = 5 \quad \text{(By cross multiplying \& Transposing)}$$
$$k = 1$$

Therefore, x-axis divides it in the ratio 1:1.

$$\text{Division point} = \left(\frac{-4(1)+1}{1+1}, \frac{5(1)+5}{1+1}\right)$$
$$= \left(\frac{-4+1}{2}, \frac{5+5}{2}\right)$$
$$= \left(\frac{-3}{2}, 0\right)$$

Q6. If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

Difficulty Level: Medium

Reasoning:

The coordinates of the point P(x, y) which divides the line segment joining the points A(x1, y1) and B(x2, y2), internally, in the ratio m1 : m2 is given by the Section Formula.

$$P(x, y) = \left[\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right]$$

Known:

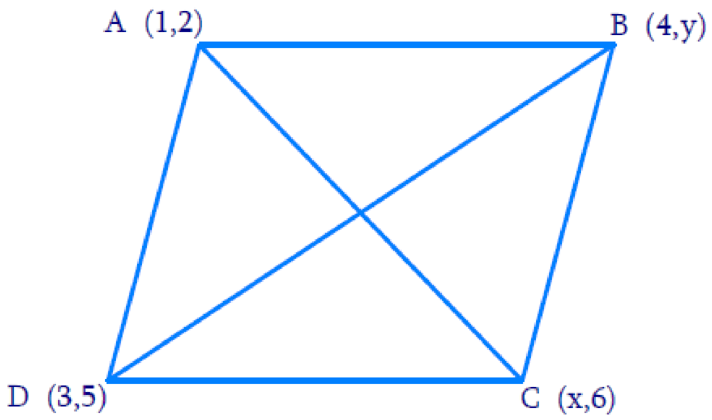
The x and y co-ordinates of the vertices of the parallelogram.

Unknown:

The missing x and y co-ordinate.

Solution:

From the Figure,



Given,

- Let A (1, 2), B (4, y), C(x, 6), and D (3, 5) are the vertices of a parallelogram ABCD.
- Since the diagonals of a parallelogram bisect each other, Intersection point O of diagonal AC and BD also divides these diagonals

Therefore, O is the mid-point of AC and BD.

If O is the mid-point of AC, then the coordinates of O are

$$\left(\frac{1+x}{2}, \frac{2+6}{2}\right) \Rightarrow \left(\frac{x+1}{2}, 4\right)$$

If O is the mid-point of BD, then the coordinates of O are

$$\left(\frac{4+3}{2}, \frac{5+y}{2}\right) \Rightarrow \left(\frac{7}{2}, \frac{5+y}{2}\right)$$

Since both the coordinates are of the same point O,

$$\therefore \frac{x+1}{2} = \frac{7}{2} \text{ and } 4 = \frac{5+y}{2}$$

$$\Rightarrow x+1 = 7 \text{ and } 5+y = 8 \qquad \text{(By cross multiplying \& transposing)}$$

$$\Rightarrow x = 6 \text{ and } y = 3$$

Q7. Find the coordinates of a point A, where AB is the diameter of circle whose center is (2, −3) and B is (1, 4)

Difficulty Level: Medium

Reasoning:

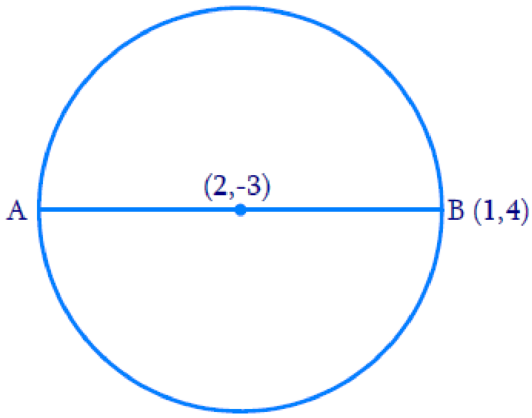
The coordinates of the point P(x, y) which divides the line segment joining the points A(x1, y1) and B(x2, y2), internally, in the ratio m1 : m2 is given by the Section Formula.

Known:

The x and y co-ordinates of the center of the circle and one end of the diameter B.

Unknown:
The coordinates of a point A.

Solution:
From the Figure,



Given,

- Let the coordinates of point A be (x, y).
- Mid-point of AB is C (2, −3), which is the center of the circle.

$$\therefore (2,-3)=\left(\frac{x+1}{2},\frac{y+4}{2}\right)$$

$$\Rightarrow \frac{x+1}{2}=2 \text{ and } \frac{y+4}{2}=-3 \qquad \text{(By Cross multiplying \& transposing)}$$

$$\Rightarrow x+1=4 \text{ and } y+4=-6$$

$$\Rightarrow x=3 \text{ and } y=-10$$

Therefore, the coordinates of A are (3, −10)

Q8. If A and B are (−2, −2) and (2, −4), respectively, find the coordinates of P such that AP = $\frac{3}{7}$ AB and P lies on the line segment AB.

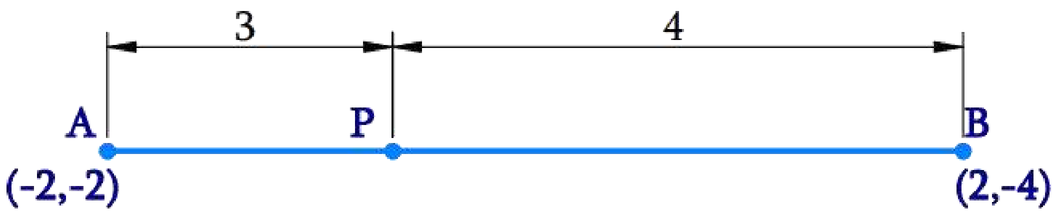
Reasoning:
The coordinates of the point P(x, y) which divides the line segment joining the points A(x1, y1) and B(x2, y2), internally, in the ratio m1 : m2 is given by the Section Formula.

$$P(x,y)=\left[\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right]$$

Known:
The x and y co-ordinates of the points A and B.
The ratio in which P divides AB.

Unknown:
Co-ordinates of P

Solution:
From the Figure,



Given,

- The coordinates of point A and B are (−2, −2) and (2, −4) respectively.
- $AP = \frac{3}{7}AB$

Hence $\frac{AB}{AP} = \frac{7}{3}$

We know that $AB = AP + PB$ from figure,

$$\frac{AP + PB}{AP} = \frac{3+4}{3}$$

$$1 + \frac{PB}{AP} = 1 + \frac{4}{3}$$

$$\frac{PB}{AP} = \frac{4}{3}$$

Therefore, AP:PB = 3:4

Point P(x, y) divides the line segment AB in the ratio 3:4. Using Section Formula,

$$\text{Coordinates of } P(x, y) = \left(\frac{3\times 2 + 4\times(-2)}{3+4}, \frac{3\times(-4) + 4\times(-2)}{3+4}\right)$$

$$= \left(\frac{6-8}{7}, \frac{-12-8}{7}\right)$$

$$= \left(-\frac{2}{7}, -\frac{20}{7}\right)$$

Q9. Find the coordinates of the points which divide the line segment joining A (−2, 2) and B (2, 8) into four equal parts.

