HOLIDAYS HOMEWORK

Maths

1. HCF of p and $q = p$

2. Least Composite no. $= 4$
   Least Prime no. $= 2$
   Their HCF $: 2$
   Their LCM $= 4$
   Ratio $= 2/4 = 1/2 = 1:2$

3. Let us assume in contrary that $\sqrt{2}+\sqrt{5}$ is a rational no.
   $\therefore \sqrt{2}+\sqrt{5} = \frac{p}{q}$, where p and q are co-prime integers and $q \neq 0$.

$$\sqrt{5} = \frac{p}{q} - \sqrt{2}$$

Squaring on both sides -

$$5 = \frac{p^2}{q^2} + 2 - 2 \times \frac{p}{q} \times \sqrt{2}$$

$$5 - 2 = \frac{p^2}{q^2} - 2\sqrt{2}\frac{p}{q}$$

$$3 = \frac{p^2}{q^2} - \frac{2\sqrt{2}p}{q}$$

$$3 = \frac{p^2 - 2\sqrt{2}pq}{q^2}$$

$$3q^2 = p^2 - 2\sqrt{2}pq$$

$$3q^2 - p^2 = -2\sqrt{2}pq$$

$$\frac{3q^2 - p^2}{pq} = -2\sqrt{2}$$

$$\frac{3q^2 - p^2}{-2pq} = \sqrt{2}$$

$$\frac{p^2 - 3q^2}{2pq} = \sqrt{2}$$

$\frac{p^2 - 3q^2}{2pq}$ is rational but $\sqrt{2}$ is irrational.

Rational No. $\neq$ Irrational No.

It contradicts to our assumption that p and q are co-prime.

$\therefore \sqrt{2}+\sqrt{5}$ is an irrational no.

4. LCM (48, 72, 108)

2	48 - 72 - 108
2	24 - 36 - 54
2	12 - 18 - 27
2	6 - 9 - 27
3	3 - 9 - 27
3	1 - 3 - 9
3	1 - 1 - 3
	1 - 1 - 1

LCM:- 432 s

$$60 \overline{)\,432\,}(7 \text{ mins.}$$
$$\underline{-420}$$
$$\underline{\times 12} \text{ seconds}$$

Time when lights will change together next :- 7:07:12 s.

5. $p(x) = x^2 - ax - b$

$$a + b = \frac{-(-a)}{1} = a$$

$$ab = \frac{-b}{1} = -b$$

$$a^2 + b^2 = (a+b)^2 - 2ab$$
$$= (a)^2 - 2 \times (-b)$$
$$= a^2 + 2b$$

6. $x^2 - 3x - m(m+3)$

$$x^2 - 3x - m^2 - 3m$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = 3 \pm \sqrt{(-3)^2 - 4 \times 1(m^2 - 3m)} / 2$$

$$x = 3 \pm \sqrt{9 - 4m^2 + 12m} / 2$$

$$x = 3 \pm \sqrt{(3-2m)^2} / 2$$

$$x = \frac{3 \pm 3 - 2m}{2}$$

$$x = \frac{3 + 3 - 2m}{2}$$
$$x = \frac{6 - 2m}{2}$$
$$x = 3 - m$$

$$x = \frac{3 - (3-2)m}{2}$$
$$x = \frac{3 - 3 + 2m}{2}$$
$$x = \frac{2m}{2} = m$$

7. $ky^2 - 11y + (k-23) = 0$

$$\alpha + \beta = \frac{-(-11)}{k} = \frac{11}{k}$$

$$\alpha\beta = \frac{k-23}{k}$$

A.T.Q.

$$\frac{11}{k} = \frac{13}{21} + \frac{k-23}{k}$$

$$\frac{11}{k} - \frac{k-23}{k} = \frac{13}{21}$$

$$\frac{11-k+23}{k} = \frac{13}{21}$$

$$21(34-k) = 13k$$

$$714 - 21k = 13k$$

$$-21k - 13k = -714$$

$$-34k = -714$$

$$k = \frac{714}{34}$$

$$k = 21$$

8. $p(x) = x^2 - (k+6)x + 2(2k-1)$

$$\alpha + \beta = \frac{-[-(k+6)]}{1} = k+6$$

$$\alpha\beta = \frac{2(2k-1)}{1} = 4k-2$$

A.T.Q.

$$\alpha + \beta = \frac{1}{2} \times \alpha\beta$$

$$k+6 = \frac{1}{2}(4k-2)$$

$$k+6 = \frac{1}{\cancel{2}}(\cancel{2}(2k-1))$$

$$k+6 = 2k-1$$

$$k - 2k = -1-6$$

$$\cancel{-}k = \cancel{-}7$$

$$k = 7$$

9. V. Imp $\quad ax - by = a^2 - b^2$ —— ②

$x + y = a + b$ —— ①

Multiplying eqⁿ ① by a and eqⁿ ② by 1.

$$\cancel{ax} + ay = a(a+b) = a^2 + ab$$
$$\cancel{ax} - by = a^2 - b^2$$
$$- \quad + \quad\quad -$$

$$y(a+b) = \cancel{a^2} + ab - \cancel{a^2} + b^2$$

$$y\cancel{(a+b)} = b\cancel{(a+b)}$$

$$y = b$$

Put $y = b$ in ① -

$$x + \cancel{b} = a + \cancel{b}$$
$$x = a$$

10. Let walking speeds of two people = x km/hr, y km/hr
Since they walk ~~in opposite direction~~ towards each-other, their effective speeds will be added. (CASE-I)
∴ Their effective speed = $(x+y)$ km/hr

Time = 2 hrs

We know that,

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\therefore \text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\text{Time} = \frac{16}{(x+y)} \text{ —— ①}$$

(CASE-2) When both travel ~~towards ea~~ in the same direction with same speed.

Effective speed = $(x-y)$ km/hr (Let $x > y$)

$$\text{Time taken} = \frac{\text{Distance}}{\text{Speed}}$$

$$= \frac{16}{(x-y)} \text{ —— ②}$$

From ① -

$$2 = \frac{16}{(x+y)} \quad (\text{Time} = 2\text{hrs})$$

$$2(x+y) = 16$$

$$x + y = 8 \text{ —— ③}$$

From 2 -

$$8 = \frac{16}{(x-y)} \quad [\text{Time} = 8\text{hrs}]$$

$$8(x-y) = 16$$

$$x-y = \frac{16}{8} = 2 \text{ ——— } (4)$$

Solving 3 & 4 -

$$x + \cancel{y} = 8$$
$$x - \cancel{y} = 2$$
$$2x = 10$$

$$x = 5 \text{ km/hr}$$

Put $x = 5$ in 3 -

$$5 + y = 8$$
$$y = 3 \text{ km/hr}$$

∴ walking speeds of two persons = 5km/hr, 3 km/hr.

