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Chapter 8: Introduction to Trigonometry

Exercise 8.1 (Page 181 of Grade 10 NCERT Textbook)

Q1. In ΔABC, right-angled at B, AB = 24 cm, BC = 7 cm, determine:

(i) $\sin A, \cos A$

(ii) $\sin C, \cos C$

Difficulty level: Easy

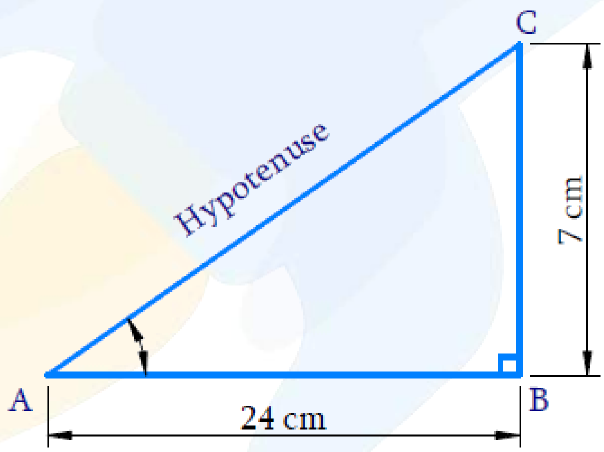
What is the known?

Two sides of a right-angled triangle ΔABC

What is the unknown?

Sine and cosine of angle A and C.

Reasoning:



Applying Pythagoras theorem for ΔABC, we can find hypotenuse (side AC). Once hypotenuse is known, we can find sine and cosine angle using trigonometric ratios.

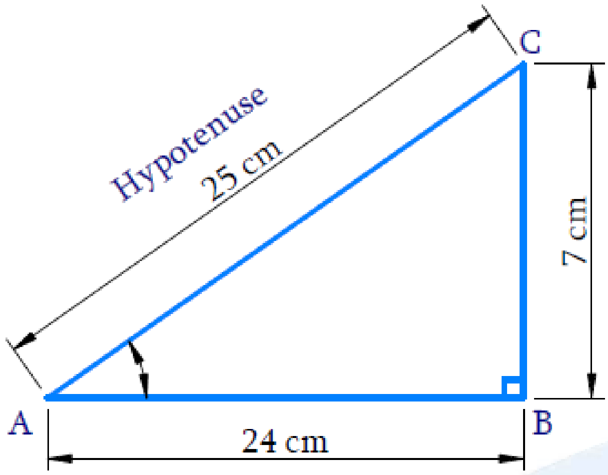
Solution:

In ΔABC, we obtain.

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ &= (24cm)^2 + (7cm)^2 \\ &= (576 + 49)\ cm^2 \\ &= 625\ cm^2 \end{aligned}$$

$\therefore$ Hypotenuse $AC = \sqrt{625}\ cm = 25\ cm$

(i)



$$\sin A = \frac{\text{side opposite to } \angle A}{\text{hypotenuse}} = \frac{BC}{AC}$$

$$\sin A = \frac{7\,\text{cm}}{25\,\text{cm}} = \frac{7}{25}$$

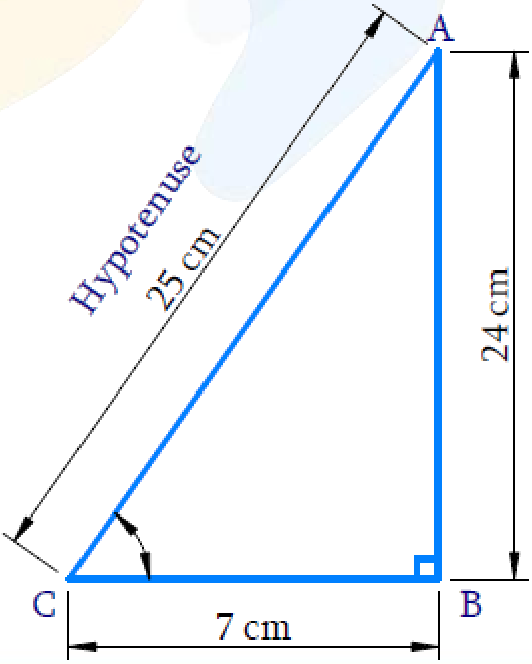
$$\sin A = \frac{7}{25}$$

$$\cos A = \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AB}{AC}$$

$$= \frac{24\,\text{cm}}{25\,\text{cm}} = \frac{24}{25}$$

$$\cos A = \frac{24}{25}$$

(ii)



$$\sin C = \frac{\text{side opposite to } \angle C}{\text{hypotenuse}} = \frac{AB}{AC}$$

$$\sin C = \frac{24\,\text{cm}}{25\,\text{cm}} = \frac{24}{25}$$

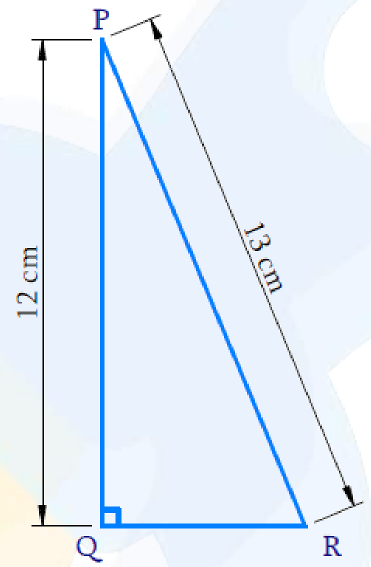
$$\sin C = \frac{24}{25}$$

$$\cos C = \frac{\text{side adjacent to } \angle C}{\text{hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{7\,\text{cm}}{25\,\text{cm}} = \frac{7}{25}$$

$$\cos C = \frac{7}{25}$$

Q2. In the given figure, find $\tan P - \cot R$.



Difficulty level: Medium

What is the known/given?

PQ = 12 cm and PR = 13 cm.

What is the unknown?

One side of right-angled triangle ΔPQR

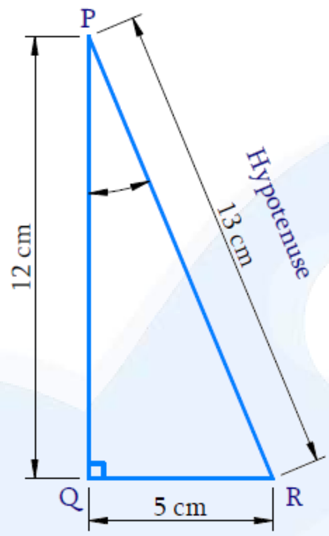
Reasoning:

Using Pythagoras theorem, we can find the length of the third side. Then the required trigonometric ratios.

Solution:

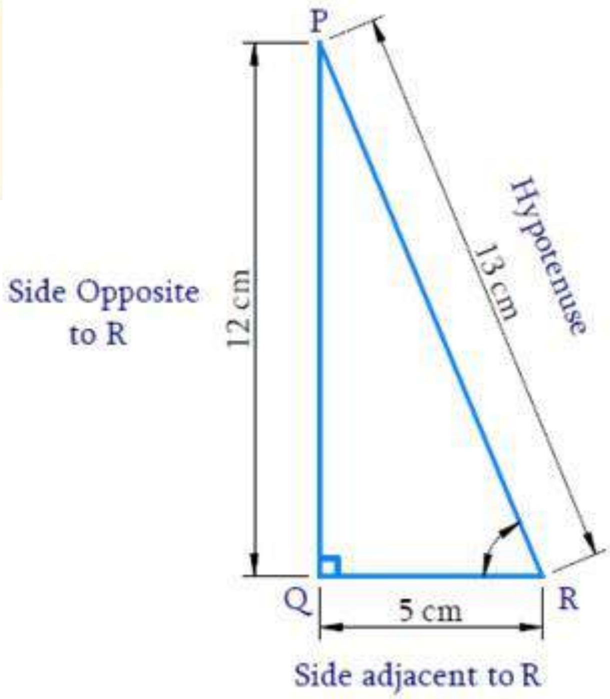
Apply Pythagoras theorem for ΔPQR we obtain:

$$PR^2 = PQ^2 + QR^2$$
$$QR^2 = PR^2 - PQ^2$$
$$QR^2 = (13cm)^2 - (12cm)^2$$
$$QR^2 = 169cm^2 - 144cm^2$$
$$QR^2 = 25cm^2$$
$$QR = 5cm$$



$$\tan P = \frac{\text{side opposite to } \angle P}{\text{side adjacent to } \angle P} = \frac{QR}{PQ} = \frac{5cm}{12cm}$$

$$\tan P = \frac{5}{12}$$



$$\cot R = \frac{\text{side adjacent to } \angle R}{\text{side opposite to } \angle R} = \frac{QR}{PQ} = \frac{5\,\text{cm}}{12\,\text{cm}}$$

$$\cot R = \frac{5}{12}$$

$$\tan P - \cot R = \frac{5}{12} - \frac{5}{12}$$

$$\tan P - \cot R = 0$$

Q3. If $\sin A = \frac{3}{4}$ calculate cos A and tan A.

Difficulty level: Medium

What is the known/given?