Difficulty Level: Medium

Reasoning:

The coordinates of the point P(x, y) which divides the line segment joining the points A(x1, y1) and B(x2, y2), internally, in the ratio m1 : m2 is given by the Section Formula.

Known:

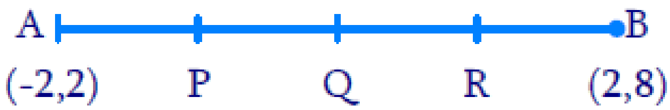
The x and y co-ordinates of the points A and B.

Unknown:

The coordinates of the points which divide the line segment joining A (−2, 2) and B (2, 8) into four equal parts.

Solution:

From the Figure,



By observation, that points P, Q, R divides the line segment A (−2, 2) and B (2, 8) into four equal parts

Point P divides the line segment AQ into two equal parts

$$\text{Hence, Coordinates of P} = \left(\frac{1\times2+3\times(-2)}{1+3}, \frac{1\times8+3\times2}{1+3}\right)$$

$$= \left(-1, \frac{7}{2}\right)$$

Point Q divides the line segment AB into two equal parts

$$\text{Coordinates of Q} = \left(\frac{2+(-2)}{2}, \frac{2+8}{2}\right)$$

$$= (0, 5)$$

Point R divides the line segment BQ into two equal parts

$$\text{Coordinates of R} = \left(\frac{3\times2+1\times(-2)}{3+1}, \frac{3\times8+1\times2}{3+1}\right)$$

$$= \left(1, \frac{13}{2}\right)$$

Q10. Find the area of a rhombus if its vertices are (3, 0), (4, 5), (−1, 4) and (−2, −1) taken in order. [**Hint:** Area of a rhombus = (product of its diagonals)]

Difficulty Level: Medium

Reasoning:

A rhombus has all sides of equal length and opposite sides are parallel.

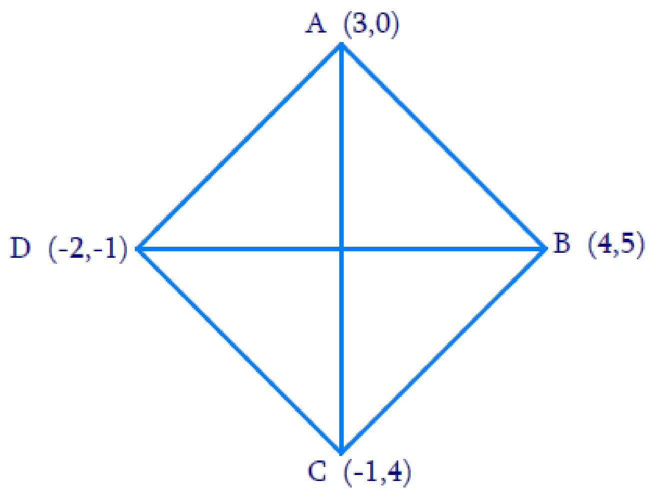
Known:

The x and y co-ordinates of the vertices of the rhombus.

Unknown:

The area of the rhombus

Solution:



From the Figure,

Given,

- Let A(3, 0), B(4, 5), C(−1, 4) and D(−2, −1) are the vertices of a rhombus ABCD.

We know that the distance between the two points is given by the Distance Formula,

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \qquad \text{... Equation (1)}$$

Therefore, distance between A (3, 0) and C (−1, 4) is given by

$$\text{Length of diagonal AC} = \sqrt{[3-(-1)]^2 + (0-4)^2}$$
$$= \sqrt{16+16} = 4\sqrt{2}$$

Therefore, distance between B (4, 5) and D (−2, −1) is given by

$$\text{Length of diagonal BD} = \sqrt{[4-(-2)]^2 + (5-(-1)^2}$$
$$= \sqrt{36+36}$$
$$= 6\sqrt{2}$$

$$\text{Area of the rhombus ABCD} = \frac{1}{2} \times \text{(Product of lengths of diagonals)}$$
$$= \frac{1}{2} \times AC \times BD$$

Therefore, area of rhombus

$$ABCD = \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$$
$$= 24 \text{ square units}$$

Chapter 7: Coordinate Geometry

Exercise 7.3 (Page 170 of Grade 10 NCERT Textbook)

Q1. Find the area of the triangle whose vertices are:

(i) (2, 3), (−1, 0), (2, −4)

(ii) (ii) (−5, −1), (3, −5), (5, 2)

Reasoning:

Let ABC be any triangle whose vertices are A(x1, y1), B(x2, y2) and C(x3, y3).

$$\text{Area of a triangle} = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

Known:

The x and y co-ordinates of the vertices of the triangle.

Unknown:

The area of the triangle.

Solution.

(i) Given,

- Let A(x_1, y_1) = (2, 3)
- Let B(x_2, y_2) = (−1 , 0)
- Let C(x_3, y_3) = (2, −4)

$$\text{Area of a triangle} = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\} \qquad \text{...Equation}(1)$$

By substituting the values of vertices, A, B, C in the Equation (1),

$$\text{Area of the given triangle} = \frac{1}{2}[2\{0-(-4)\} + (-1)\{(-4)-(3)\}+2(3-0)]$$

$$= \frac{1}{2}\{8 + 7 + 6\}$$

$$= \frac{21}{2} \text{ Square units}$$

(ii) Given,

- Let A(x_1, y_1) = (−5, −1)
- Let B(x_2, y_2) = (3, −5)
- Let C(x_3, y_3) = (5, 2)

$$\text{Area of a triangle} = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\} \qquad \text{...Equation}(1)$$

By substituting the values of vertices, A, B, C in the Equation (1), Area of the given triangle = $\frac{1}{2}[(-5)\{(-5)-2\} + 3(2-(-1)) + 5\{-1-(-5)\}]$

$$= \frac{1}{2}\{35 + 9 + 20\}$$

$$= 32 \text{ square units}$$

Q2. In each of the following find the value of 'k', for which the points are collinear.

(i) (7, −2), (5, 1), (3, k)

(ii) (8, 1), (k, −4), (2, −5)

Difficulty Level: Medium

Reasoning:

Three or more points are said to be collinear if they lie on a single straight line.

Known:

The x and y co-ordinates of the points.