Q11:- Let digit at ones place = x
and let digit at tens place = y
Original no. = $x + 10y$
On reversing the digits,
new no. = $10x + y$
A.T.Q. (Case I)

$$x + 10y = 7(x+y)$$
$$x + 10y = 7x + 7y$$
$$x - 7x = 7y - 10y$$
$$\cancel{-}6x = \cancel{-}3y$$
$$2x = y \text{ ——— } (1)$$

A.T.Q. (Case-2)

$$10x + y = x + 10y \cancel{+18} - 18$$
$$10x - x + y - 10y = -18$$
$$9x - 9y = -18$$
$$\cancel{9}(x-y) = \cancel{9} \times -2$$
$$x - y = -2 \text{ ——— } (2)$$

OR

$$\frac{480}{(x-8)} = \frac{480}{x} + 3$$

$$\frac{480}{x-8} - \frac{480}{x} = 3$$

$$\frac{\cancel{480x} - \cancel{480x} + 3840}{x(x-8)} = 3$$

$$\frac{3840}{x^2-8x} = 3$$

$$3840 = 3(x^2-8x)$$

$$3(x^2-8x) = 3840$$

$$3x^2 - 24x - 3840 = 0$$

$$3(x^2 - 8x - 1280) = 0$$

$$x^2 - 8x - 1280 = 0$$

$$D = (-8)^2 - 4 \times 1 \times -1280$$

$$= 64 + 5120$$

$$= 5184$$

$$\begin{array}{r|l} & 72 \\ \hline 7 & \overline{51}\,\overline{84} \\ & -49 \\ \hline 142 & 284 \\ & -284 \\ \hline & \times \end{array}$$

$$x = \frac{-(-8) \pm \sqrt{5184}}{2 \times 1}$$

$$= \frac{8 \pm 72}{2}$$

$$x = \frac{8-72}{2} = \frac{-64}{2} = -32$$

$$x = \frac{8+72}{2} = \frac{80}{2} = 40$$

Neglecting $x = -32$

$\therefore x = 40.$

$\therefore$ Usual / original speed of the train = 40 km/hr.

Put the value of y from ① in ② -

$$x - 2x = -2$$
$$\cancel{-}x = \cancel{-}2$$
$$x = 2$$

Put $x = 2$ in ① -

$$y = 2 \times 2 = 4$$

$\therefore$ Original No. $= 2 + 10 \times 4 = 42$

Q12:- $mx^2 - 2(m-1)x + (m+2) = 0$

Here, $a = m$, $b = -2(m-1)$, $c = m+2$

For two real and equal roots -

$$D = 0$$
$$b^2 - 4ac = 0$$
$$\Rightarrow \left[-2(m-1)\right]^2 - 4 \times m \times (m+2) = 0$$
$$\Rightarrow 4(m-1)^2 - 4m(m+2) = 0$$
$$\Rightarrow 4(m^2 + 1 - 2m) - 4m^2 - 8m = 0$$
$$\Rightarrow \cancel{4m^2} + 4 - 8m - \cancel{4m^2} - 8m = 0$$
$$\Rightarrow -16m + 4 = 0$$
$$\Rightarrow \cancel{-}16m = \cancel{-}4$$
$$\Rightarrow m = \frac{4}{16}$$
$$\Rightarrow m = \frac{1}{4}$$

Q13:- Total distance = 480 km

Let original speed of the train = x km/hr.

New speed of the train = $(x-8)$ km/hr

Time taken by train to cover 480 km with original speed = $\frac{480}{x}$ km/hr

Time taken by train to cover 480 km with decreased speed = $\frac{480}{(x-8)}$ km/hr.

A.T.Q.

$$\frac{480}{x} + 3 = \frac{480}{x-8}$$

Q14:- Let two numbers = x, y

A.T.Q. (Case-I)

$$x + y = 34 \quad \text{——— (1)}$$

A.T.Q. (Case-II)

$$(x-3)(y+2) = 260$$

$$x(y+2) - 3(y+2) = 260$$

$$xy + 2x - 3y - 6 = 260$$

$$2x - 3y + xy = 260 + 6$$

$$2x - 3y + xy = 266$$

$$\cancel{x(34-x}$$

$$2x - 3(34-x) + x(34-x) = 266$$

$$2x - 102 + 3x + 34x - x^2 = 266$$

$$-x^2 + 39x = 266 + 102$$

$$-x^2 + 39x = 368$$

$$-x^2 + 39x - 368 = 0$$

$$-(x^2 - 39x + 368) = 0$$

$$x^2 - 39x + 368 = 0$$

$$x^2 - (16+23)x + 368 = 0$$

$$x^2 - 16x - 23x + 368 = 0$$

$$x(x-16) - 23(x-16) = 0$$

$x - 16 = 0$	$x - 23 = 0$
$x = 16$	$x = 23$

$\begin{array}{r} 34 \\ +5 \\ \hline 39 \end{array}$

When $x = 16$,

$$y = 34 - 16 = 18$$

When $x = 23$,

$$y = 34 - 23 = 11$$

$\therefore$ No.'s are = 16, 18

OR

23, 11

15. Let a be the first term and d be the common ~~tert~~ difference.

$a_3 + a_7 = 0$

$a + 2d + a + 6d = 0$

$2a + 8d = 0$

$2(a + 4d) = 0$

$a + 4d = 0$

$\therefore a_5 = a + 4d = 0.$

16. The given A.P. is :

$12, 8, 4, \ldots, -84$

$a = -84$

$d = 4$

$a_{11} = a + 10d$

$= -84 + 10 \times 4$

$= -84 + 40$

$= -44$

17. Let a be the first term and d be the common difference.

A.T.Q.

$a_{16} = 2a_8 + 1$

$a + 15d = 2(a + 7d) + 1$

$a + 15d = 2a + 14d + 1$

$a - 2a + 15d - 14d = 1$

$-a + d = 1$

$-(a - d) = 1$

$a - d = -1$

$-d = -1 - a$

$d = 1 + a$ —— ①

$a_{12} = 47$

$a + 11d = 47$

$a + 11(1 + a) = 47$

$a + 11 + 11a = 47$

$12a + 11 = 47$

$12a = 47 - 11$

$12a = 36$

$a = \frac{\cancel{36}\,3}{\cancel{12}\,1} = 3$

Put $a = 3$ in ① -

$d = 1 + 3$

$d = 4$

$a_n = a + (n-1)d$

$= 3 + (n-1)(4)$

$= 3 + 4n - 4$

$= 4n - 1$

18. Let a be the first term and d be the common difference.

$S_6 = 36.$

$\Rightarrow \frac{6}{2}\left[2a + 5d\right] = 36$

$\Rightarrow 3\,(2a + 5d) = 36$

$\Rightarrow 2a + 5d = 12$ —— 1

$S_{16} = 256$

$\Rightarrow \frac{16}{2}\left[2a + 15d\right] = 256$

$\Rightarrow 8\left[2a + 15d\right] = 256$

$\Rightarrow 2a + 15d = 256/8 = 32$ —— 2

Solving 1 & 2 -

$$\begin{aligned} \cancel{2a} + 5d &= 12 \\ \cancel{2a} + 15d &= 32 \\ - \quad - \quad &\quad - \\ \hline -10d &= -20 \end{aligned}$$