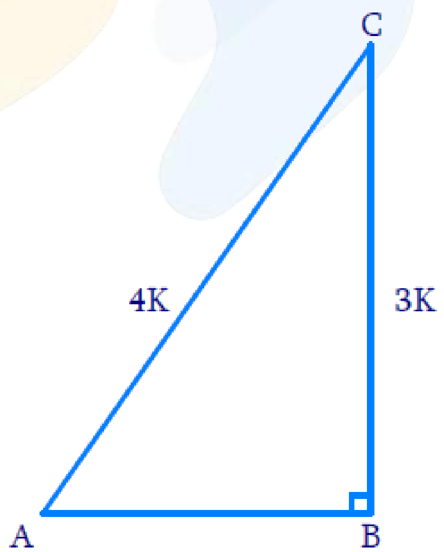
Sine of $\angle A$.

What is the unknown?

Cosine and tangent of $\angle A$

Reasoning:

Using sin A, we can find the ratio of the length of two sides of the right-angled triangle. Then by using Pythagoras theorem, the third side and required trigonometric ratios.



Solution:

Let ΔABC be a right-angled triangle, right angled at point B.

Given that

$$\sin A = \frac{3}{4}$$
$$\Rightarrow \frac{BC}{AC} = \frac{3}{4}$$

Let BC be 3k. Therefore, AC will be 4k where k is a positive integer.

Applying Pythagoras theorem for ΔABC, we obtain:

$$AC^2 = AB^2 + BC^2$$
$$AB^2 = AC^2 - BC^2$$
$$AB^2 = (4k)^2 - (3k)^2$$
$$AB^2 = 16k^2 - 9k^2$$
$$AB^2 = 7k^2$$
$$AB = \sqrt{7}k$$

$$\cos A = \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{7}k}{4k}$$
$$= \frac{\sqrt{7}}{4}$$

$$\tan A = \frac{\text{side opposite to } \angle A}{\text{side adjacent to } \angle A} = \frac{BC}{AB} = \frac{3k}{\sqrt{7}k}$$
$$= \frac{3}{\sqrt{7}}$$

Thus, $\cos A = \frac{\sqrt{7}}{4}$ and $\tan A = \frac{3}{\sqrt{7}}$

Q4. Given 15 cot A = 8, find sin A and sec A.

Difficulty level: Medium

What is the known/given?

Cotangent of $\angle A$

What is the unknown?

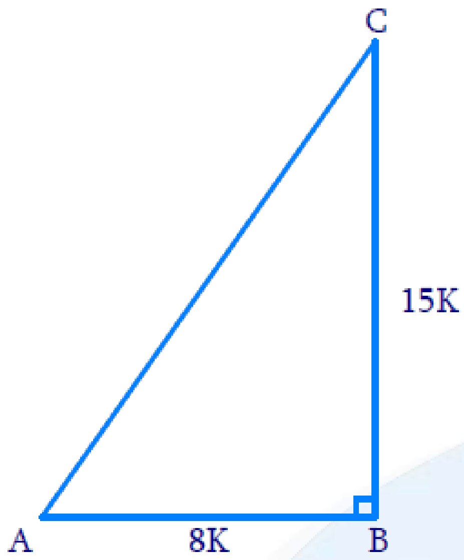
Sine and Secant of $\angle A$.

Reasoning:

Using cot A, we can find the ratio of the length of two sides of the right-angled triangle. Then by using Pythagoras theorem, the third side and required trigonometric ratios.

Solution:

Let us consider a right-angled ΔABC, right angled at B.



$$\cot A = \frac{\text{side adjacent to } \angle A}{\text{side opposite to } \angle A} = \frac{AB}{BC}$$

It is given that

$$\cot A = \frac{8}{15} \Rightarrow \frac{AB}{BC} = \frac{8}{15}$$

Let AB be 8 k. Therefore, BC will be 15 k where k is a positive integer.

Apply Pythagoras theorem in ΔABC, we obtain.

$$AC^2 = AB^2 + BC^2$$
$$AC^2 = (8k)^2 + (15k)^2$$
$$AC^2 = 64k^2 + 225k^2$$
$$AC^2 = 289k^2$$
$$AC = 17k$$

$$\sin A = \frac{\text{side opposite to } \angle A}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{15k}{17k}$$
$$= \frac{15}{17}$$

$$\sec A = \frac{\text{hypotenuse}}{\text{side adjacent to } \angle A} = \frac{AC}{AB} = \frac{17k}{8k}$$
$$= \frac{17}{8}$$

Thus, $\sin A = \frac{15}{17}$ and $\sec A = \frac{17}{8}$

Q5. Given $\sec\theta = \frac{13}{12}$, calculate all other trigonometric ratios.

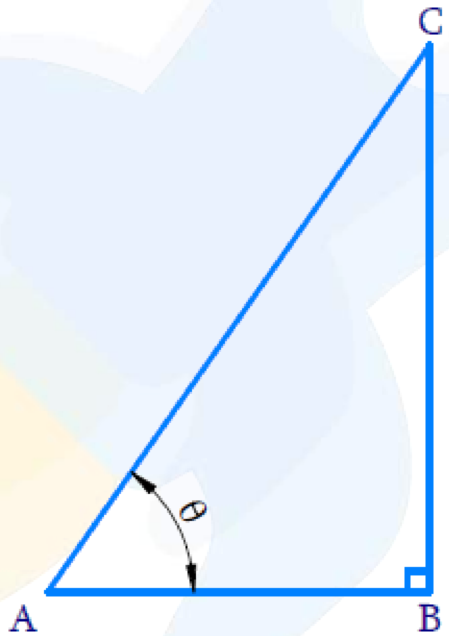
Difficulty level: Medium

What is the known/given?
Secant of θ

What is the unknown?
Other trigonometric ratios.

Reasoning:
Using $\text{Sec}\,\theta$, we can find the ratio of the length of two sides of the right-angled triangle. Then by using Pythagoras theorem, the third side and required trigonometric ratios.

Solution:
Let ΔABC be a right-angled triangle, right angled at point B.



It is given that:

$$\sec\theta = \frac{\text{hypotenuse}}{\text{side adjacent to } \angle\theta} = \frac{AC}{AB} = \frac{13}{12}$$

Let AC = 13 k and AB = 12 k where k is a positive integer.

Apply Pythagoras theorem in ΔABC, we obtain:

$$AC^2 = AB^2 + BC^2$$
$$BC^2 = AC^2 - AB^2$$
$$BC^2 = (13k)^2 - (12k)^2$$
$$BC^2 = 169k^2 - 144k^2$$
$$BC^2 = 25k^2$$
$$BC = 5k$$

$$\sin\theta = \frac{\text{side opposite to } \angle\theta}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{5}{13}$$

$$\cos\theta = \frac{\text{side adjacent to } \angle\theta}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{12}{13}$$

$$\tan\theta = \frac{\text{side opposite to } \angle\theta}{\text{side adjacent to } \angle\theta} = \frac{BC}{AB} = \frac{5}{12}$$
$$\cot\theta = \frac{\text{side adjacent to } \angle\theta}{\text{side opposite to } \angle\theta} = \frac{AB}{BC} = \frac{12}{5}$$
$$\operatorname{cosec}\theta = \frac{\text{hypotenuse}}{\text{side opposite to } \angle\theta} = \frac{AC}{BC} = \frac{13}{5}$$

Q6. If $\angle A$ and $\angle B$ are acute angles such that $\cos A = \cos B,$ then show that $\angle A = \angle B.$

Difficulty level: Medium

What is the known/given?

$\angle A$ and $\angle B$ are acute angles and $\cos A = \cos B.$

What is the unknown?

To show that $\angle A = \angle B$

Reasoning:

Using cos A and cos B, we can find the ratio of the length of two sides of the right-angled triangle with respective angles. Then compare both the ratios.

Solution:

In the right-angled triangle ABC, ∠A and ∠B are acute angles and ∠C is right angle.

$$\cos A = \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AC}{AB}$$

$$\cos B = \frac{\text{side adjacent to } \angle B}{\text{hypotenuse}} = \frac{BC}{AB}$$

Given that $\cos A = \cos B$

Therefore,

$$\frac{AC}{AB} = \frac{BC}{AB}$$

$$AC = BC$$

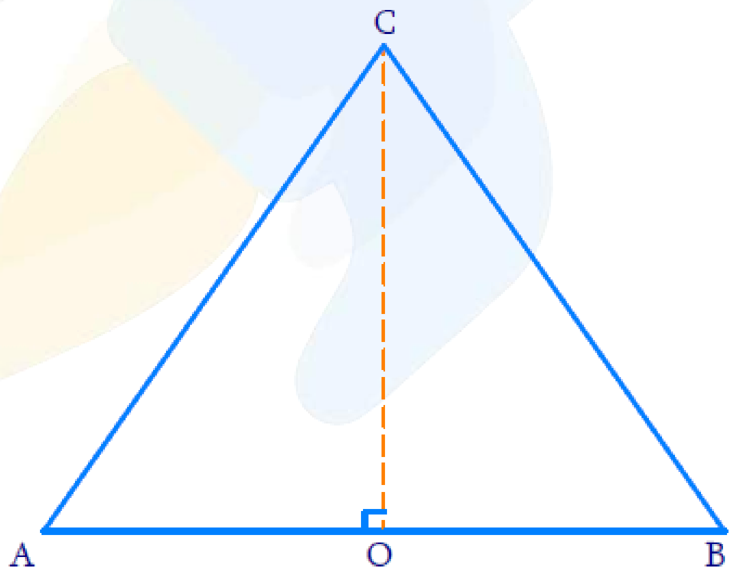
Hence, $\angle A = \angle B$, (angles opposite to equal sides of triangle are equal.)