Unknown:

The value of 'k', for which the points are collinear.

Solution:

(i) Given,

- Let A(x_1, y_1) = (7, −2)
- Let B(x_2, y_2) = (5 , 1)
- Let C(x_3, y_3) = (3, k)

Area of a triangle = $\frac{1}{2}\{x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)\}=0$ for Collinear Points

By substituting the values of vertices, A, B, C in the Equation (1),

$$\frac{1}{2}[7\{1-k\} + 5\{k-(-2)\} + 3\{(-2)-1\}] = 0$$

$$7-7k + 5k + 10-9 = 0$$

$$-2k + 8 = 0$$

$$k = 4$$

Hence the given points are collinear for $k = 4$

(ii) Given,

- Let A(x_1, y_1) = (8, 1)
- Let B(x_2, y_2) = (k , −4)
- Let C(x_3, y_3) = (2, −5)

Area of a triangle = $\frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\} = 0$ for

Collinear Points

By substituting the values of vertices, A, B, C in the Equation (1),

$$\frac{1}{2}[8\{-4-(-5)\} + k\{(-5)-(1)\} + 2\{1-(-4)\}] = 0$$

$8 - 6k + 10 = 0$ (By Transposing)

$6k = 18$

$k = 3$

Hence the given points are collinear for $k = 3$

Q3. Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, −1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.

Reasoning:

Let ABC be any triangle whose vertices are A(x1, y1), B(x2, y2) and C(x3, y3).

Area of a triangle = $\frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

Known:

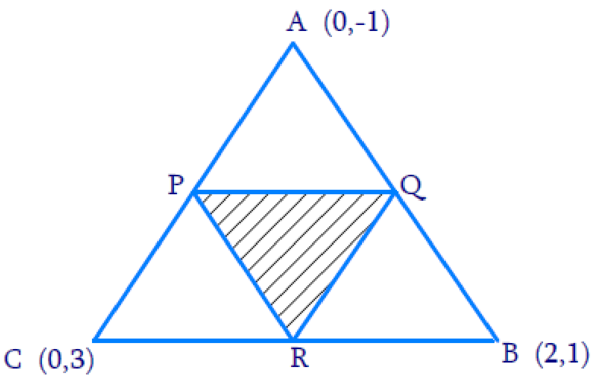
The x and y co-ordinates of the vertices of the triangle.

Unknown:

The ratio of this area to the area of the given triangle.

Solution:

From the given figure,



Given,

- Let A(x_1, y_1) = (0, −1)
- Let B(x_2, y_2) = (2 , 1)
- Let C(x_3, y_3) = (0, 3)

Area of a triangle $= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$ ...Equation (1)

By substituting the values of vertices, A, B, C in (1).

Let P, Q, R be the mid-points of the sides of this triangle.

Coordinates of P, Q, and R are given by

$$P = \left(\frac{0+2}{2}, \frac{-1+1}{2}\right) = (1, 0)$$
$$Q = \left(\frac{0+0}{2}, \frac{3-1}{2}\right) = (0, 1)$$
$$R = \left(\frac{2+0}{2}, \frac{1+3}{2}\right) = (1, 2)$$

By substituting the values of Points P, Q, R

$$\begin{aligned} \text{Area of } \Delta PQR &= \frac{1}{2}\{(2-1) + 1(1-0) + 0(0-2)\} \\ &= \frac{1}{2}(1+1) \\ &= 1 \text{ Square units} \end{aligned}$$

By substituting the values of Points A, B, C

$$\begin{aligned} \text{Area if } \Delta ABC &= \frac{1}{2}[0(1-3) + 2\{3-(-1)\} + 0(-1-1)] \\ &= \frac{1}{2}\{8\} \\ &= 4 \text{ Square units} \end{aligned}$$

Therefore, Ratio of this area ΔPQR to the area of the triangle $\Delta ABC = 1:4$

Q4. Find the area of the quadrilateral whose vertices, taken in order, are (−4, −2), (−3, −5), (3, −2) and (2, 3)

Reasoning:

Let ABC be any triangle whose vertices are A(x1, y1), B(x2, y2) and C(x3, y3).

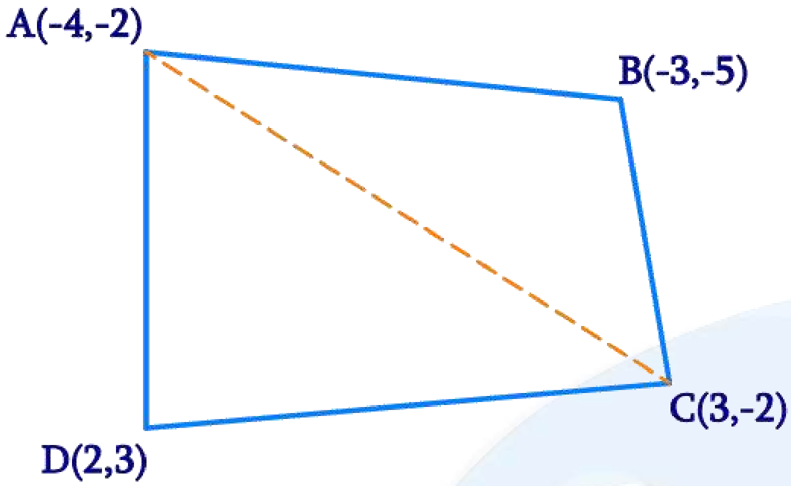
Area of a triangle $= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

Known:

The x and y co-ordinates of the vertices of the quadrilateral.

Unknown:
The area of the quadrilateral.

Solution:
From the figure,



Given,

- Let the vertices of the quadrilateral be A (−4, −2), B (−3, −5), C (3, −2), and D (2, 3).
- Join AC to form two triangles ΔABC and ΔACD.