$d = 2$

Put $d = 2$ in 1 -

$2a + 5 \times 2 = 12$

$2a + 10 = 12$

$2a = 2$

$a = 1$

$S_{10} = \frac{10}{2}\left[2 \times 1 + (10-1)2\right]$

$= 5\left[2 + 18\right]$

$= 5 \times 20$

$= 100$

19. Let a be the first term and d be the common difference.

$a = 1$

$d = 4$

$S_n = 1326$

$\Rightarrow \frac{n}{2}\left[2 \times 1 + (n-1)(4)\right] = 1326$

$\Rightarrow n(2 + 4n - 4) = 1326 \times 2$

$\Rightarrow n(-2 + 4n) = 2652$

$\Rightarrow -2n + 4n^2 = 2652$

$\Rightarrow 4n^2 - 2n - 2652 = 0$

$\Rightarrow 2(2n^2 - n - 1326) = 0$

$$\begin{array}{r} 25 \\ \times 4 \\ \hline 100 \end{array}$$

$\Rightarrow 2n^2 - n - 1326 = 0$

$\Rightarrow 2n^2 - n(52-51) - 1326 = 0$

$\Rightarrow 2n^2 - 52n + 51n - 1326 = 0$

$\Rightarrow 2n(n-26) + 51(n-26) = 0$

$\Rightarrow (2n+51)(n-26) = 0$

$2n+51=0$	$n-26=0$
$2n=-51$	$n=26$
$n=\frac{-51}{2}$	

Neglecting, $n = \frac{-51}{2}$

$\therefore n = 26$

$a_n = x$

$\Rightarrow a + (26-1)(4) = x$

$\Rightarrow a + 25 \times 4 = x$

$\Rightarrow a + 100 = x$

$\Rightarrow 1 + 100 = x$

$\Rightarrow x = 101$

20.8.21
SAME

Let a be the first term and d be the common difference.

$S_7 = 182$

$\frac{7}{2}[2a+6d] = 182$

$\Rightarrow 2a + 6d = \cancel{182}^{26} \times \frac{2}{\cancel{7}_1}$

$\Rightarrow 2(a+3d) = 52$

$\Rightarrow a + 3d = 26$

$\Rightarrow a_4 = 26$ —— (1)

$a_{17} = a + 16d$ —— (2)

Given,

$\frac{a_4}{a_{17}} = \frac{1}{5}$

$\frac{a+3d}{a+16d} = \frac{1}{5}$

$\frac{26}{a+16d} = \frac{1}{5}$

$$\begin{array}{r} a + 16d = 130 \\ a + 3d = 26 \\ - \quad\quad - \\ \hline 13d = 104 \end{array}$$

$d = 104/13 = 8$

Put $d = 8$ in (1) -

$a + 3 \times 8 = 26$

$a + 24 = 26$

$a = 2$

$\therefore$ A.P. is :-

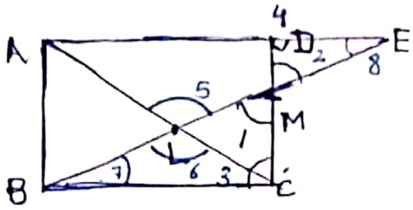
$a, a+d, a+2d, \ldots$

$\Rightarrow 2, 2+8, 2+2\times 8, \ldots$

$\Rightarrow 2, 10, 2+16, \ldots$

$\Rightarrow 2, 10, 18, \ldots$

22. Given : ||gm ABCD, in which
M is mid pt. of side CD of ||gm ABCD.
To prove :- $EL = 2BL$
Proof :- In ΔBMC and ΔEMD -



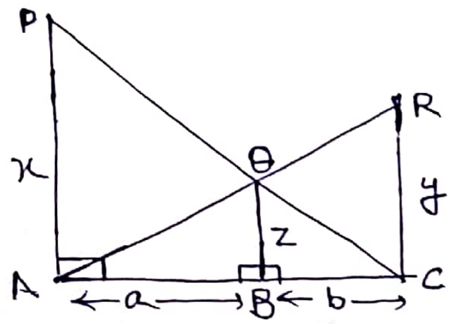
$\angle 1 = \angle 2$ (V.O.∠s)
$CM = DM$ (given)
$\angle 3 = \angle 4$ (Alt. int ∠s)
By ASA congruence rule.
$\Delta BMC \cong \Delta EMD$.
$\therefore BC = DE$ (CPCT) —— ①
~~Similar In Δ~~
$AE = AD + DE = BC + BC = 2BC$ ($\because AD = BC, DE = BC$) —— ②
In ΔBLC and ΔELA -
$\angle 6 = \angle 5$ (V.O.∠s)
$\angle 7 = \angle 8$ (Alt. int ∠s)
By AA similarity criterion.
$\Delta BLC \sim \Delta ELA$

$$\frac{BL}{EL} = \frac{BC}{EA} \text{ (CPST)}$$

$$\frac{BL}{EL} = \frac{\cancel{BC}}{2\cancel{BC}}$$

$2BL = EL$
Hence proved

23. $PA \perp AC$ and $QB \perp AC$
$\angle PAC = 90°$ and $\angle QBC = 90°$
In ΔQBC and ΔPAC -



$\angle QBC = \angle PAC = 90°$
$\angle QCB = \angle PCA$ (Common)
$\Delta QBC \sim \Delta PAC$
By AA similarity criterion

$$\frac{QB}{PA} = \frac{BC}{AC}$$

Let $AB = a$ and $BC = b$

$$\frac{z}{x} = \frac{b}{a+b} \text{ —— ①}$$

Similarly, we have, $\Delta QAB \sim \Delta RAC$.

$$\therefore \frac{QB}{RC} = \frac{AB}{AC} \Rightarrow \frac{z}{y} = \frac{a}{a+b} \text{ —— ②}$$

Add ① & ② -

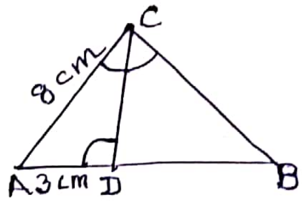
$$\frac{z}{x} + \frac{z}{y} = \frac{b}{a+b} + \frac{a}{a+b}$$

$$\Rightarrow z\left(\frac{1}{x}+\frac{1}{y}\right) = \frac{a+b}{a+b} = 1$$

Hence, $\frac{1}{x}+\frac{1}{y} = \frac{1}{z}$

Hence proved

24. In ΔACB and ΔADC -



$\angle A = \angle A$ (common)

$\angle BCA = \angle ADC$ (given)

By AA similarity criterion.