Alternatively,

Reasoning:

Using cos A and cos B, we can find the ratio of the length of two sides of the right-angled triangle with respective angles. Then by using Pythagoras theorem, relation between the sides.

Let us consider a triangle ABC in which CO ⊥ AB.



It is given that

$$\cos A = \cos B$$

$$\frac{AO}{AC} = \frac{BO}{BC}$$

$$\frac{AO}{BO} = \frac{AC}{BC}$$

Let $\frac{AO}{BO}=\frac{AC}{BC}=k$

$$AO = k.BO \qquad (i)$$

$$AC = k.BC \qquad (ii)$$

By applying Pythagoras theorem in ΔCAO and ΔCBO, we get.

$$AC^2 = AO^2 + CO^2 \qquad \text{from } \Delta CAO$$

$$CO^2 = AC^2 - AO^2 \qquad (iii)$$

$$BC^2 = BD^2 + CO^2 \qquad \text{from } \Delta CBO$$

$$CO^2 = BC^2 - BO^2 \qquad (iv)$$

From equation (iii) and equation (iv), we get

$$AC^2 - AO^2 = BC^2 - BO^2$$

$$(kBC)^2 - (kBO)^2 = BC^2 - BO^2$$

$$k^2BC^2 - k^2BO^2 = BC^2 - BO^2$$

$$k^2(BC^2 - BO^2) = BC^2 - BO^2$$

$$k^2 = \frac{BC^2 - BO^2}{BC^2 - BO^2} = 1$$

$$k = 1$$

Putting this value in equation (ii) we obtain

$$AC = BC$$

$\angle A = \angle B$ (angles opposite to equal sides of triangle are equal.)

Q7. If $\cot\theta = \frac{7}{8}$, evaluate: (i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$, (ii) $\cot^2\theta$

Difficulty Level: Medium

What is the known/given?

$\cot\theta = \frac{7}{8}$

What is the unknown?

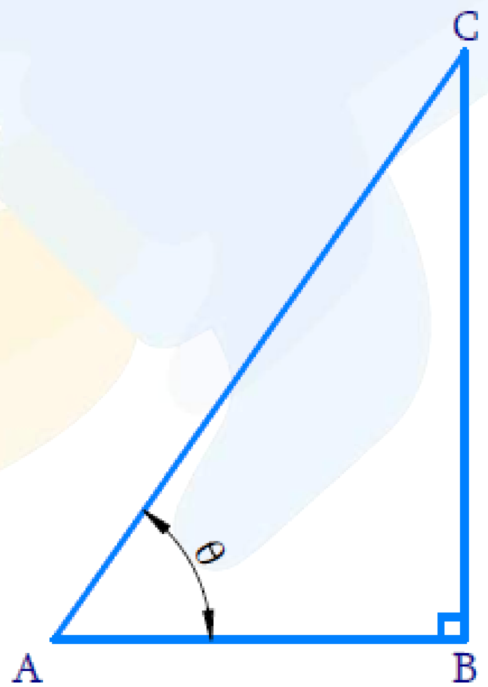
Value of (i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$, and (ii) $\cot^2\theta$

Reasoning:

Using $\cot\theta = \frac{7}{8}$, we can find the ratio of the length of two sides of the right-angled triangle. Then by using Pythagoras theorem, the third side and required trigonometric ratios.

Solution:

Let ΔABC, in which angle B is right angle.



$$\cot\theta = \frac{\text{side adjacent to } \theta}{\text{side opposite to } \theta} = \frac{AB}{BC} = \frac{7}{8}$$

Let $AB = 7k$ and $BC = 8k$, where k is a positive integer.

By applying Pythagoras theorem in ΔABC, we get.

$$AC^2 = AB^2 + BC^2$$
$$= (7k)^2 + (8k)^2$$
$$= 49k^2 + 64k^2$$
$$= 113k^2$$
$$AC = \sqrt{113k^2}$$
$$= \sqrt{113}k$$

Therefore,

$$\sin\theta = \frac{\text{side opposite to }\theta}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}$$

$$\cos\theta = \frac{\text{side adjacent to }\theta}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}$$

(i) $\dfrac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta} \qquad \left[\because (a+b)(a-b) = (a^2-b^2)\right]$$

$$= \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2}$$

$$= \frac{1-\frac{64}{113}}{1-\frac{49}{113}}$$

$$= \frac{49/113}{64/113}$$

$$= \frac{49}{64}$$

(ii) $\cot^2\theta$

$$\cot^2\theta = \left(\frac{7}{8}\right)^2$$
$$= \frac{49}{64}$$

Q8. If 3 cot A = 4, check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

Difficulty Level: Medium

What is the known/given?
Cotangent of angle A

What is the unknown?
whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$

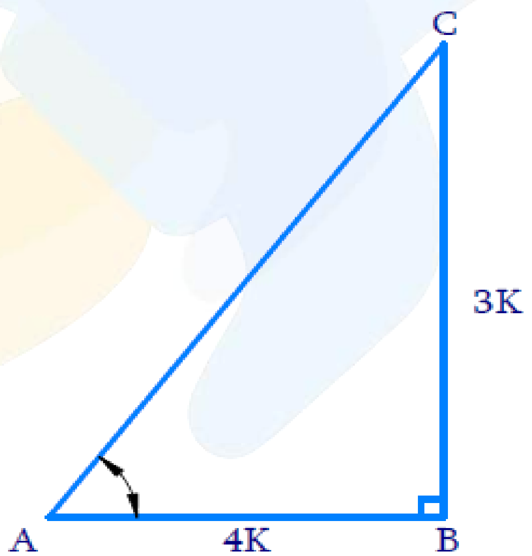
Reasoning:
Using $3\cot A = 4$, we can find the ratio of the length of two sides of the right-angled triangle. Then by using Pythagoras theorem, the third side and required trigonometric ratios.

Solution:

$$3\cot A = 4$$

$$\cot A = \frac{4}{3}$$

Let ΔABC, in which angle B is right angle.



$$\cot A = \frac{\text{side adjacent to } \angle A}{\text{side opposite to } \angle A} = \frac{AB}{BC} = \frac{4}{3}$$

Let $AB = 4k$ and $BC = 3k$ where k is a positive integer.

By applying Pythagoras theorem in ΔABC, we get.

$$AC^2 = AB^2 + BC^2$$
$$= (4k)^2 + (3k)^2$$
$$= 16k^2 + 9k^2$$
$$= 25k^2$$
$$AC = \sqrt{25k^2}$$
$$= 5k$$

Therefore,

$$\tan A = \frac{\text{side opposite to } \angle A}{\text{side adjacent to } \angle A} = \frac{BC}{AB} = \frac{3k}{4k} = \frac{3}{4}$$

$$\sin A = \frac{\text{side opposite to } \angle A}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$$

$$\cos A = \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$$

$$L.H.S = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$
$$= \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2}$$
$$= \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}}$$
$$= \frac{16 - 9}{16 + 9}$$
$$= \frac{7}{25}$$

$$R.H.S = \cos^2 A - \sin^2 A$$
$$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$
$$= \frac{16}{25} - \frac{9}{25}$$
$$= \frac{16 - 9}{25}$$
$$= \frac{7}{25}$$

Therefore, $\dfrac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$

Q9. In the triangle ABC right-angled at B, if $\tan A = \dfrac{1}{\sqrt{3}}$ find the value of:

(i) $\sin A \cos C + \cos A \sin C$

(ii) $\cos A \cos C - \sin A \sin C$

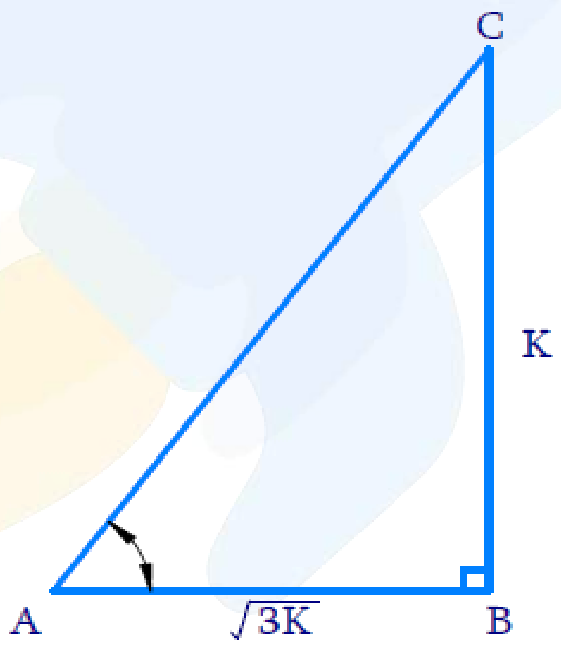
Difficulty level: Medium

Reasoning:

Using $\tan A = \dfrac{1}{\sqrt{3}}$, we can find the ratio of the length of two sides of the right-angled triangle. Then by using Pythagoras theorem, the third side and required trigonometric ratios.