We know that,

$$\text{Area of a triangle} = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\} \quad \text{...Equation}(1)$$

By substituting the values of vertices, A, B, C in the Equation (1),

$$\text{Area of } \Delta ABC = \frac{1}{2}[(-4)\{(-5)-(-2)\}+(-3)\{(-2)-(-2)\}+3\{(-2)-(-5)\}]$$

$$= \frac{1}{2}(12+0+9)$$

$$= \frac{21}{2} \text{ Square units}$$

By substituting the values of vertices, A, C, D in the Equation (1),

$$\Delta ACD = \frac{1}{2}[(-4)\{(-2)-3\}+3\{(3)-(-2)+2\{(-2)\}-(-2)\}]$$

$$= \frac{1}{2}\{20+15+0\}$$

$$= \frac{35}{2} \text{ Square units}$$

$$\text{Area of ABCD} = \text{Area of } \Delta ABC + \text{Area of } \Delta ACD$$

$$= \left(\frac{21}{2}+\frac{35}{2}\right) \text{ Square units}$$

$$= 28 \text{ square units}$$

Q5. You have studied in Class IX that a median of a triangle divides it into two triangles of equal areas. Verify this result for ΔABC whose vertices are A (4, −6), B (3, −2) and C (5, 2)

Difficulty Level: Medium

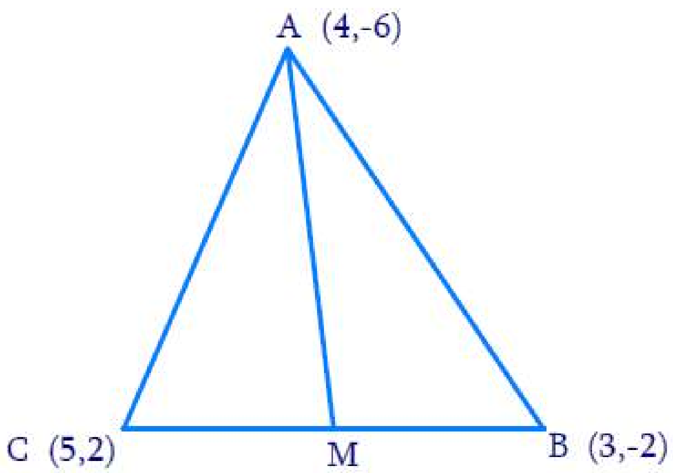
Reasoning:
Let ABC be any triangle whose vertices are A(x1, y1), B(x2, y2) and C(x3, y3).

$$\text{Area of a triangle} = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

Known:
The x and y co-ordinates of the vertices of the triangle.

Unknown:
To verify that a median of a triangle divides it into two triangles of equal areas.

Solution:
From the figure,



Given,

- Let the vertices of the triangle be A (4, −6), B (3, −2), and C (5, 2).
- Let M be the mid-point of side BC of ΔABC.

Therefore, AM is the median in ΔABC.

$$\text{Coordinates of point M} = \left(\frac{3+5}{2} + \frac{-2+2}{2}\right)$$
$$= (4, 0)$$

$$\text{Area of a triangle} = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\} \qquad \text{...Equation}(1)$$

By substituting the values of vertices, A, B, M in the Equation (1)

$$\text{Area of } \Delta ABM = \frac{1}{2}[(4)\{(-2)-(0)\}+(3)\{(0)-(-6)\}+(4)\{(-6)\}-(-2)\}$$
$$= \frac{1}{2}(-8+18-16)$$
$$= 3 \text{ Square units}$$

By substituting the values of vertices, A, D, C in the Equation (1)

$$\text{Area of } \Delta ABD = \frac{1}{2}[(4)\{(-2)-(0)\}+(3)\{(0)-(-6)\}+(4)\{(-6)\}-(-2)\}$$
$$= \frac{1}{2}(-8+18-16)$$
$$= 3 \text{ Square units}$$

However, area cannot be negative. Therefore, area of ΔAMC is 3 square units.

Hence, clearly, median AM has divided ΔABC in two triangles of equal areas.

Chapter 7: Coordinate Geometry

Exercise 7.4 (Page 171 of Grade 10 NCERT Textbook)

Q1. Determine the ratio in which the line $2x + y - 4 = 0$ divides the line segment joining the points $A\ (2, -2)$ and $B\ (3, 7)$.

Difficulty Level: Medium

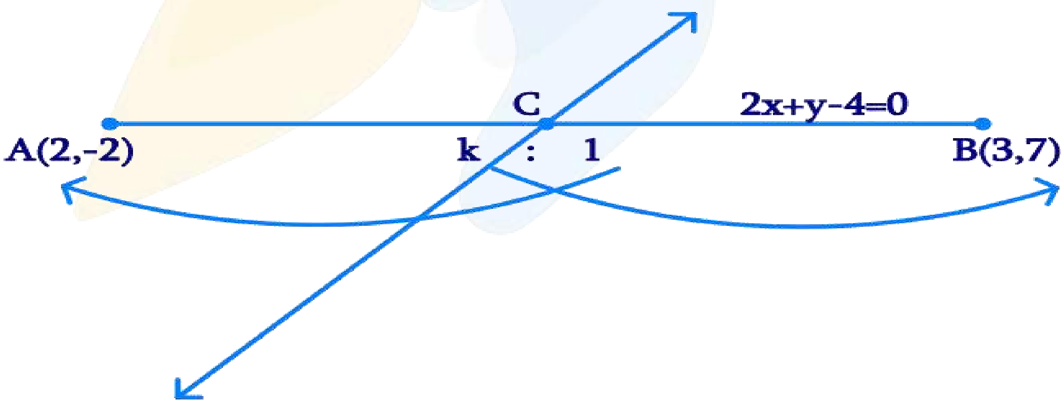
Reasoning:
The coordinates of the point P(x, y) which divides the line segment joining the points A(x1, y1) and B(x2, y2), internally, in the ratio m1 : m2 is given by the Section Formula.

$$P(x, y) = \left[\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right]$$

What is known/given?
The x and y co-ordinates of the points A and B.
The equation of the straight line i.e. $2x + y - 4 = 0$

What is the unknown?
The ratio in which the line $2x + y - 4 = 0$ divides the line segment joining the points $A\ (2, -2)$ and $B\ (3, 7)$

Solution:
From the Figure,



Given,

- Let the given line $2x + y - 4 = 0$ divide the line segment joining the points $A(2, \ -2)$ and $B(3, \ 7)$ in a ratio $k : 1$ at point C.

Coordinates of the point of division, $C(x, y) = \left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1} \right)$

This point C also lies on $2x + y - 4 = 0$ ………Equation (1)

By substituting the values of $C(x, y)$ in Equation (1),

$$\therefore 2\left(\frac{3k+2}{k+1}\right) + \left(\frac{7k-2}{k+1}\right) - 4 = 0$$

$$\frac{6k+4+7k-2-4k-4}{k+1} = 0 \quad \text{(By Cross multiplying \& Transposing)}$$

$$9k - 2 = 0$$

$$k = \frac{2}{9}$$

Therefore, the ratio in which the line $2x + y - 4 = 0$ divides the line segment joining the points A $(2, -2)$ and B $(3, 7)$ is 2:9 internally.

Q2. Find a relation between x and y if the points (x, y), $(1, 2)$ and $(7, 0)$ are collinear.

Difficulty Level: Medium

Reasoning:

Three or more points are said to be collinear if they lie on a single straight line .

What is the known/given?

The x and y co-ordinates of the points.

What is the unknown?

The relation between x and y if the points (x, y), $(1, 2)$ and $(7, 0)$ are collinear.