$\Delta ACB \sim \Delta ADC$.

$$\frac{AB}{AC} = \frac{AC}{AD}$$

$$\& \ \frac{AB}{8} = \frac{8}{3}$$

$$3AB = 64$$

$$AB = \frac{64}{3}$$

$$BD = AB - AD$$

$$= \frac{64}{3} - 3$$

$$= \frac{64-9}{3}$$

$$= \frac{55}{3} = 18\frac{1}{3} \text{ cm.}$$

25. Distance of pt. $P(x,y)$ from origin $(0,0)$ is $=\sqrt{(0-x)^2+(0-y)^2}$

$$=\sqrt{x^2+y^2}$$

26. $PQ = 10$ units

$$PQ^2 = 100$$

$$(x-9)^2+(4-10)^2 = 100$$

$$x^2+81-18x+36 = 100$$

$$x^2-18x+117 = 100$$

$$x^2-18x+117-100 = 0$$

$$x^2-18x+17 = 0$$

$$x^2-(17+1)x+17 = 0$$

$$x^2-17x-x+17 = 0$$

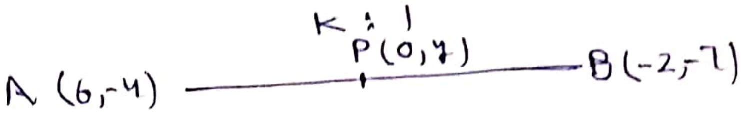
$$x(x-17)-1(x-17) = 0$$

$x-17=0$ | $x-1=0$

$x=17$ | $x=1$

$\therefore$ The values of x are- 1, 17.

27. Let P(0, y) divides line segment AB in the ratio K:1.



By using section formula-

$$(0,y) = \left(\frac{-2K+6}{K+1}, \frac{-7K-4}{K+1}\right)$$

Equating x-coordinate:-

$$\frac{-2K+6}{K+1} = 0$$

$$-2K+6=0$$

$$-2K=-6$$

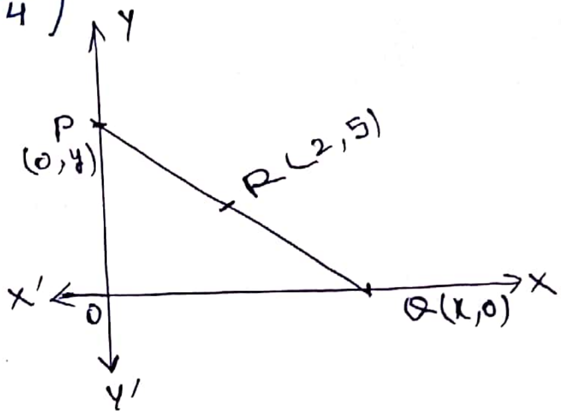
$$K=3$$

$\therefore$ Pt. of intersection is $\left(0, \frac{-7\times 3-4}{3+1}\right)$

$$= \left(0, \frac{-21-4}{4}\right)$$

$$= \left(0, \frac{-25}{4}\right)$$

28. Let P(0, y) and Q(x, 0). R is mid pt. of P and Q



$$(2,5) = \left(\frac{0+x}{2}, \frac{y+0}{2}\right)$$

$$(2,5) = \left(\frac{x}{2}, \frac{y}{2}\right)$$

Equating both axes coordinate

$\frac{x}{2} = 2$	$\frac{y}{2} = 5$
$x = 4$	$y = 10.$

The coordinates of P and Q are -
P(0,10), Q(4,0).

29. $\sin A - \cos A = 0$

$\Rightarrow \sin A = \cos A$

$A = 45°$

$\sin^4 A + \cos^4 A$

$\Rightarrow (\sin 45°)^4 + (\cos 45°)^4$

$$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4$$

$$= \frac{1}{4} + \frac{1}{4} = \frac{\cancel{2}^{1}}{\cancel{4}_{2}} = \frac{1}{2}$$

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30. $\sec M + \tan M = p$ (given)

we are to prove that -

$\sin M = (p^2-1)/(p^2+1)$

RHS. $\dfrac{p^2-1}{p^2+1}$

$$= \frac{(\sec M+\tan M)^2-1}{(\sec M+\tan M)^2+1}$$

$$= \frac{\sec^2 M+\tan^2 M+2\sec M\tan M-1}{\sec^2 M+\tan^2 M+2\sec M\tan M+1}$$

$$= \frac{(\sec^2 M-1)+\tan^2 M+2\sec M\tan M}{\sec^2 M+\tan^2 M+2\sec M\tan M+1}$$

$$= \frac{\tan^2 M+\tan^2 M+2\sec M\tan M}{1+\tan^2 M+\tan^2 M+2\sec M\tan M+1}$$

$$= \frac{2\tan^2 M+2\sec M\tan M}{(1+\tan^2 M)+(1+\tan^2 M)+2\sec M\tan M}$$

$$= \frac{2\tan M(\tan M+\sec M)}{\sec^2 M+\sec^2 M+2\sec M\tan M}$$

$$= \frac{2\tan M(\tan M+\sec M)}{2\sec^2 M+2\sec M\tan M}$$

$$= \frac{2\tan M(\cancel{\tan M+\sec M})}{2\sec M(\cancel{\sec M+\tan M})}$$

$$= \frac{\cancel{2}\tan M}{\cancel{2}\sec M}$$

$$= \frac{\frac{\sin M}{\cos M}}{\frac{1}{\cos M}}$$

$$= \frac{\sin M}{\cancel{\cos M}} \times \frac{\cancel{\cos M}}{1}$$

$= \sin M$

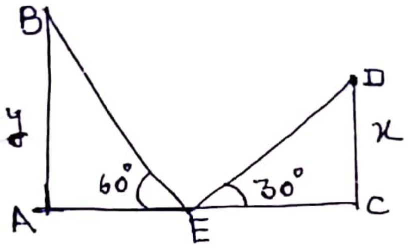
$=$ ~~RH~~ LHS

LHS = RHS

Hence proved

~~32~~.

31. Let AB and CD be two towers and E be the mid pt. of AC.



$AB = y$

$CD = x$

In rt.∠d △BAE -

$$\frac{AB}{AE} = \tan 60°$$

$$\frac{y}{AE} = \sqrt{3}$$

$$\sqrt{3}\,AE = y$$

$$AE = \frac{y}{\sqrt{3}} \quad \text{—— (1)}$$

In rt.∠d △DCE -

$$\frac{CD}{CE} = \tan 30°$$

$$\frac{x}{CE} = \frac{1}{\sqrt{3}}$$

$$CE = \sqrt{3}\,x \quad \text{—— (2).}$$

Since CE and AE are equal.