Solution:

(i) Let ΔABC be a right-angled triangle $\tan A = \dfrac{1}{\sqrt{3}}$



$$\tan A = \frac{\text{side opposite to } \angle A}{\text{side adjacent to } \angle A} = \frac{BC}{AB} = \frac{1}{\sqrt{3}}$$

Let $BC = k \text{ and } AB = \sqrt{3}k$ where k is a positive real number.

By applying Pythagoras theorem for ΔABC

$$AC^2 = AB^2 + BC^2$$
$$= \left(\sqrt{3}k\right) + (k)^2$$
$$= 3k^2 + k^2$$
$$= 4k^2$$
$$AC = \sqrt{4k^2}$$
$$= 2k$$

Therefore,

$$\sin A = \frac{\text{side opposite to } \angle A}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{1}{2}$$

$$\cos A = \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$$

$$\sin C = \frac{\text{side opposite to } \angle C}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$$

$$\cos C = \frac{\text{side adjacent to } \angle C}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{1}{2}$$

(i) $\sin A \cos C + \cos A \sin C$

By substituting the values of the trigonometric functions in the above equation.

$$\sin A \cos C + \cos A \sin C == \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$
$$= \frac{1}{4} + \frac{3}{4}$$
$$= \frac{1+3}{4}$$
$$= \frac{4}{4}$$
$$= 1$$

(ii) $\cos A \cos C - \sin A \sin C$

By substituting the values of the trigonometric functions in the above equation.

$$\cos A \cos C - \sin A \sin C = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$
$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$$
$$= 0$$

Q10. In ΔPQR, right-angled at Q, $PR + QR = 25$ cm and $PQ = 5$ cm. Determine the values of sin P, cos P and tan P.

Difficulty level: Medium

Reasoning:
Using Pythagoras theorem, we can find the length of the all three sides. Then the required trigonometric ratios

Solution:
Given, Δ PQR is right-angled at Q.

$$PQ = 5cm$$
$$PR + QR = 25cm$$

Let $PR = x\ cm$

Therefore,

$$\begin{aligned} QR &= 25\text{cm} - PR \\ &= (25 - x)\text{cm} \end{aligned}$$

By applying Pythagoras theorem for Δ PQR, we obtain.

$$\begin{aligned} PR^2 &= PQ^2 + QR^2 \\ x^2 &= (5)^2 + (25 - x)^2 \\ x^2 &= 25 + 625 - 50x + x^2 \\ 50x &= 650 \\ x &= \frac{650}{50} \\ &= 13 \end{aligned}$$

Therefore,

$$PR = 13cm$$
$$QR = (25 - 13)\ cm$$
$$= 12\ cm$$

By substituting the values obtained above in the trigonometric functions below.

$$\sin P = \frac{\text{side opposite to } \angle P}{\text{hypotenuse}} = \frac{QR}{PR} = \frac{12}{13}$$
$$\cos P = \frac{\text{side adjacent to } \angle P}{\text{hypotenuse}} = \frac{PQ}{PR} = \frac{5}{13}$$
$$\tan P = \frac{\text{side opposite to } \angle P}{\text{side adjacent to } \angle P} = \frac{QR}{PQ} = \frac{12}{5}$$

Q11. State whether the following are true or false. Justify your answer.

(i) The value of tan A is always less than 1.

(ii) $\sec A = \frac{12}{5}$ for some value of angle A.

(iii) cos A is the abbreviation used for the cosecant of angle A.

(iv) cot A is the product of cot and A.

(v) $\sin\theta = \frac{4}{3}$, for some angle θ.

Difficulty level: Medium

Solution:

(i) False, because sides of a right-angled triangle may have any length. So tan A may have any value.

(ii)

$$\sec A = \frac{\text{hypotenuse}}{\text{side adjacent to } \angle A}$$

As hypotenuse is the largest side, the ratio on RHS will be greater than 1. Hence $\sec A > 1$. Thus, the given statement is true.

(iii) Abbreviation used for cosecant of $\angle A$ is cosec A and cos A is the abbreviation used for cosine of $\angle A$. Hence the given statement is false.

(iv) cot A is not the product of cot and A. It is the cotangent of $\angle A$. Hence, the given statement is false.

(v) $\sin\theta = \dfrac{4}{3}$

We know that in a right-angled triangle,

$$\sin\theta = \frac{\text{side adjacent to } \angle\theta}{\text{hypotenuse}}$$

In a right-angled triangle, hypotenuse is always greater than the remaining two sides. Also, the value of Sine should be less than 1. Therefore, such value of $\sin\theta$ is not possible. Hence the given statement is false.

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Chapter 8: Introduction to Trigonometry

Exercise 8.2 (Page 187 of Grade 10 NCERT Textbook)

Q1. Evaluate the following:

(i) $\sin 60^0 \cos 30^0 + \sin 30^0 \cos 60^0$

(ii) $2\tan^2 45^0 + \cos^2 30^0 - \sin^2 60^0$

(iii) $\dfrac{\cos 45^0}{\sec 30^0 + \operatorname{cosec} 30^0}$

(iv) $\dfrac{\sin 30^0 + \tan 45^0 - \operatorname{cosec} 60^0}{\sec 30^0 + \cos 60^0 - \cot 45^0}$

(v) $\dfrac{5\cos^2 60^0 + 4\sec^2 30^0 - \tan^2 45^0}{\sec^2 30^0 + \cos^2 30^0}$

Difficulty level: Medium

Reasoning:

We know that,

Exact Values of Trigonometric Functions				
Angle (θ)		sin (θ)	cos (θ)	tan (θ)
Degrees	Radians			
0°	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	Not Defined

Solution:

(i)

$$\sin 60^0 \cos 30^0 + \sin 30^0 \cos 60^0 = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{3}{4} + \frac{1}{4}$$

$$= \frac{3+1}{4}$$

$$= \frac{4}{4}$$

$$= 1$$

(ii)

$$2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ = 2\left(\tan 45^\circ\right)^2 + \left(\cos 30^\circ\right)^2 - \left(\sin 60^\circ\right)^2$$

$$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 2 + \frac{3}{4} - \frac{3}{4}$$

$$= 2$$

(iii)

$$\frac{\cos 45^0}{\sec 30^0 + \operatorname{cosec} 30^0} = \frac{\left(\frac{1}{\sqrt{2}}\right)}{\left(\frac{2}{\sqrt{3}}\right) + \left(\frac{2}{1}\right)}$$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2 + 2\sqrt{3}}{\sqrt{3}}}$$

$$= \frac{1 \times \sqrt{3}}{\sqrt{2} \times \left(2 + 2\sqrt{3}\right)}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}\left(\sqrt{3} + 1\right)}$$

Multiplying numerator and denominator by $\sqrt{2}\left(\sqrt{3} - 1\right)$, we get

$$= \frac{\sqrt{3}}{2\sqrt{2}\left(\sqrt{3} + 1\right)} \times \frac{\sqrt{2}(\sqrt{3} - 1)}{\sqrt{2}(\sqrt{3} - 1)}$$

$$= \frac{3\sqrt{2} - \sqrt{6}}{4\left(3 - 1\right)}$$

$$= \frac{3\sqrt{2} - \sqrt{6}}{8}$$

(iv)

$$\frac{\sin 30^0 + \tan 45^0 - \operatorname{cosec} 60^0}{\sec 30^0 + \cos 60^0 + \cot 45^0} = \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1}$$

$$= \frac{\frac{3}{2} - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{3}{2}}$$

$$= \frac{\frac{3\sqrt{3} - 4}{2\sqrt{3}}}{\frac{4 + 3\sqrt{3}}{2\sqrt{3}}}$$

$$= \frac{3\sqrt{3} - 4}{3\sqrt{3} + 4}$$

Multiplying numerator and denominator by $\left(3\sqrt{3} - 4\right)$, we get

$$= \frac{(3\sqrt{3} - 4)\,(3\sqrt{3} - 4)}{(3\sqrt{3} + 4)\,(3\sqrt{3} - 4)}$$

$$= \frac{27 + 16 - 24\sqrt{3}}{27 - 16}$$

$$= \frac{43 - 24\sqrt{3}}{11}$$

(v)

$$\frac{5\cos^2 60^0 + 4\sec^2 30^0 - \tan^2 45^0}{\sin^2 30^0 + \cos^2 30^0} = \frac{5\times\left(\frac{1}{2}\right)^2 + 4\times\left(\frac{2}{\sqrt{3}}\right)^2 - (-1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{\left(\frac{5}{4} + \frac{16}{3} - 1\right)}{\left(\frac{1}{4} + \frac{3}{4}\right)}$$

$$= \frac{\left(\frac{15+64-12}{12}\right)}{\left(\frac{3+1}{4}\right)}$$

$$= \frac{\left(\frac{67}{12}\right)}{\left(\frac{4}{4}\right)}$$

$$= \frac{67}{12}$$

Q2. Choose the correct option and justify your choice:

(i) $\frac{2\tan 30^0}{1+\tan^2 30^0}$

$(A)\ \sin 60^0$ $(B)\ \cos 60^0$ $(C)\ \tan 60^0$ $(D)\ \sin 60^0$

(ii) $\frac{1-\tan^2 45^0}{1+\tan^2 45^0}$

$(A)\ \tan 90^0$ $(B)\ 1$ $(C)\ \sin 45^0$ $(D)\ 0^0$

(iii) $\sin 2A = 2\sin A$ is true when $A =$

$(A)\ 0^0$ $(B)\ 30^0$ $(C)\ 45^0$ $(D)\ 60^0$

(iv) $\frac{2\tan 30^0}{1-\tan^2 30^0}$

$(A)\ \cos 60^0$ $(B)\ \sin 60^0$ $(C)\ \tan 60^0$ $(D)\ \sin 30^0$

Difficulty level: Medium

Reasoning:

We know that,

Exact Values of Trigonometric Functions				
Angle (θ)		sin (θ)	cos (θ)	tan (θ)
Degrees	Radians			
0°	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	Not Defined

Solution:

(i) $\frac{2\tan 30^0}{1+\tan^2 30^0}$

By substituting the values of given trigonometric ratios in the above equation, we get.

$$= \frac{2\times\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{2\times\frac{1}{\sqrt{3}}}{1+\frac{1}{3}}$$

$$= \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$$

$$= \frac{2}{\sqrt{3}}\times\frac{3}{4}$$

$$= \frac{\sqrt{3}}{2}$$

Out of the given options only $\sin 60^0 = \frac{\sqrt{3}}{2}$. Hence, option (A) is correct.