Solution:

Given,

- Let $A(x1, y1) = (x, y)$
- Let $B(x2, y2) = (1, 2)$
- Let $C(x3, y3) = (7, 0)$

If the given points are collinear, then the area of triangle formed by these points will be 0.

Area of a triangle $= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$ ……Equation (1)

By substituting the values of vertices, A, B, C in the Equation (1),

$$\text{Area} = \frac{1}{2}[x(2-0)+1(0-y)+7(y-2)]$$

$$0 = \frac{1}{2}[2x - y + 7y - 14]$$

$$0 = \frac{1}{2}(2x + 6y - 14)$$

$$2x + 6y - 14 = 0$$

$$x + 3y - 7 = 0$$

This is the required relation between *x* and *y*.

Q3. Find the centre of a circle passing through the points $(6, -6)$, $(3, -7)$ and $(3, 3)$.

Difficulty Level: Medium

Reasoning:

The distance between the two points can be measured using the Distance Formula which is given by:

$$\text{Distance Formula} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

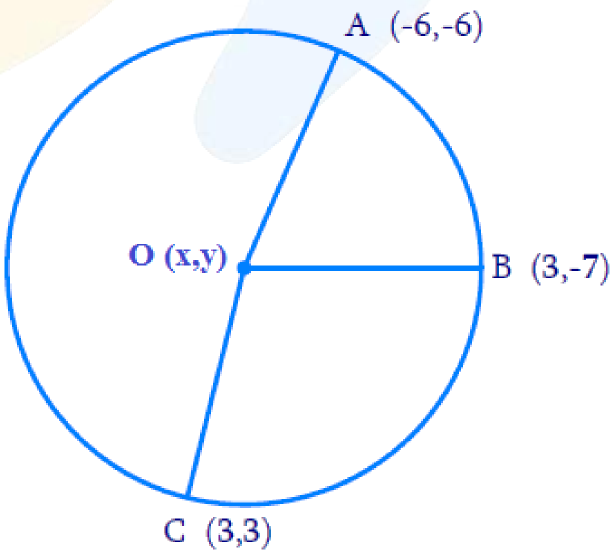
What is the known/given?

The x and y co-ordinates of the points.

What is the unknown?

The centre of the circle passing through the points $(6, -6)$, $(3, -7)$ and $(3, 3)$.

Solution:



From the Figure,

Given,

- Let $O(x, y)$ be the centre of the circle.
- Let the points $(6, -6)$, $(3, -7)$, and $(3, 3)$ be representing the points A, B, and C on the circumference of the circle.

Distance from centre O to A, B, C are found below using the Distance formula mentioned in the Reasoning.

$$\therefore OA = \sqrt{(x-6)^2 + (y+6)^2}$$
$$OB = \sqrt{(x-3)^2 + (y+7)^2}$$
$$OC = \sqrt{(x-3)^2 + (y-3)^2}$$

From the Figure that, $OA = OB$ (radii of the same circle)

$$\sqrt{(x-6)^2 + (y+6)^2} = \sqrt{(x-3)^2 + (y+7)^2}$$
$$x^2 + 36 - 12x + y^2 + 36 + 12y = x^2 + 9 - 6x + y^2 + 49 + 14y \qquad \text{(Squaring on both sides)}$$
$$-6x - 2y + 14 = 0$$
$$3x + y = 7 \qquad \ldots\ldots \text{Equation (1)}$$

Similarly, $OA = OC$ (radii of the same circle)

$$\sqrt{(x-6)^2 + (y+6)^2} = \sqrt{(x-3)^2 + (y-3)^2}$$
$$x^2 + 36 - 12x + y^2 + 36 + 12y = x^2 + 9 - 6x + y^2 + 9 - 6y \qquad \text{(Squaring on both sides)}$$
$$-6x + 18y + 54 = 0$$
$$-3x + 9y = -27 \qquad \ldots\ldots \text{Equation(2)}$$

On adding Equation (1) and Equation (2), we obtain

$$10y = -20$$
$$y = -2$$

From Equation (1),

we obtain

$$3x - 2 = 7$$
$$3x = 9$$
$$x = 3$$

Therefore, the centre of the circle is (3, −2).

Q4. The two opposite vertices of a square are $(-1,2)$ and $(3,2)$. Find the coordinates of the other two vertices.

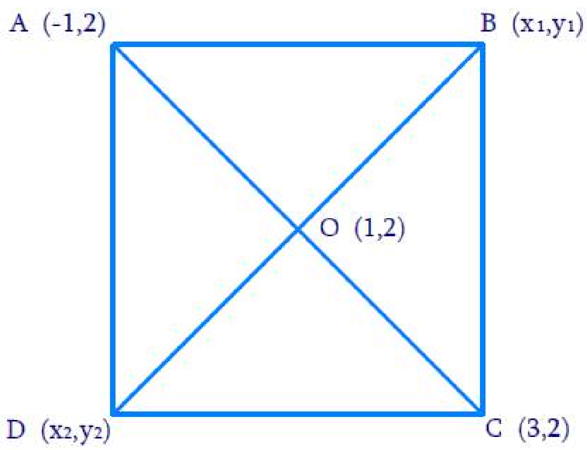
Difficulty Level: Medium

Reasoning:

What is the known/given?
The x and y co-ordinates of the two opposite vertices of a square.

What is the unknown?
The coordinates of the other two vertices.

Solution:
From the Figure,



Given,

- Let ABCD be a square having known vertices $A(-1,\ 2)$ and $C(3,\ 2)$ as vertices A and C respectively.
- Let $B(x_1, y_1)$ be one unknown vertex

We know that the sides of a square are equal to each other.

$\therefore$ AB = BC

By Using Distance formula to find distance between points AB & BC,

$$\sqrt{(x_1-(-1))^2+(y_1-2)^2}=\sqrt{(x_1-3)^2+(y_1-2)^2}$$

$$x_1^2+2x_1+1+y_1^2-4y_1+4=x_1^2+9-6x_1+y_1^2+4-4y_1 \quad \text{(By Simplifying \& Transposing)}$$

$$8x_1=8$$

$$x_1=1$$

We know that in a square, all interior angles are of 90°.