∴ Equating (1) & (2) -

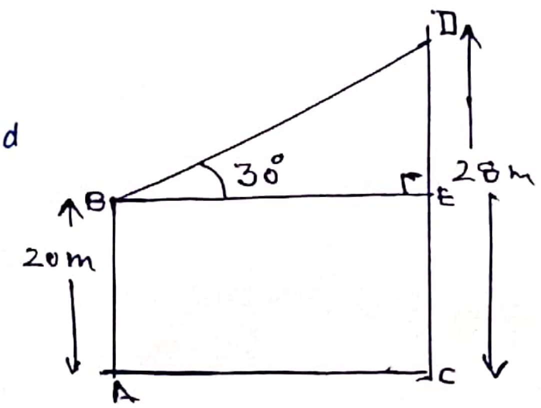
$$\sqrt{3}\,x = \frac{y}{\sqrt{3}}$$

$$3x = y$$

$$\frac{x}{y} = \frac{1}{3}$$

$$x : y = 1 : 3$$

32. Let AB be a pole of height 20m and CD be the pole of height 28m and both poles are tied with wire BD.



$AB = CE = 20m$

$DE = CD - CE = (28 - 20)\,m = 8m$

In rt.∠d △DEB -

$$\frac{DE}{BD} = \sin 30°$$

$$\frac{8}{BD} = \frac{1}{2}$$

$$BD = 16\,m$$

∴ Length of wire = 16m

In rt.∠d △DEB —

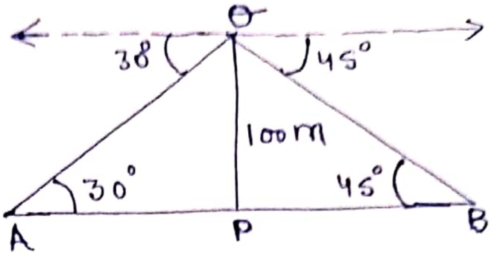
$$\frac{DE}{BE} = \tan 30°$$

$$\frac{8}{BE} = \frac{1}{\sqrt{3}}$$

$$BE = 8\sqrt{3}\,m.$$

∴ Distance b/w two poles = $8\sqrt{3}\,m$

33. Let PQ be the tower and A and B be the two positions of the car.



In rt.∠d ΔQPB -

$$\frac{QP}{BP} = \tan 45°$$

$$\frac{100}{BP} = 1$$

$$BP = 100\,m$$

Now, in rt.∠d ΔQPA -

$$\frac{QP}{AP} = \tan 30°$$

$$\frac{100}{AP} = \frac{1}{\sqrt{3}}$$

$$AP = 100\sqrt{3}\ m$$

∴ Distance b/w the cars = $AP + BP$

$$= 100\sqrt{3} + 100$$

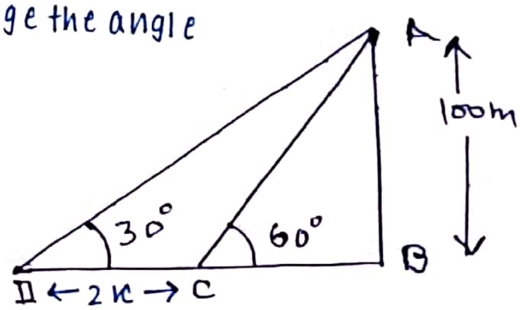
$$= 100(\sqrt{3}+1)m.$$

$$= 100(1.732+1)$$

$$= 100 \times 2.732$$

$$= 273.2\ m.$$

34. Let the speed of boat be x m per minute. And CD is the distance which man travelled to change the angle of elevation.



Therefore,

$CD = 2x$ [Distance = Speed x Time]

In rt.∠d ΔABC -

$$\frac{AB}{BC} = \tan 60°$$

$$\frac{100}{BC} = \sqrt{3}$$

$$BC = \frac{100}{\sqrt{3}}$$

$$CD = BD - BC.$$

$$2x = 100\sqrt{3} - \frac{100}{\sqrt{3}}$$

$$2x = \frac{300 - 100}{\sqrt{3}}$$

$$x = \frac{200}{2\sqrt{3}}$$

$$x = \frac{100}{\sqrt{3}}$$

$$x = \frac{100}{1.73} = 57.80$$

Hence, speed of boat is 57.80 m/per min.

35. Const:- Join OP

Proof:- In ΔOAP and ΔOBP,

$OA = OB$ (Radii of circle)

$OP = OP$ (Common)

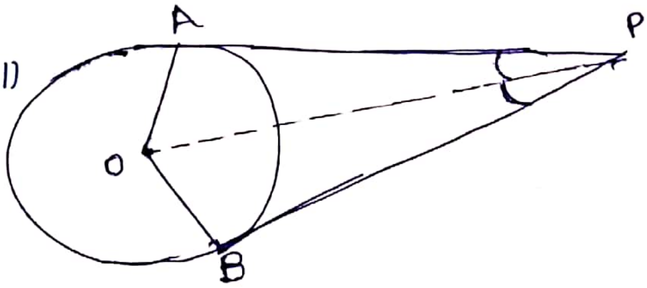
$PA = PB$ (tangents are equal)

By SSS congruence rule.

$\Delta OAP \cong \Delta OBP$.

$\therefore \angle APO = \angle BPO$ (CPCT).

Hence, OP bisects $\angle APB$.

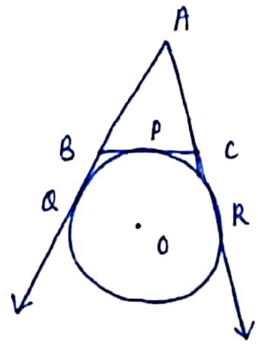


36. We know that tangents drawn from an external pt. to a circle are equal.

$\therefore BP = BQ$ (tangents from pt. B) —— (i)

$CP = CR$ (tangents from pt. C) —— (ii)

$AQ = AR$ (tangents from pt. A) —— (iii)



Now, Perimeter of $\Delta ABC = AB + BC + CA$

$= AB + BP + PC + AC$

$= AB + BQ + CR + AC$ [Using (i) and (ii)]

$= AQ + AR = 2AQ$ [Using (iii)]

$AQ = \frac{1}{2}$ (Perimeter of ΔABC)

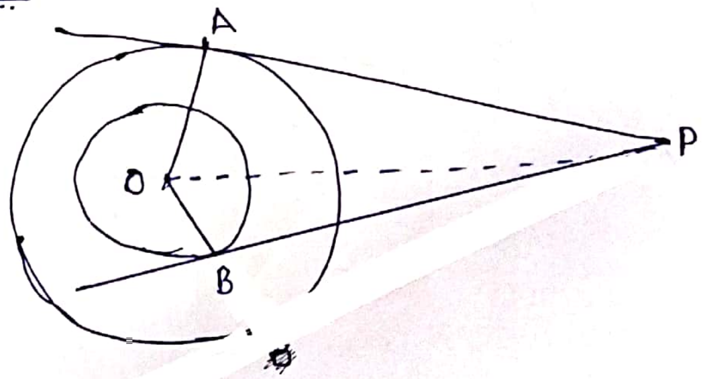
Hence proved

37. Const:- Join OP

We have,

$OA \perp AP$ and $OB \perp BP$

(tangents at any pt. to a circle is $\perp$ar to the radius through the pt. of contact)



In rt. $\angle$d ΔOAP, we get.

$OP^2 = OA^2 + AP^2$

$OP = \sqrt{OA^2 + AP^2} = \sqrt{8^2 + 15^2}$ cm

$= \sqrt{289} = 17$ cm

In rt. $\angle$d ΔOBP, we get -

$OP^2 = OB^2 + BP^2$

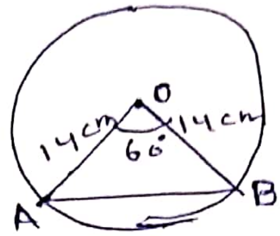
$BP = \sqrt{OP^2 - OB^2} = \sqrt{17^2 - 5^2}$ cm $= \sqrt{264}$ cm

Thus, length of $BP = \sqrt{264} = 16.25$ cm (Approx.)