(ii)

$$\frac{1-\tan^2 45^0}{1+\tan^2 45^0}$$

By substituting the values of given trigonometric ratios for $\tan 45^0$.

$$= \frac{1-(1)^2}{1+(1)^2}$$

$$= \frac{1-1}{1+1}$$

$$= \frac{0}{2}$$

$$= 0$$

Hence, option (D) is correct.

(iii)

$$\sin 2A = 2\sin A$$

By substituting $A = 0^0, 30^0, 45^0$ and 60^0, we get

For $A = 0^0$

$$\sin 2A = \sin 2\times 0^0$$
$$= \sin 0^0$$
$$= 0$$
$$2\sin A = 2\times \sin 0^0$$
$$= 2\times 0^0$$
$$= 0$$
$$\sin 2A = 2\sin A \qquad (\text{When } A = 0^0)$$

For $A = 30^0$

$$\sin 2A = \sin 2\times 30^0$$
$$= \sin 60^0$$
$$= \frac{\sqrt{3}}{2}$$
$$2\sin A = 2\times \sin 30^0$$
$$= 2\times \frac{1}{2}$$
$$= 1$$
$$\sin 2A \neq 2\sin A \qquad (\text{When } A = 30^0)$$

For $A = 45^0$

$$\sin 2A = \sin 2 \times 45^0$$
$$= \sin 90^0$$
$$= 1$$
$$2\sin A = 2 \times \sin 45^0$$
$$= 2 \times \frac{1}{\sqrt{2}}$$
$$= \sqrt{2}$$
$$\sin 2A \neq 2\sin A \qquad \text{(When } A = 45^0\text{)}$$

For $A = 60^0$

$$\sin 2A = \sin 2 \times 60^0$$
$$= \sin 120^0$$
$$= \frac{\sqrt{3}}{2}$$
$$2\sin A = 2 \times \sin 60^0$$
$$= 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$$
$$\sin 2A \neq 2\sin A \qquad \text{(When } A = 60^0\text{)}$$

Hence Option (A) is correct

(iv)

$$\frac{2\tan 30^0}{1 - \tan^2 30^0}$$

By substituting the values of given trigonometric ratios for $\tan 30^0$, we get

$$= \frac{2 \times \left(\frac{1}{\sqrt{3}}\right)}{1 - \left(\frac{1}{\sqrt{3}}\right)^2}$$
$$= \frac{\left(\frac{2}{\sqrt{3}}\right)}{\left(1 - \frac{1}{3}\right)}$$
$$= \frac{\left(\frac{2}{\sqrt{3}}\right)}{\left(\frac{2}{3}\right)}$$
$$= \frac{2}{\sqrt{3}} \times \frac{3}{2}$$
$$= \sqrt{3}$$

Out of the given option only $\tan 60^0 = \sqrt{3}$.

Hence option (C) is correct.

Q3. If $\tan(A+B)=\sqrt{3}$ and $\tan(A-B)=\frac{1}{\sqrt{3}}$; $0^0 < (A+B) \le 90^0$, $A > B$, find A and B.

Difficulty level: Medium

Solution:
Given that

$$\tan(A+B)=\sqrt{3} \text{ and, } \tan(A-B)=\frac{1}{\sqrt{3}}$$

$$\text{Since, } \tan 60^0 = \sqrt{3} \text{ and } \tan 30^0 = \frac{1}{\sqrt{3}}$$

Therefore,

$$\tan(A+B) = \tan 60^0$$

$$(A+B) = 60^0 \qquad \text{(i)}$$

$$\tan(A-B) = \tan 30^0$$

$$(A-B) = 30^0 \qquad \text{(ii)}$$

On adding both equations (i) and (ii), we obtain:

$$A+B+A-B = 60^0 + 30^0$$

$$2A = 90^0$$

$$A = 45^0$$

By substituting the value of A in equation (i) we obtain

$$A+B=60^0$$

$$45^0 + B = 60^0$$

$$B = 60^0 - 45^0 = 15^0$$

Therefore, $\angle A = 45^0$ and $\angle B = 15^0$ $(A > B)$

Q4. State whether the following are true or false. Justify your answer.

(i) $\sin(A+B) = \sin A + \sin B$.

(ii) The value of sin θ increases as θ increases.

(iii) The value of cos θ increases as θ increases.

(iv) $\sin\theta = \cos\theta$ for all values of θ.

(v) cot A is not defined for A = 0°.

Difficulty level: Medium

Solution:

$$\sin(A+B)=\sin A+\sin B.$$

For the purpose of verification, Let $A=30^0$ and $B=60^0$

$$\begin{aligned}\text{L.H.S} &= \sin(A+B)\\ &= \sin(30^0+60^0)\\ &= \sin 90^0\\ &= 1\end{aligned}$$

$$\begin{aligned}\text{R.H.S} &= \sin A+\sin B\\ &= \sin 30^0+\sin 60^0\\ &= \frac{1}{2}+\frac{\sqrt{3}}{2}\\ &= \frac{1+\sqrt{3}}{2}\end{aligned}$$

Since, $\sin(A+B)\neq\sin A+\sin B.$

Hence, the given statement is not true

(ii) The value of $\sin\theta$ increases from 0 to 1 as θ increases from 0^0 to 90^0

$$\sin 0^0=0$$

$$\sin 30^0=\frac{1}{2}=0.5$$

$$\sin 45^0=\frac{1}{\sqrt{2}}=0.707$$

$$\sin 60^0=\frac{\sqrt{3}}{2}=0.866$$

$$\sin 90^0=1$$

Hence, the given statement is true.

(iii) The value of $\cos\theta$ decreases from 1 to 0 as θ increases from 0^0 to 90^0

$$\cos 0^0=1$$

$$\cos 30^0=\frac{\sqrt{3}}{2}=0.866$$

$$\cos 45^0=\frac{1}{\sqrt{2}}=0.707$$

$$\cos 60^0=\frac{1}{2}=0.5$$

$$\cos 90^0=0$$

Hence, the given statement is false.

(iv)

This is true when $\theta = 45^0$

As $\sin 45^0 = \frac{1}{\sqrt{2}}$ and $\cos 45^0 = \frac{1}{\sqrt{2}}$

It is not true for other values of θ

As,

$$\sin 30^0 = \frac{1}{2} \text{ and } \cos 30^0 = \frac{\sqrt{3}}{2}$$

$$\sin 60^0 = \frac{\sqrt{3}}{2} \text{ and } \cos 60^0 = \frac{1}{\sqrt{2}}$$

$$\sin 90^0 = 1 \text{ and } \cos 90^0 = 0$$

Hence, the given statement is false.

(v)

$$\cot A = \frac{\cos A}{\sin A}$$

$$\therefore \cot 0^0 = \frac{\cos 0^0}{\sin 0^0} = \frac{1}{0} = \text{ undefined}$$

Hence the given statement is true.