In ΔABC

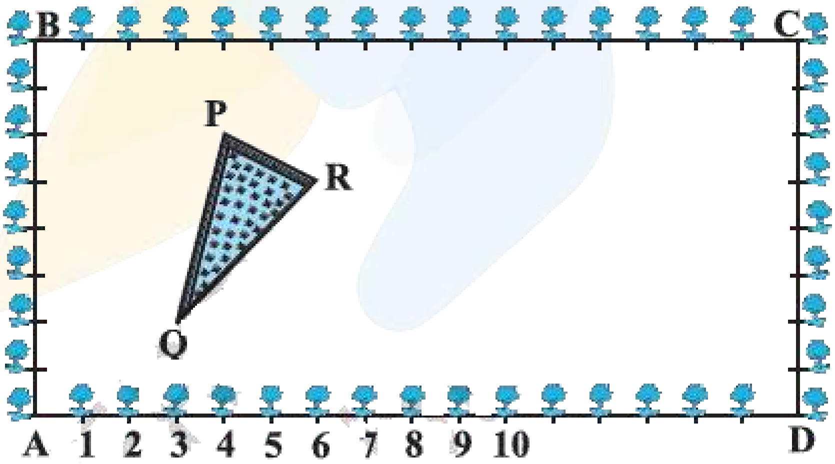
$$AB^2 + BC^2 = AC^2 \quad \text{[By Pythagoras theorem]}$$

Distance formula is used to find distance between AB, BC and AC

$$\left(\sqrt{(1+1)^2+(y_1-2)^2}\right)^2+\left(\sqrt{(1-3)^2+(y_1-2)^2}\right)^2=\left(\sqrt{(3+1)^2+(2-2)^2}\right)^2$$
$$4+{y_1}^2+4-4y_1+4+{y_1}^2-4y_1+4=16$$
$$2{y_1}^2+16-8y_1=16$$
$$2{y_1}^2-8y_1=0$$
$$y_1(y_1-4)=0$$
$$y_1=0 \text{ or } 4$$

Hence the required vertices are $B\ (1,0)$ and $D\ (1,\ 4)$

Q5. The class X students of a secondary school in Krishinagar have been allotted a rectangular plot of land for their gardening activity. Saplings of Gulmohar are planted on the boundary at a distance of 1 m from each other. There is a triangular grassy lawn in the plot as shown in the following figure. The students are to sow seeds of flowering plants on the remaining area of the plot.



(i) Taking A as origin, find the coordinates of the vertices of the triangle.

(ii) What will be the coordinates of the vertices of ΔPQR if C is the origin? Also calculate the areas of the triangles in these cases. What do you observe?

Difficulty Level: Hard

What is the known/given?

Saplings of Gulmohar are planted on the boundary at a distance of 1 m from each other.

What is the unknown?

- The coordinates of the vertices of the triangle.
- The coordinates of the vertices of ΔPQR if C is the origin.

Solution:

(i) Given,

- Taking A as origin, we will take AD as *x*-axis and AB as *y*- axis.
- It can be observed from the figure that the coordinates of point P, Q, and R are (4, 6), (3, 2), and (6, 5) respectively.
- Let $P(x_1, y_1) = (4, 6)$
- Let $Q(x_2, y_2) = (3, 2)$
- Let $R(x_3, y_3) = (6, 5)$

Area of a triangle $= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)$ ……Equation(1)

By substituting the values of vertices P, Q, R in the Equation (1),

$$\text{Area of triangle PQR} = \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

$$= \frac{1}{2}[4(2-5) + 3(5-6) + 6(6-2)]$$

$$= \frac{1}{2}[-12 - 3 + 24]$$

$$= \frac{9}{2}\text{Square units}$$

(ii) Given,

- Taking C as origin, CB as x-axis, and CD as y-axis
- The coordinates of vertices P, Q, and R are (12, 2), (13, 6), and (10, 3) respectively.
- Let $P(x1, y1) = (12, 2)$
- Let $Q(x2, y2) = (13, 6)$
- Let $R(x3, y3) = (10,3)$

Area of a triangle $= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)$ ……Equation (1)

By substituting the values of vertices P, Q, R in the Equation (1), Area of triangle PQR $= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$

$$= \frac{1}{2}\left[12(6-3) + 13(3-2) + 10(2-6)\right]$$

$$= \frac{1}{2}\left[36 + 13 - 40\right]$$

$$= \frac{9}{2} \text{ Square units}$$

It can be observed that the area of the triangle is same in both the cases.

Q6. The vertices of a ΔABC are A (4, 6), B (1, 5) and C (7, 2). A line is drawn to intersect sides AB and AC at D and E respectively, such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$.

Calculate the area of the ΔADE and compare it with the area of ΔABC.
(Recall Converse of basic proportionality theorem and Theorem 6.6 related to Ratio of areas of two similar triangles)

Difficulty Level: Hard

Reasoning:

Converse of Basic Proportionality Theorem: If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

Theorem 6.6: The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

What is the known/given?

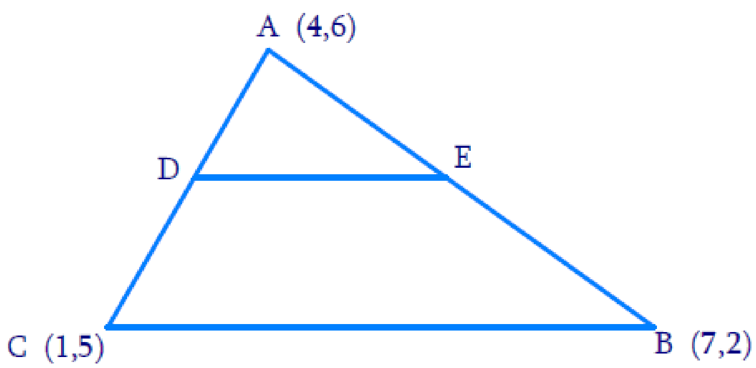
The x and y coordinates of the vertices of the triangle.

$\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$.

where AB and AC are sides of the triangle intersected at D and E.

What is the unknown?

The area of the ΔADE is to be calculated and compared with the area of ΔABC.



Solution:

From the figure,

Given

$$\frac{AD}{AB}=\frac{AE}{AC}=\frac{1}{4}$$

Therefore, D and E are two points on side AB and AC respectively such that they divide side AB and AC in a ratio of 1:3.