38. Radius = 14 cm (OA = OB)

$\theta = 60°$

In ΔAOB, $\angle A = \angle B$ (Angles opp. to equal sides)



$\therefore \angle A = \angle B = \angle O = 60°$

$\therefore \Delta ABO$ is equilateral Δ.

$\Rightarrow$ Area of minor segment = Area of sector OABO − Area of ΔAOB

$$= \frac{\theta}{360} \times \pi r^2 - \frac{\sqrt{3}}{4} \times (\text{side})^2$$

$$= \frac{6\cancel{0}}{36\cancel{0}} \times \frac{22}{7} \times 14 \times 14 - \frac{\sqrt{3}}{4} \times (14)^2$$

$$= (102.66 - 84.87)\ cm^2$$

$$= 17.79\ cm^2$$

Area of major segment = Area of circle − Area of minor segment

$$= \pi r^2 - 17.79$$

$$= \frac{22}{\cancel{7}} \times \cancel{14}^{2} \times 14 - 17.79$$

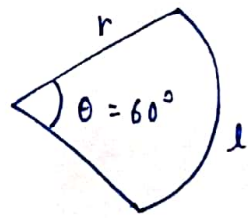
$$= 44 \times 14 - 17.79$$

$$= 616 - 17.79$$

$$= 598.21\ cm^2$$

39. Length (l) = 22 cm

$\theta = 60°$



Length of arc $= \frac{\theta}{360} \times 2\pi r$

$$22 = \frac{\cancel{60}^{1}}{\cancel{360}_{\cancel{6}\ 3}} \times \cancel{2}^{1} \times \frac{22}{7} \times r$$

$$22 = \frac{1}{6} \times 2 \times \frac{22}{7} \times r$$

$$r = \cancel{22} \times \cancel{6}^{3} \times 7 / \cancel{22} \times \cancel{2}_{1}$$

$r = 21$ cm.

40. $r = 6$ cm

Given time :- 7:20 a.m. to 7:55 a.m.

$= 35$ min $= \frac{35}{60}$ hrs.

In 1 hr, the hour hand rotates by $30°$

Thus, central angle $(\theta) = 30 \times \frac{35}{60}$

$= 17.5°$

41. $l = 2$ km $= 2 \times 1000$ m $= 2000$ m

$b = 40$ m

$h = 3$ m

Vol. of water flowing into the sea in one hour = vol. of cuboid.

$l \times b \times h = (2000 \times 40 \times 3)$ m^3

$= 240000$ m^3

vol. of water flowing into sea in one minute

$= \frac{240000}{60}$ $m^3 = 4000 m^3$.

∴ vol. of water flowing into sea in two minutes

$= 4000 \times 2$

$= 8000 m^3$.

42. Let h be the height of the cone and r be the radius of base of the cone.

The vol. of wooden toy $= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{1}{3}\pi r^2 (h + 2r)$

$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 (h + 7)$

$= \frac{77}{6}(h + 7)$

Acc. to question,

$\frac{77}{6}(h+7) = 166$

$\frac{77}{6}(h+7) = \frac{1001}{6}$

$h = 6$

The height of wooden toy $= 6 + 3.5$ cm $= 9.5$ cm

CSA of hemispherical part $= 2 \times \frac{22}{7} \times 3.5 \times 3.5 = 77$ cm^2

Hence, cost of painting the hemispherical part of the toy $= 77 \times 10 = ₹770$.

43. Radius of sphere, $r = 18/2 = 9$ cm

Radius of cylindrical vessel

$R = 36/2 = 18$ cm

Let the rise of water level be H.

Now,

volume of rise in water = Vol. of sphere

$$\pi R^2 h = \frac{4}{3}\pi H^3$$

$$h = \frac{4\pi H^3}{3R^2}$$

$$h = \frac{4 \times 9 \times 9 \times 9}{3 \times 18 \times 18}$$

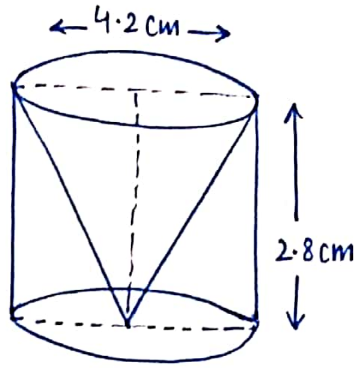
$$h = 3 \text{ cm.}$$

44. Height of cone = height of cylinder

$h = 2.8$ cm

Radius of base, $r = 4.2/2 = 2.1$ cm

Slant height of cone, $l = \sqrt{r^2 + h^2}$



$$\sqrt{(2.1)^2 + (2.8)^2}$$

$$= \sqrt{4.41 + 7.84}$$

$$= \sqrt{12.25}$$

$$= 3.5 \text{ cm}$$

Now, the TSA of remaining solid = CSA of cylinder + CSA of cone + Area of base

$$= 2\pi Hh + \pi rl + \pi r^2$$

$$= \pi H (2h + l + r)$$

$$= \frac{22}{7} \times 2.1 \times (2 \times 2.8 \times 3.5 + 2.1)$$

$$= 22 \times 0.3 \times (5.6 + 5.6)$$

$$= 6.6 \times 11.2$$

$$= 73.92 \text{ cm}^2$$

So, the TSA of remaining solid = 73.92 cm².

45.

Class	Frequency	x_i	$d_i = x_i - a$	$f_i d_i$
0-10	4	5	-30	-120
10-20	4	15	-20	-80
20-30	7	25	-10	-70
30-40	10	35-a	0	0
40-50	12	45	10	120
50-60	8	55	20	160
60-70	5	65	30	150
	$\Sigma f_i = 50$			$\Sigma f_i d_i = 160$

$$\bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

$$= 35 + \frac{160}{50}$$

$$= \frac{1750 + 160}{50}$$

$$= \frac{1910}{50}$$

$$= 38.2$$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 40 + \left(\frac{12 - 10}{2 \times 12 - 10 - 8}\right) \times 10$$

$$= 40 + \frac{2}{24 - 10 - 8} \times 10$$

$$= 40 + \frac{2}{\cancel{6}\ 3} \times \cancel{10}\ 5$$

$$= 40 + \frac{10}{3}$$

$$= \frac{120 + 10}{3}$$

$$= \frac{130}{3}$$

$$= 43.3$$

Class	Frequency (fi)	Cumulative Frequency (cf)
0 - 10	4	4
10 - 20	4	8
20 - 30	7	15 — cf
30 - 40	10 — f	25
40 - 50	12	37
50 - 60	8	45
60 - 70	5	50

$n = 50$

$\frac{n}{2} = \frac{\cancel{50}^{25}}{\cancel{2}_{1}} = 25$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 30 + \left(\frac{25 - 15}{10}\right) \times 10$$

$$= 30 + \left(\frac{10}{\cancel{10}}\right) \times \cancel{10}$$

$$= 40.$$

46.