Chapter 8: Introduction to Trigonometry

Exercise 8.3 (Page 189 of Grade 10 NCERT Textbook)

Q1. Evaluate:

(i) $\dfrac{\sin 18^0}{\cos 72^0}$ (ii) $\dfrac{\tan 26^0}{\cot 64^0}$ (iii) $\cos 48^0 - \sin 42^0$ (iv) $\operatorname{cosec} 31^0 - \sec 59^0$

Difficulty level: Medium

Reasoning:

$$\sin\left(90^0 - \theta\right) = \cos\theta$$
$$\tan(90^0 - \theta) = \cot\theta$$
$$\sec(90^0 - \theta) = \operatorname{cosec}\theta$$

Solution:

(i)

$$\frac{\sin 18^0}{\cos 72^0}$$

Since,

$$\sin\left(90^0 - \theta\right) = \cos\theta$$

Here $\theta = 72^0$

$$= \frac{\sin(90^0 - 72^0)}{\cos 72^0}$$
$$= \frac{\cos 72^0}{\cos 72^0}$$
$$= 1$$

(ii)

$$\frac{\tan 26^0}{\cot 64^0}$$

Since

$$\tan(90^0 - \theta) = \cot\theta$$

Here $\theta = 64^0$

$$= \frac{\tan(90^0 - 64^0)}{\cot 64^0}$$
$$= \frac{\cot 64^0}{\cot 64^0}$$
$$= 1$$

(iii)

$$\cos 48^0 - \sin 42^0$$

Since,

$$\sin(90^0 - \theta) = \cos\theta$$

Here $\theta = 48^0$

$$= \cos 48^0 - \sin\left(90^0 - 48^0\right)$$
$$= \cos 48^0 - \cos 48^0$$
$$= 0$$

(iv)

$$\text{cosec} 31^0 - \sec 59^0$$

Since,

$$\sec(90^0 - \theta) = \text{cosec}\,\theta$$

Here $\theta = 31^0$

$$= \text{cosec} 31^0 - \sec\left(90^0 - 31^0\right)$$
$$= \text{cosec} 31^0 - \text{cosec} 31^0$$
$$= 0$$

Q2. Show that:

(i) $\tan 48° \tan 23° \tan 42° \tan 67° = 1$

(ii) $\cos 38° \cos 52° - \sin 38° \sin 52° = 0$

Difficulty level: Medium

Reasoning:

$$\sin\left(90^0 - \theta\right) = \cos\theta$$
$$\tan(90^0 - \theta) = \cot\theta$$

Solution:

(i)Taking L.H.S

$$= \tan 48° \tan 23° \tan 42° \tan 67°$$

Since $\tan(90^0 - \theta) = \cot\theta$

$$= \tan(90^0 - 42^0)\tan(90^0 - 67^0)\tan 42^0 \tan 67^0$$
$$= \cot 42^0 \cot 67^0 \tan 42^0 \tan 67^0$$
$$= (\cot 42^0 \tan 42^0)(\cot 67^0 \tan 67^0)$$
$$= \left(\frac{1}{\tan 42^0} \times \tan 42^0\right)\left(\frac{1}{\tan 67^0} \times \tan 67^0\right)$$
$$= 1 \times 1$$
$$= 1$$
$$= \text{R.H.S}$$

Hence, $\tan 48° \tan 23° \tan 42° \tan 67° = 1$

(ii) Taking L.H.S

$$= \cos 38° \cos 52° - \sin 38° \sin 52°$$

Since, $\sin(90^0 - \theta) = \cos\theta$

$$= \cos 38^0 \cos 52^0 - \sin(90^0 - 57^0)\sin(90^0 - 38^0)$$
$$= \cos 38^0 \cos 52^0 - \cos 52^0 \cos 38^0$$
$$= 0$$
$$= \text{R.H.S}$$

Hence, $\cos 38^0 \cos 52^0 - \sin 38^0 \sin 52^0 = 0$

Q3. If $\tan 2A = \cot(A - 18°)$, where *2A* is an acute angle, find the value of A.

Difficulty level: Medium

Reasoning:

$$\tan(90^0 - \theta) = \cot\theta$$

Solution:

Given that: $\tan 2A = \cot(A - 18°) \ldots.(i)$

But $\tan 2A = \cot(90^0 - 2A)$

By substituting this in equation (i) we get:

$$\cot\left(90^0 - 2A\right) = \cot\left(A - 18^0\right)$$
$$90^0 - 2A = A - 18^0$$
$$3A = 108^0$$
$$A = \frac{108^0}{3} =$$
$$A = 36^0$$

Q4. If $\tan A = \cot B$, prove that $A + B = 90^0$.

Difficulty level: Easy

Reasoning:

$$\tan(90^0 - \theta) = \cot\theta$$

Solution:

Given that: $\tan A = \cot B$ (i)

We know that $\tan A = \cot\left(90^0 - A\right)$

By substituting this in equation (i) we get:

$$\cot(90^0 - A) = \cot B$$
$$90^0 - A = B$$
$$A + B = 90^0$$

Q5. If $\sec 4A = \operatorname{cosec}\left(A - 20°\right)$, where 4A is an acute angle, find the value of A.

Difficulty level: Easy

Reasoning:

$$\sec A = \operatorname{cosec}\left(90^0 - A\right)$$

Solution:

Given that: $\sec 4A = \operatorname{cosec}\left(A - 20^{\circ}\right)$ ....(i)

Since, $\sec A = \operatorname{cosec}\left(90^0 - A\right)$

By using property in equation (i) we get:

$$\operatorname{cosec}\left(90^0-4A\right)=\operatorname{cosec}\left(A-20^0\right)$$
$$90^0-4A=A-20^0$$
$$5A=110^0$$
$$A=\frac{110^0}{5}$$
$$A=22^0$$

Q6. If A, B and C are interior angles of a triangle ABC, then show that $\sin\left(\frac{B+C}{2}\right)=\cos\frac{A}{2}$

Difficulty level: Medium

Reasoning:

$$\sin\left(90^0-\theta\right)=\cos\theta$$

Solution:

We know that for ΔABC,

$$\angle A+\angle B+\angle C=180^0$$
$$\angle B+\angle C=180^0-\angle A$$

On dividing both sides by 2, we get:

$$\frac{\angle B+\angle C}{2}=\frac{180^0-\angle A}{2}$$
$$\frac{\angle B+\angle C}{2}=90^0-\frac{\angle A}{2}$$

Applying sine angles on both the sides:

$$\sin\left(\frac{B+C}{2}\right)=\sin\left(90^0-\frac{A}{2}\right)$$

Since

$$\sin(90^0-\theta)=\cos\theta$$
$$\therefore \sin\left(\frac{B+C}{2}\right)=\cos\left(\frac{A}{2}\right)$$

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Q7. Express $\sin 67^0 + \cos 75^0$ in terms of trigonometric ratios of angles between 0^0 and 45^0.

Difficulty level: Medium

Reasoning:

$$\cos\left(90^0 - \theta\right) = \sin\theta$$

Solution:

Given that: $\sin 67^0 + \cos 75^0 \ldots.(i)$

Since $\cos\left(90^0 - \theta\right) = \sin\theta$

By using property in equation (i) we get:

$$= \sin\left(90^0 - 23^0\right) + \cos\left(90^0 - 15^0\right)$$
$$= \cos 23^0 + \sin 15^0$$

Hence, the expression $\cos 23^0 + \sin 15^0$ has trigonometric ratios of angles between 0^0 and 45^0.

Chapter 8: Introduction to Trigonometry

Exercise 8.4 (Page 193 of Grade 10 NCERT Textbook)

Q1. Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.

Difficulty level: Medium

Reasoning:

$$\text{cosec}^2 A = 1 + \cot^2 A$$
$$\sec^2 A = 1 + \tan^2 A$$

Solution:

Consider a ΔABC with $\angle B = 90°$

Using the Trigonometric Identity,

$$\text{cosec}^2 A = 1 + \cot^2 A$$
$$\frac{1}{\text{cosec}^2 A} = \frac{1}{1+\cot^2 A} \quad \text{(By taking reciprocal both the sides)}$$
$$\sin^2 A = \frac{1}{1+\cot^2 A} \quad \left(\text{As } \frac{1}{\text{cosec}^2 A} = \sin^2 A\right)$$

Therefore,

$$\sin A = \pm \frac{1}{\sqrt{1+\cot^2 A}}$$

For any sine value with respect to an angle in a triangle, sine value will never be negative. Since, sine value will be negative for all angles greater than 180°.

Therefore, $\sin A = \frac{1}{\sqrt{1+\cot^2 A}}$

We know that, $\tan A = \frac{\sin A}{\cos A}$

However, Trigonometric Function, $\cot A = \frac{\cos A}{\text{Sin} A}$

Therefore, Trigonometric Function, $\tan A = \frac{1}{\cot A}$

Also, $\sec^2 A = 1 + \tan^2 A$ (Trigonometric Identity)

$$= 1 + \frac{1}{\cot^2 A}$$

$$= \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\sec A = \frac{\sqrt{\cot^2 A + 1}}{\cot A}$$

Q2. Write all the other trigonometric ratios of $\angle A$ in terms of sec A.

Difficulty level: Medium

Reasoning:

$$\sin^2 A + \cos^2 A = 1$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sec^2 A = 1 + \tan^2 A$$

Solution:

We know that,

Trigonometric Function, $\cos A = \dfrac{1}{\sec A}$ ...Equation (1)

Also,

$\sin^2 A + \cos^2 A = 1$ (Trigonometric identity)

$\sin^2 A = 1 - \cos^2 A$ (By transposing)

Using value of cos A from Equation (1) and simplifying further,

$$\sin A = \sqrt{1 - \left(\frac{1}{\sec A}\right)^2}$$

$$= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}}$$

$$= \frac{\sqrt{\sec^2 A - 1}}{\sec A} \quad \text{...Equation (2)}$$

$\tan^2 A + 1 = \sec^2 A$ (Trigonometric identity)

$\tan^2 A = \sec^2 A - 1$ (By transposing)

Trigonometric Function,

$$\tan A = \sqrt{\sec^2 A - 1} \qquad \text{... Equation } (3)$$

$$\cot A = \frac{\cos A}{\sin A}$$

$$= \frac{\dfrac{1}{\sec A}}{\dfrac{\sqrt{\sec^2 A - 1}}{\sec A}} \qquad \ldots\left(\text{By substituting Equations } (1) \text{ and } (2)\right)$$

$$= \frac{1}{\sqrt{\sec^2 A - 1}}$$

$$\operatorname{cosec} A = \frac{1}{\sin A}$$

$$= \frac{\sec A}{\sqrt{\sec^2 A - 1}} \qquad \left(\text{By substituting Equation } (2) \text{ and simplifying}\right)$$