By Section formula

$$P(x,y)=\left[\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right] \qquad \ldots\ldots(1)$$

$$\text{Coordinates of Point D}=\left(\frac{1\times1+3\times4}{1+3},\frac{1\times5+3\times6}{1+3}\right)$$

$$=\left(\frac{13}{4},\frac{23}{4}\right)$$

$$\text{Coordinates of point E}=\left(\frac{1\times7+3\times4}{1+3},\frac{1\times2+3\times6}{1+3}\right)$$

$$=\left(\frac{19}{4},\frac{20}{4}\right)$$

$$\text{Area of triangle}=\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right] \quad \ldots(2)$$

By substituting the vertices A, D, E in (2),

$$\text{Area of }\Delta ADE=\frac{1}{2}\left[4\left(\frac{23}{4}-\frac{20}{4}\right)+\frac{13}{4}\left(\frac{20}{4}-6\right)+\frac{19}{4}\left(6-\frac{23}{4}\right)\right]$$

$$=\frac{1}{2}\left[3-\frac{13}{4}+\frac{19}{16}\right]$$

$$=\frac{1}{2}\left[\frac{48-52+19}{16}\right]$$

$$=\frac{15}{32}\text{ Square units}$$

By substituting the vertices A, B, C in (2)

$$\text{Area of } \Delta ADE = \frac{1}{2}\left[4\left(\frac{23}{4}-\frac{20}{4}\right)+\frac{13}{4}\left(\frac{20}{4}-6\right)+\frac{19}{4}\left(6-\frac{23}{4}\right)\right]$$

$$= \frac{1}{2}\left[3-\frac{13}{4}+\frac{19}{16}\right]$$

$$= \frac{1}{2}\left[\frac{48-52+19}{16}\right]$$

$$= \frac{15}{32} \text{ Square units}$$

Clearly, the ratio between the areas of ΔADE and ΔABC is 1:16.

Alternatively, we know that if a line segment in a triangle divides its two sides in the same ratio, then the line segment is parallel to the third side of the triangle [**Converse of Basic Proportionality Theorem**]. These two triangles so formed (here ΔADE and ΔABC) will be similar to each other.

Hence, the ratio between the areas of these two triangles will be the square of the ratio between the sides of these two triangles. [**Theorem 6.6**]

Therefore, ratio between the areas of ΔADE and

$$\Delta ABC = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$$

Q7. Let A (4, 2), B (6, 5) and C (1, 4) be the vertices of ΔABC.

(i) The median from A meets BC at D. Find the coordinates of point D.
(ii) Find the coordinates of the point P on AD such that AP: PD = 2:1
(iii) Find the coordinates of point Q and R on medians BE and CF respectively such that BQ: QE = 2:1 and CR: RF = 2:1.
(iv) What do you observe?
(v) If A (x_1, y_1), B (x_2, y_2), and C (x_3, y_3) are the vertices of ΔABC, find the coordinates of the centroid of the triangle.

Difficulty Level: Hard

Reasoning:

A median of a triangle is a line segment joining the vertex to the midpoint of the opposite side, thus bisecting that side. Every triangle has exactly three medians, one from each vertex, and they all intersect each other at the triangle's centroid.

What is known/given?

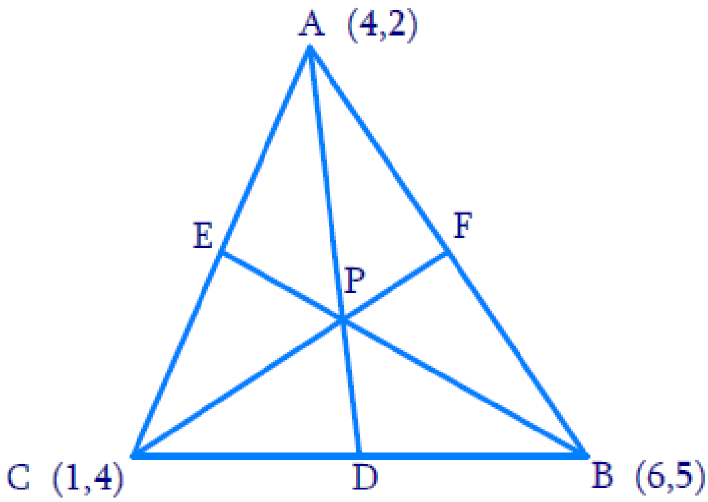
The x and y coordinates of the vertices of the triangle.

What is unknown?

The coordinates of point D, P, Q, R and the co-ordinates of the centroid.

Solution:

From the figure,



Given,

- Let $A(x_1, y_1) = (4, 2)$
- Let $B(x_2, y_2) = (6, 5)$
- Let $C(x_3, y_3) = (1, 4)$

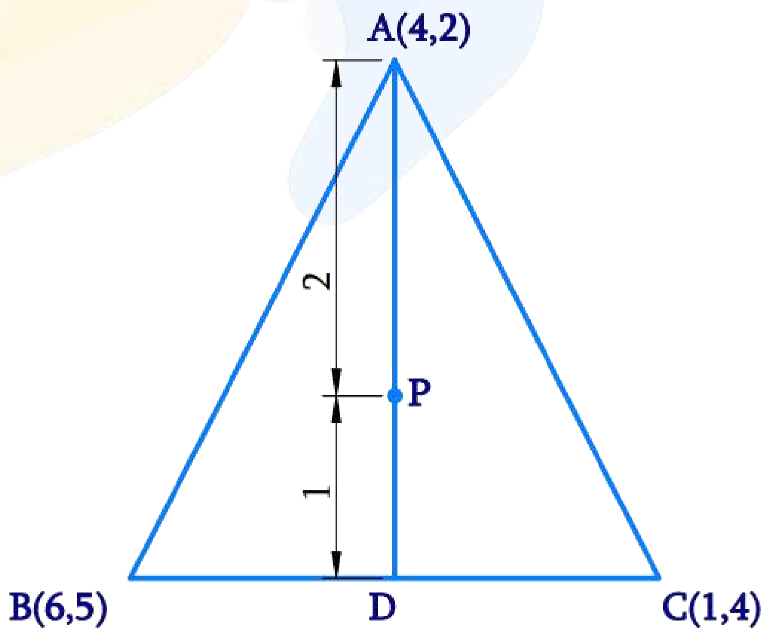
(i)

- Let AD be the median of the triangle ABC
- Hence D is the midpoint of BC which can be found using the Mid Point formula.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\text{Coordinates of D} = \left(\frac{6+1}{2}, \frac{5+4}{2}\right) = \left(\frac{7}{2}, \frac{9}{2}\right)$$

(ii) From the Figure,



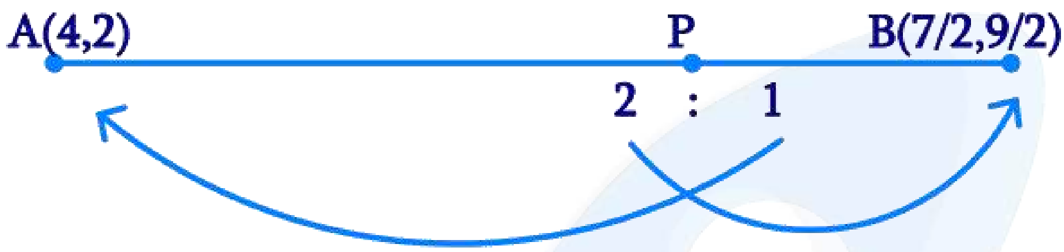
Point P divides the side AD in a ratio m: n = 2: 1.