Class	Frequency (fi)	Cumulative Frequency (cf)
0 - 10	10	10
10 - 20	x	10 + x
20 - 30	25	35 + x
30 - 40	30	65 + x
40 - 50	y	65 + x + y
50 - 60	10	75 + x + y
	100	

$75 + x + y = 100$

$x + y = 25$

$f = 30,\ h = 30,\ cf = 35 + x,\ \frac{n}{2} = 50$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$32 = \left(\frac{50 - 35 - x}{30}\right) \times 10$$

$$32 - 30 = \frac{15 - x}{3}$$

$6 = 15 - x$

$x = 9$

$\therefore y = 16$

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47. Given that mean = 14 and median = 15
we know that mode = 3Median − 2Mean

$$\text{Mode} = 3(15) - 2(14) = 45 - 28 = 17.$$

48. Given, no. of red ball = 5
No. of green ball = n
$\therefore$ Total balls = $n + 5$
Now $P(\text{red ball}) = \frac{5}{n+5}$

and $P(\text{green ball}) = \frac{n}{n+5}$

Acc. to Question,

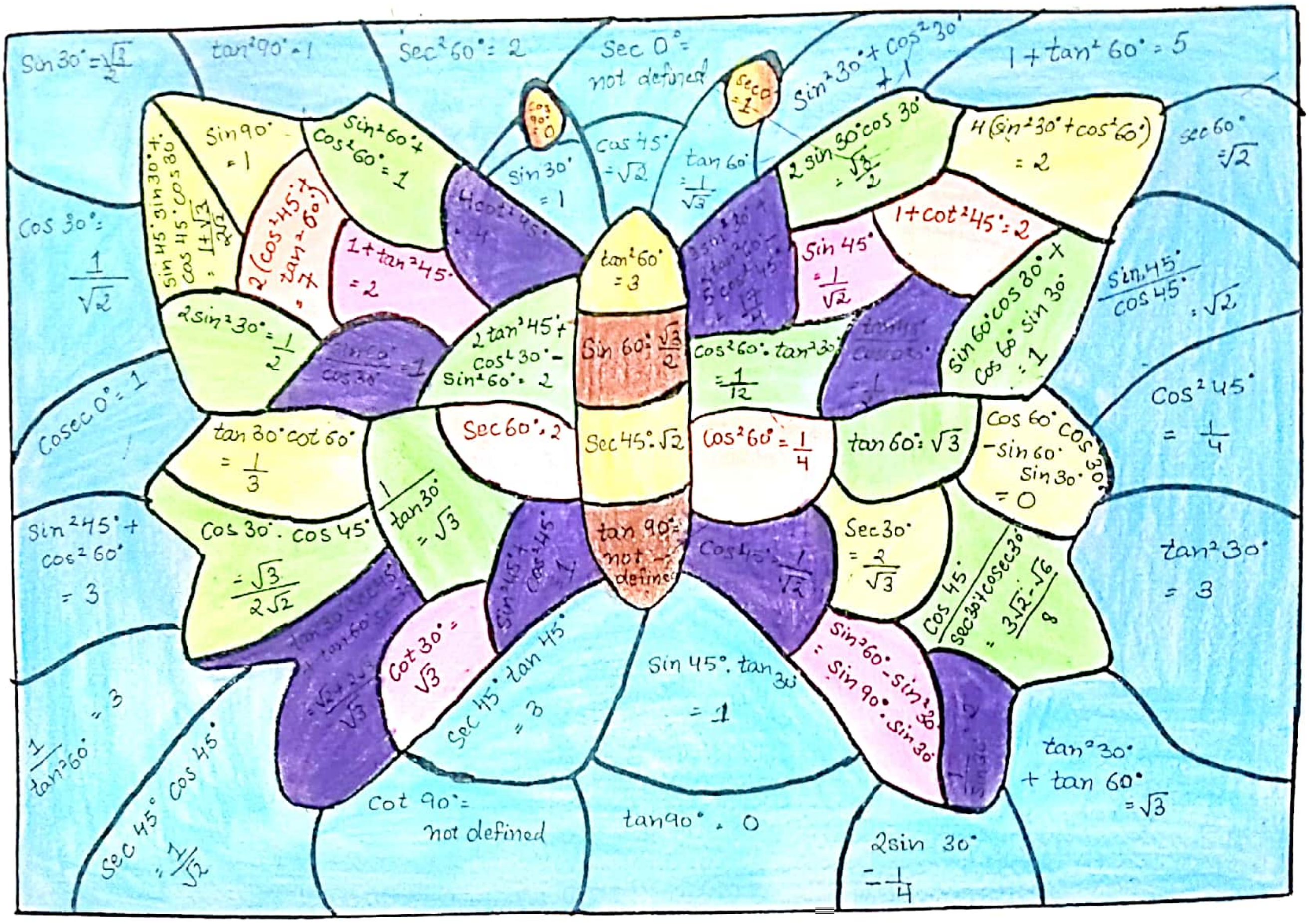
$$\frac{n}{n+5} = \frac{3 \times 5}{n+5}$$

$$n = 15$$

So, no. of green balls = 15.