Q3. Evaluate

(i) $\dfrac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$

(ii) $\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$

Difficulty level: Medium

Reasoning:

$\sin^2 A + \cos^2 A = 1$

$\sin(90^\circ - \theta) = \cos\theta$

$\cos(90^\circ - \theta) = \sin\theta$

Solution:

(i) $\dfrac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$

$$= \frac{\left[\sin\left(90^\circ - 27\right)\right]^2 + \sin^2 27}{\left[\cos\left(90^\circ - 73^\circ\right)\right]^2 + \cos^2 73^\circ}$$

$$= \frac{\cos^2 27^\circ + \sin^2 27^\circ}{\sin^2 73^\circ + \cos^2 73^\circ} \qquad \left(\sin(90^\circ - \theta) = \cos\theta \;\; \& \;\; \cos(90^\circ - \theta) = \sin\theta\right)$$

$$= \frac{1}{1} \qquad \left(\text{By Identity } \sin^2 A + \cos^2 A = 1\right)$$

$$= 1$$

(ii) $\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$

$= \sin 25^\circ \left[\cos\left(90^\circ - 25^\circ\right)\right] + \cos 25^\circ \left[\sin\left(90^\circ - 25^\circ\right)\right]$

$= \sin 25^\circ . \sin 25^\circ + \cos 25^\circ . \cos 25^\circ \quad \left[\because \sin\left(90^\circ - \theta\right) = \cos\theta \ \& \ \cos\left(90^\circ - \theta\right) = \sin\theta\right]$

$= \sin^2 25^\circ + \cos^2 25^\circ$

$= 1 \quad$ (By Identity $\sin^2 A + \cos^2 A = 1$)

Q4. Choose the correct option. Justify your choice.

(i) $9 \sec 2A - 9 \tan 2A =$ __________

(A) 1
(B) 9
(C) 8
(D) 0

(ii) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \text{cosec}\,\theta)$

(A) 0
(B) 1
(C) 2
(D) -1

(iii) $(\sec A + \tan A)(1 - \sin A) =$ __________

(A) sec A
(B) sin A
(C) cosec A
(D) cos A

(iv) $\dfrac{1+\tan^2 A}{1+\cot^2 A}$

(A) sec2A
(B) -1
(C) cot2A
(D) tan2A

Difficulty level: Medium

Reasoning:

$\sin^2 A + \cos^2 A = 1$

$\operatorname{cosec}^2 A = 1 + \cot^2 A$

$\sec^2 A = 1 + \tan^2 A$

Solution:

(i) $9\sec^2 A - 9\tan^2 A$

$= 9(\sec^2 A - \tan^2 A)$

$= 9 \times 1$ [By the identity, $1 + \sec^2 A = \tan^2 A$, Hence $\sec^2 A - \tan^2 A = 1$]

$= 9$

(ii) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$ ……..Equation (1)

We know that the trigonometric functions,

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$\cot(x) = \frac{\cos(x)}{\sin(x)} = \frac{1}{\tan(x)}$$

And

$$\sec(x) = \frac{1}{\cos(x)}$$

$$\operatorname{cosec}(x) = \frac{1}{\sin(x)}$$

By substituting the above function in Equation (1),

$$\Rightarrow \left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)\left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)$$

$$= \left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)\left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)$$ (By taking LCM and multiplying)

$$= \frac{(\sin\theta + \cos\theta)^2 - (1)^2}{\sin\theta\cos\theta}$$ (Using $a^2 - b^2 = (a+b)(a-b)$)

$$= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta}$$

$$= \frac{1 + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta}$$ (Using identify $\sin^2\theta + \cos^2\theta = 1$)

$$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2$$

Hence, option (C) is correct.

(iii) $(\sec A + \tan A)(1 - \sin A)$ ......(1)

We know that the trigonometric functions,

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

And

$$\sec(x) = \frac{1}{\cos(x)}$$

By substituting the above function in Equation (1),

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$$

$$= \left(\frac{1 + \sin A}{\cos A}\right)(1 - \sin A)$$

$$= \frac{1 - \sin^2 A}{\cos A}$$

$$= \frac{\cos^2 A}{\cos A} \quad (\text{By identify } \sin^2\theta + \cos^2\theta = 1, \text{Hence } 1 - \sin^2\theta = \cos^2\theta)$$

$$= \cos A$$

Hence, option (D) is correct.

(iv) $\dfrac{1 + \tan^2 A}{1 + \cot^2 A}$

We know that the trigonometric functions,

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$\cot(x) = \frac{\cos(x)}{\sin(x)}$$

$$= \frac{1}{\tan(x)}$$

By substituting the above function in Equation (1),

$$\frac{1+\tan^2 A}{1+\cot^2 A}=\frac{1+\frac{\sin^2 A}{\cos^2 A}}{1+\frac{\cos^2 A}{\sin^2 A}}$$

$$=\frac{\frac{\cos^2 A+\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A+\cos^2 A}{\sin^2 A}}$$

$$=\frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$$

$$=\frac{\sin^2 A}{\cos^2 A}$$

$$=\tan^2 A$$

Hence, option (D)is correct.

Q5. Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

(i) $(\operatorname{cosec}\theta-\cot\theta)^2=\frac{1-\cos\theta}{1+\cos\theta}$

(ii) $\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=2\sec A$

(iii) $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\operatorname{cosec}\theta$

(iv) $\frac{1+\sec A}{\sec A}=\frac{\sin^2 A}{1-\cos A}$

(v) $\frac{\cos A-\sin A+1}{\cos A+\sin A+1}=\operatorname{cosec}A+\cot A$

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}}=\sec A+\tan A$

(vii) $\frac{\sin\theta-2\sin^3\theta}{2\cos\theta-\cos\theta}=\tan\theta$

(viii) $(\sin A+\operatorname{cosec}A)^2+(\cos A+\sec A)^2=7+\tan^2 A+\cot^2 A$

(ix) $(\operatorname{cosec}A-\sin A)(\sec A-\cos A)=\frac{1}{\tan A+\cot A}$

(x) $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\frac{1-\tan A}{1-\cot A}\right)^2=\tan^2 A$

Difficulty level: Medium

Reasoning:

$$\sin^2 A + \cos^2 A = 1$$
$$\text{cosec}^2 A = 1 + \cot^2 A$$
$$\sec^2 A = 1 + \tan^2 A$$

Solution:

(i) $(\text{cosec}\theta - \cot\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

$\text{L.H.S} = (\text{cosec}\theta - \cot\theta)^2 \qquad \ldots\ldots\ldots(1)$

We know that the trigonometric functions,

$$\cot(x) = \frac{\cos(x)}{\sin(x)} = \frac{1}{\tan(x)}$$
$$\text{cosec}(x) = \frac{1}{\sin(x)}$$

By substituting the above function in Equation (1)

$$(\text{cosec}\theta - \cot\theta)^2 = \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$$
$$= \frac{(1-\cos\theta)^2}{(\sin\theta)^2}$$
$$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$$
$$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta} \qquad \left(\text{By Identity } \sin^2 A + \cos^2 A = 1 \text{Hence, } 1-\cos^2 A = \sin^2 A\right)$$
$$= \frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)} \qquad \left[\text{Using } a^2 - b^2 = (a+b)(a-b)\right]$$
$$= \frac{1-\cos\theta}{1+\cos\theta}$$
$$= \text{RHS}$$

(ii) $\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=2\sec A$

$$\text{L.H.S}=\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}$$
$$=\frac{\cos^2 A+(1+\sin A)^2}{(1+\sin A)(\cos A)}$$
$$=\frac{\cos^2 A+1+\sin^2 A+2\sin A}{(1+\sin A)(\cos A)}$$
$$=\frac{\sin^2 A+\cos^2 A+1+2\sin A}{(1+\sin A)(\cos A)}$$
$$=\frac{1+1+2\sin A}{(1+\sin A)(\cos A)} \qquad \left(\text{By identify } \sin^2 A+\cos^2 A=1\right)$$
$$=\frac{2+2\sin A}{(1+\sin A)(\cos A)}$$
$$=\frac{2(1+\sin A)}{(1+\sin A)(\cos A)}$$
$$=\frac{2}{\cos A}$$
$$=2\sec A$$
$$=\text{R.H.S}$$

(iii) $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\ \operatorname{cosec}\theta$

$$\text{LHS}=\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta} \qquad \ldots\ldots(1)$$

We know that the trigonometric functions,

$$\tan(x)=\frac{\sin(x)}{\cos(x)}$$
$$\cot(x)=\frac{\cos(x)}{\sin(x)}=\frac{1}{\tan(x)}$$

By substituting the above relations in Equation (1),

$$=\frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\cos\theta}{\sin\theta}}$$

$$=\frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$=\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}+\frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$=\frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^2\theta}{\cos\theta}-\frac{\cos^2\theta}{\sin\theta}\right]$$

$$=\frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta}\right]$$

Using $a^3-b^3=(a-b)\left(a^2+ab+b^2\right)$

$$=\frac{1}{(\sin\theta-\cos\theta)}\left[\frac{(\sin\theta-\cos\theta)\left(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta\right)}{\sin\theta\cos\theta}\right]$$

$$=\frac{(1+\sin\theta\cos\theta)}{(\sin\theta\cos\theta)} \qquad \text{(By Identity } \sin^2A+\cos^2A=1\text{)}$$

$$=\frac{1}{\sin\theta\cos\theta}+\frac{\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$=1+\sec\theta\operatorname{cosec}\theta$

$=$R.H.S.