By Section formula

$$P(x,y)=\left[\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right] \qquad .....\text{Equation } (1)$$

$$\text{Coordinates of } P=\left(\frac{2\times\frac{7}{2}+1\times4}{2+1},\frac{2\times\frac{9}{2}+1\times2}{2+1}\right)=\left(\frac{11}{3},\frac{11}{3}\right)$$

(iii) From the figure,



Median BE of the triangle will divide the side AC in two equal parts.

Therefore, E is the mid-point of side AC. The co-ordinates of E can be calculated using the Mid-Point Formula.

$$M=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$\text{Coordinates of } E=\left(\frac{4+1}{2},\frac{2+4}{2}\right)=\left(\frac{5}{2},3\right)$$

Point Q divides the side BE in a ratio 2:1.

By Section formula

$$P(x,y)=\left[\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right] \qquad .....\text{Equation } (1)$$

$$\text{Coordinates of } Q=\left(\frac{2\times\frac{5}{2}+1\times6}{2+1},\frac{2\times3+1\times5}{2+1}\right)=\left(\frac{11}{3},\frac{11}{3}\right)$$

Median CF of the triangle will divide the side AB in two equal parts.

Therefore, F is the mid-point of side AB. The co-ordinates of F can be calculated using the Mid-Point Formula.

$$\text{Coordinates of } F=\left(\frac{4+6}{2},\frac{2+5}{2}\right)=\left(5,\frac{7}{2}\right)$$

Point R divides the side CF in a ratio 2: 1

By Section formula

$$P(x,y)=\left[\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right] \qquad \text{.....Equation } (1)$$

$$\text{Coordinates of } R=\left(\frac{2\times 5+1\times 1}{2+1},\frac{2\times \frac{7}{2}+1\times 4}{2+1}\right)$$

$$=\left(\frac{11}{3},\frac{11}{3}\right)$$

(iv) It can be observed that the coordinates of point P, Q, R are the same. Therefore, all these are representing the same point on the plane i.e., the centroid of the triangle.

(v) Consider a triangle, ΔABC, having its vertices as $A(x_1,y_1)$, $B(x_2,y_2)$, and $C(x_3,y_3)$

Median AD of the triangle will divide the side BC in two equal parts.

Therefore, D is the mid-point of side BC. The co-ordinates of D can be calculated using the Mid-Point Formula.

$$\text{Coordinates of } D=\left(\frac{x_2+x_3}{2},\frac{y_2+y_3}{2}\right)$$

Let the centroid of this triangle be O.

Point O divides the side AD in a ratio 2:1.

$$\text{Coordinates of } O=\left(\frac{2\times\frac{x_2+x_3}{2}+1\times x_1}{2+1},\frac{2\times\frac{y_2+y_3}{2}+1\times y_1}{2+1}\right)$$

$$=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$$

Q.8. ABCD is a rectangle formed by the points A (− 1, − 1), B (− 1, 4), C (5, 4) and D (5, − 1). P, Q, R and S are the mid-points of AB, BC, CD, and DA respectively. Is the quadrilateral PQRS is a square? A rectangle? Or a rhombus? Justify your answer.

Difficulty Level: Medium

Reasoning:

The distance between the two points can be measured using the Distance Formula which is given by:

$$\text{Distance Formula} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

What is the known/given?

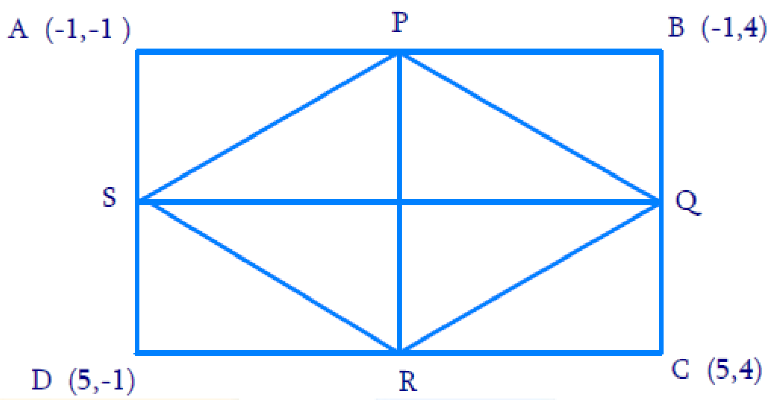
The x and y coordinates of the vertices of the rectangle.

What is the unknown?

To find whether quadrilateral PQRS is a square? A rectangle? Or a rhombus?

Solution:

From the figure below,



P is the mid-point of side AB. The co-ordinates of P can be calculated using the Mid-Point Formula as $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Therefore, the coordinates of P are $\left(\frac{-1-1}{2}, \frac{-1+4}{2}\right) = \left(-1, \frac{3}{2}\right)$

Similarly, the coordinates of Q, R and S are calculated using the Mid-Point Formula as : $(2,4)$, $\left(5, \frac{3}{2}\right)$ and $(2,-1)$ respectively.

We know that the distance between the two points is given by the Distance Formula,

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \quad \ldots\ldots\text{Equation (1)}$$

Distance between two points P and Q,

Length of PQ

$$= \sqrt{(-1-2)^2 + \left(\frac{3}{2} - 4\right)^2}$$

$$= \sqrt{9 + \frac{25}{4}}$$

$$= \sqrt{\frac{61}{4}}$$

Distance between two points Q and R,

Length of QR

$$= \sqrt{(2-5)^2 + \left(4 - \frac{3}{2}\right)^2}$$

$$= \sqrt{9 + \frac{25}{4}}$$

$$= \sqrt{\frac{61}{4}}$$

Distance between two points R and S,

Length of RS

$$= \sqrt{(5-2)^2 + \left(\frac{3}{2} + 1\right)^2}$$

$$= \sqrt{9 + \frac{25}{4}}$$

$$= \sqrt{\frac{61}{4}}$$

Distance between two points S and P,

Length of SP

$$= \sqrt{(2+1)^2 + \left(-1 - \frac{3}{2}\right)^2}$$

$$= \sqrt{9 + \frac{25}{4}}$$

$$= \sqrt{\frac{61}{4}}$$

Distance between two points P and R which form the diagonal are

Length of PR

$$= \sqrt{(-1-5)^2 + \left(\frac{3}{2} - \frac{3}{2}\right)^2}$$

$$= 6$$

Distance between two points Q and S which form the diagonal is calculated using the Distance Formula as,

Length of QS

$$= \sqrt{(2-2)^2 + (4+1)^2}$$

$$= 5$$

It can be observed that all sides of the given quadrilateral are of the same measure. However, the diagonals are of different lengths. Therefore, PQRS is a rhombus.