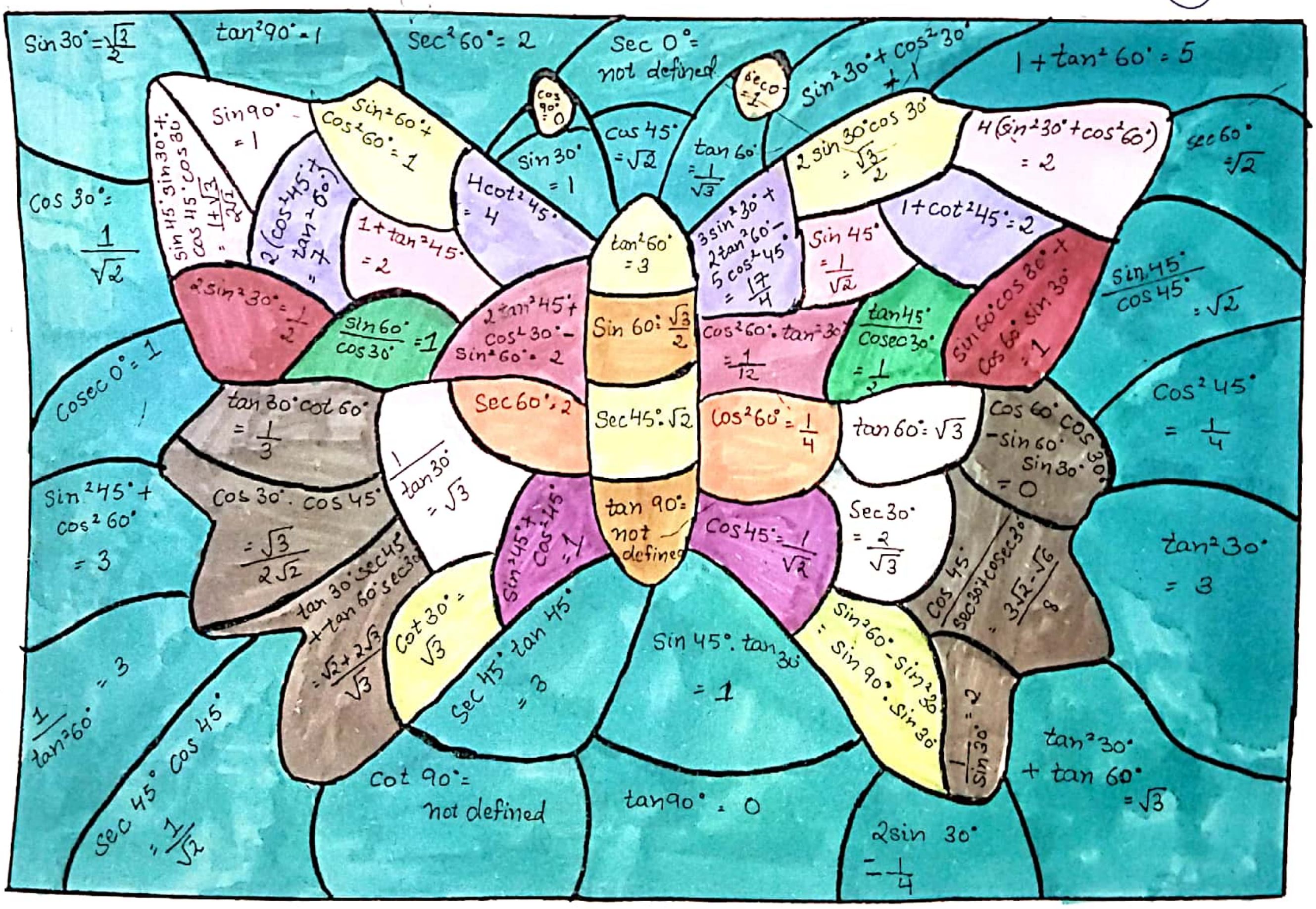
49. $P(\text{multiple of } 3 \text{ and } 4) = \frac{1}{15}.$

50. Perfect square no.'s from 1 to 49 = 1, 4, 9, 16, 25, 36, 49.
No. of favourable outcomes = 7
Total. no. of outcomes = 49.
$\therefore P(\text{perfect square no.}) = \frac{7}{49} = \frac{1}{7}.$



Sin 30° = √3/2
tan²90° = 1
Sec²60° = 2
Sec 0° = not defined
Sin²30° + Cos²30° + 1
1 + tan²60° = 5
Cos 30° = 1/√2
Sin 90° = 1
Sin²60° + Cos²60° = 1
Cos 90° = 0
Sec 0° = 1
Sin 30° = 1
Cos 45° = √2
tan 60° = 1/√3
2 sin 30° cos 30° = √3/2
4(Sin²30° + cos²60°) = 2
sec 60° = √2
Sin 45° Sin 30° + Cos 45° Cos 30° = (1+√3)/(2√2)
2(Cos²45° + tan²60°) = 7
4cot²45° = 4
1 + tan²45° = 2
tan²60° = 3
3sin²30° + 2tan²60° − 5cos²45° = 17/4
Sin 45° = 1/√2
1 + cot²45° = 2
Sin 45°/Cos 45° = √2
2sin²30° = 1/2
Sin 60°/Cos 30° = 1
2tan²45° + Cos²30° − Sin²60° = 2
Sin 60° = √3/2
Cos²60°. tan²30° = 1/12
tan 45°/Cosec 30° = 1/2
Sin 60° cos 30° + Cos 60° Sin 30° = 1
Cosec 0° = 1
Cos²45° = 1/4
tan 30° cot 60° = 1/3
Sec 60° = 2
Sec 45° = √2
Cos²60° = 1/4
tan 60° = √3
Cos 60° Cos 30° − Sin 60° Sin 30° = 0
Sin²45° + cos²60° = 3
Cos 30°. Cos 45° = √3/(2√2)
1/tan 30° = √3
Sin²45° + Cos²45° = 1
tan 90° = not defined
Cos 45° = 1/√2
Sec 30° = 2/√3
Cos 45°/(sec 30° + cosec 30°) = (3√2 − √6)/8
tan²30° = 3
tan 30° sec 45° + tan 60° sec 30° = (√2 + 2√3)/√3
Cot 30° = √3
Sec 45° tan 45° = 3
Sin 45°. tan 30° = 1
Sin²60° − Sin²30° = Sin 90°. Sin 30°
1/Sin 30° = 2
1/tan²60° = 3
tan²30° + tan 60° = √3
Sec 45° Cos 45° = 1/√2
Cot 90° = not defined
tan 90° = 0
2sin 30° = 1/4

25



Sin 30° = √3/2
tan²90° = 1
Sec²60° = 2
Sec 0° = not defined
Sin²30° + Cos²30° + 1
1 + tan²60° = 5
Cos 30° = 1/√2
Sin 90° = 1
Sin²60° + Cos²60° = 1
Cos 90° = 0
Sec 0° = 1
Sin 30° = 1
Cos 45° = √2
tan 60° = 1/√3
2 sin 30° cos 30° = √3/2
4(Sin²30° + cos²60°) = 2
sec 60° = √2
Sin 45° Sin 30° + Cos 45° Cos 30° = (1+√3)/(2√2)
2(Cos²45° + tan²60°) = 7
4cot²45° = 4
1 + tan²45° = 2
tan²60° = 3
3sin²30° + 2tan²60° − 5cos²45° = 17/4
Sin 45° = 1/√2
1 + cot²45° = 2
Sin 45°/Cos 45° = √2
2sin²30° = 1/2
Sin 60°/Cos 30° = 1
2tan²45° + Cos²30° − Sin²60° = 2
Sin 60° = √3/2
Cos²60°. tan²30° = 1/12
tan 45°/Cosec 30° = 1/2
Sin 60° cos 30° + Cos 60° Sin 30° = 1
Cosec 0° = 1
Cos²45° = 1/4
tan 30° cot 60° = 1/3
Sec 60° = 2
Sec 45° = √2
Cos²60° = 1/4
tan 60° = √3
Cos 60° Cos 30° − Sin 60° Sin 30° = 0
Sin²45° + cos²60° = 3
Cos 30°. Cos 45° = √3/(2√2)
1/tan 30° = √3
Sin²45° + Cos²45° = 1
tan 90° = not defined
Cos 45° = 1/√2
Sec 30° = 2/√3
Cos 45°/(sec 30° + cosec 30°) = (3√2 − √6)/8
tan²30° = 3
tan 30° sec 45° + tan 60° sec 30° = (√2 + 2√3)/√3
Cot 30° = √3
Sec 45° tan 45° = 3
Sin 45°. tan 30° = 1
Sin²60° − Sin²30° = Sin 90°. Sin 30°
1/Sin 30° = 2
1/tan²60° = 3
tan²30° + tan 60° = √3
Sec 45° Cos 45° = 1/√2
Cot 90° = not defined
tan 90° = 0
2sin 30° = 1/4