(iv) $\dfrac{1+\sec A}{\sec A}=\dfrac{\sin^2A}{1-\cos A}$

$$\text{L.H.S}=\frac{1+\sec A}{\sec A} \qquad \ldots\ldots(1)$$

We know that the trigonometric functions,

$$\sec(x)=\frac{1}{\cos(x)}$$

By substituting the above function in Equation (1),

$$\frac{1+\sec A}{\sec A}=\frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}$$

$$=\frac{\frac{\cos A+1}{\cos A}}{\frac{1}{\cos A}}$$

$$=\frac{\cos A+1}{\cos A}\times\frac{\cos A}{1}$$

$$=(1+\cos A)$$

By multiplying $(1-\cos A)$, in both denominator and numerator

$$\Rightarrow \frac{(1-\cos A)(1+\cos A)}{(1-\cos A)}$$

$$=\frac{1-\cos^2 A}{1-\cos A}$$

$$=\frac{\sin^2 A}{1-\cos A} \qquad \left[\text{By Identity } \sin^2 A+\cos^2 A=1\right]$$

$$=\text{R. H.S}$$

(v) $\frac{\cos A-\sin A+1}{\cos A+\sin A-1}=\operatorname{cosec} A+\cot A$

$$\text{L.H.S}=\frac{\cos A-\sin A+1}{\cos A+\sin A-1}$$

Diving both numerator and denominator by $\sin A$

$$\Rightarrow \frac{\frac{\cos A}{\sin A}-\frac{\sin A}{\sin A}+\frac{1}{\sin A}}{\frac{\cos A}{\sin A}+\frac{\sin A}{\sin A}-\frac{1}{\sin A}}$$

We know that the trigonometric functions,

$$\cot(x) = \frac{\cos(x)}{\sin(x)} = \frac{1}{\tan(x)}$$

$$\text{cosec}(x) = \frac{1}{\sin(x)}$$

We get

$$\Rightarrow \frac{\cot A - 1 + \text{cosec}\, A}{\cot A + 1 - \text{cosec}\, A}$$

$$\Rightarrow \frac{\cot A - (1 - \text{cosec}\, A)}{\cot A + (1 - \text{cosec}\, A)}$$

We know that, $1 + \cot^2 A = \text{Cosec}^2 A$

Hence multiplying [cot A – (1 – cosec A)] in numerator and denominator

$$\Rightarrow \frac{[(\cot A) - (1 - \text{cosec}\, A)][(\cot A) - (1 - \text{cosec}\, A)]}{[(\cot A) + (1 - \text{cosec}\, A)][(\cot A) - (1 - \text{cosec}\, A)]}$$

$$= \frac{[\cot A - (1 - \text{cosec}\, A)]^2}{(\cot A)^2 - (1 - \text{cosec} A)^2}$$

$$= \frac{\cot^2 A + (1 - \text{cosec} A)^2 - 2\cot A(1 - \text{cosec} A)}{\cot^2 A - (1 + \text{cosec}^2 A - 2\text{cosec}\, A)}$$

$$= \frac{\cot^2 A + 1 + \text{cosec}^2 A - 2\text{cosec}\, A - 2\cot A + 2\cot A\, \text{cosec} A}{\cot^2 A - (1 + \text{cosec}^2 A - 2\text{cosec}\, A)}$$

$$= \frac{2\text{cosec}^2 A + 2\cot A\ \text{cosec} A - 2\cot A - 2\text{cosec} A}{\cot^2 A - 1 - \text{cosec}^2 A + 2\text{cosec} A}$$

$$= \frac{2\text{cosec}\, A(\text{cosec}\, A + \cot A) - 2(\cot A + \text{cosec}\, A)}{\cot^2 A - \text{cosec}^2 A - 1 + 2\text{cosec}\, A}$$

$$= \frac{(\text{cosec}\, A + \cot A)(2\text{cosec} A - 2)}{-1 - 1 + 2\text{cosec}\, A}$$

$$= \frac{(\text{cosec} A + \cot A)(2\text{cosec} A - 2)}{(2\text{cosec}\, A - 2)}$$

$= \text{cosec}\ A + \cot\ A$

$=$ R.H.S

(vi) $\sqrt{\dfrac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

$$\text{LHS} = \sqrt{\frac{1+\sin A}{1-\sin A}} \qquad \ldots\ldots (1)$$

Multiplying and dividing by $\sqrt{(1+\sin A)}$

$$\Rightarrow \sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A)}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{(1-\sin^2 A)}} \qquad [\because a^2 - b^2 = (a-b)(a+b),]$$

$$= \frac{(1+\sin A)}{\sqrt{1-\sin^2 A}}$$

$$= \frac{1+\sin A}{\sqrt{\cos^2 A}}$$

$$= \frac{1+\sin A}{\cos A}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$= \sec A + \tan A$$

$$= \text{R.H.S}$$

(vii) $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos\theta - \cos\theta} = \tan\theta$

$$\text{L.H.S} = \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$$

Taking $\sin\theta$ and $\cos\theta$ common in both numerator and denominator respectively.

$$\Rightarrow \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$$

By Identity $\sin^2 A + \cos^2 A = 1$ hence, $\cos^2 A = 1 - \sin^2 A$ and substituting this in the above equation,

$$\Rightarrow \frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left\{2\left(1-\sin^2\theta\right)-1\right\}}$$

$$=\frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left(2-2\sin^2\theta-1\right)}$$

$$=\frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left(1-2\sin^2\theta\right)}$$

$$=\frac{\sin\theta}{\cos\theta}$$

$$=\tan\theta$$

=RHS

(viii) $(\sin A+\operatorname{cosec} A)^2+(\cos A+\sec A)^2=7+\tan^2 A+\cot^2 A$

$$\text{L.H.S}=(\sin A+\operatorname{cosec}A)^2+(\cos A+\sec A)^2$$

By using $(a+b)^2=a^2+2ab+b^2$

$$\Rightarrow \sin^2 A+\operatorname{cosec}^2 A+2\sin A\operatorname{cosec} A+\cos^2 A+\sec^2 A+2\cos A\sec A$$

By rearranging and using $\sec A=\frac{1}{\cos A}$ and $\operatorname{cosec} A=\frac{1}{\sin A}$

$$\Rightarrow\left(\sin^2 A+\cos^2 A\right)+\left(\operatorname{cosec}^2 A+\sec^2 A\right)+2\sin A\left(\frac{1}{\sin A}\right)+2\cos A\left(\frac{1}{\cos A}\right)$$

Hence $\left(\sin^2 A+\cos^2 A\right)=1$, $\operatorname{cosec}^2 A=\left(1+\cot^2 A\right)$ and $\left(\sec^2 A-\tan^2 A\right)=1$

$$\Rightarrow 1+1+\cot^2 A+1+\tan^2 A+2+2$$

$$=7+\tan^2 A+\cot^2 A$$

$$=\text{R.H.S}$$

(ix) $(\operatorname{cosec} A-\sin A)(\sec A-\cos A)=\frac{1}{\tan A+\cot A}$

$$\text{L.H.S}=(\operatorname{cosec} A-\sin A)(\sec A-\cos A) \qquad \ldots\ldots\ldots (1)$$

We know that the trigonometric functions,

$$\sec(x) = \frac{1}{\cos(x)}$$

$$\operatorname{cosec}(x) = \frac{1}{\sin(x)}$$

By substituting the above relations in Equation (1)

$$\Rightarrow \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$

$$= \left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right)$$

$$= \frac{\cos^2 A \sin^2 A}{\sin A \cos A}$$

$$= \frac{\sin A \cos A}{1}$$

$$= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A} \quad \left[\because \left(\sin^2 A + \cos^2 A\right) = 1\right]$$

$$= \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}} \quad \left[\text{Dividing numerator and denominator by } \left(\sin A \cos A\right)\right]$$

$$= \frac{1}{\frac{\sin^2 A}{\sin A \cos A} + \frac{\cos^2 A}{\sin A \cos A}}$$

$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$$

$$= \frac{1}{\tan A + \cot A}$$

$$= \text{RHS}$$

(x) $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\frac{1-\tan A}{1-\cot A}\right)^2=\tan^2 A$

Taking LHS, $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right)$

$$=\frac{\sec^2 A}{\operatorname{cosec}^2 A}$$

$$=\frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$$

$$=\frac{1}{\cos^2 A}\times\frac{\sin^2 A}{1}$$

$$=\tan^2 A$$

=RHS

Taking, $\left(\frac{1-\tan A}{1-\cot A}\right)^2$

$$=\left(\frac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$$

$$=\left(\frac{1-\tan A}{\frac{\tan A-1}{\tan A}}\right)^2$$

$$=\left((1-\tan A)\times\frac{\tan A}{\tan A-1}\right)^2$$

$$=(-\tan A)^2$$

$$=\tan^2 A$$

=RHS

Hence, L.H.S = R.H.S.