Date ______
Page ______

classmate
Date 20/3/24
Page

Chapter-1
Real Numbers

☆ Real Numbers - Combination of rational no.s and irrational no.s is called real no.s
eg:- 2, 3, 5, $\frac{1}{3}$, π, $\sqrt{3}$ etc.

☆ Fundamental Theorem of Arithmetic: Every composite no. can be expressed as product of primes, and this factorisation is unique

☆ Product of 2 no.s = LCM × HCF

Ex-1.1

Q1:
(i) 140

$140 = 2 \times 2 \times 5 \times 7$
$= 2^2 \times 5 \times 7$

2	140
2	70
5	35
7	7
	1

(IV) 5005

$5005 = 5 \times 7 \times 11 \times 13$

5	5005
7	1001
11	143
13	13
	1

(v) 7429

$7429 = 17 \times 19 \times 23$

17	7429
19	437
23	23
	1

(ii) 156

$156 = 2 \times 2 \times 3 \times 23$

2	156
2	78
3	69
23	23
	1

(iii) 3825

$3825 = 5 \times 5 \times 3 \times 3 \times 17$

5	3825
5	765
3	153
3	51
17	17
	1

Q2:

(iii) 336 and 54

$54 = 2 \times 3^3$

$336 = 2^4 \times 3 \times 7$

2	54
3	27
3	9
3	3
	1

2	336
2	168
2	84
2	42
3	21
7	7
	1

$LCM(54, 336) = 2^4 \times 3^3 \times 7^1 = 3024$

$HCF(54, 336) = 2^1 \times 3^1 = 6$

$LCM \times HCF = 3024 \times 6$

$= 18144$

Product of no. $= 336 \times 54$

$= 18144$

$LCM \times HCF =$ Product of no.s

Hence verified

(i) 26 & 91

$26 = 2 \times 13$

$91 = 7 \times 13$

$LCM(26, 91) = 2 \times 7 \times 13$

$= 182$

$HCF(26, 91) = 13$

$LCM \times HCF = 182 \times 13$

$= 2366$

Product of no. $= 26 \times 91$

$= 2366$

$LCM \times HCF =$ Product of no.s

Hence verified

(ii) 510 & 92

$92 = 2 \times 2 \times 23$

$= 2^2 \times 23$

$510 = 2 \times 5 \times 3 \times 17$

2	92
2	46
23	23
	1

2	510
5	255
3	51
17	17
	1

$LCM(510, 92) = 2^2 \times 23 \times 17 \times 5 \times 3 = 23460$

$HCF(510, 92) = 2$

$LCM \times HCF = 23460 \times 2 = 46920$

Product of no.s $= 510 \times 92$

$= 46920$

$LCM \times HCF =$ Product of no.s

Hence verified.

Q3:

(i) 12, 15, 21

$12 = 2 \times 2 \times 3 \times 2 \times 2 \times 3 = 2^2 \times 3^2$

$15 = 3 \times 5$

$21 = 7 \times 3$

$LCM = 2^2 \times 3^2 \times 5 \times 7 = 420$

$HCF(12, 15, 21) = 3$

(ii) 17, 23, 29

$17 = 17 \times 1$

$23 = 23 \times 1$

$29 = 29 \times 1$

$LCM(17, 23, 29) = 17 \times 23 \times 29 = 11339$

$HCF(17, 23, 29) = 1$

(iii) 8, 9, 25

$8 = 2^3$

$9 = 3^2$

$25 = 5^2$

$LCM(8, 9, 25) = 2^3 \times 3^2 \times 5^2 = 1800$

$HCF(8, 9, 25) = 1$

Q4:

$HCF(306, 657) = 9$

$LCM(306, 657) = \frac{306 \times 657}{HCF(306, 657)}$

$= \frac{\cancel{306}^{34} \times 657}{\cancel{9}_{1}}$

$= 34 \times 657$

$= 22338$

$$\begin{array}{r} 657 \\ \times 34 \\ \hline 2628 \\ 1971 \times \\ \hline 22338 \end{array}$$

Q5: If any no. ends with the digit 0, it should be divisible by 10. So it will also be divisible by 2 & 5 because $10 = 2 \times 5$.

But prime factorisation of $6^n = (2 \times 3)^n$, where 5 is not a prime factor of 6^n.

By the uniqueness of fundamental theorem of arithametic, there is no other prime factorisation of 6^n possible.

Hence, 6 cannot end with the digit 0 for any natural no.

Q6: (i) $7 \times 11 \times 13 + 13$

$= 13(7 \times 11 \times 1 + 1)$

$= 13(77 + 1)$

$= 13 \times 78$

$= 13 \times 2 \times 3 \times 13$

Since, it has more than two factors so it is composite.

(ii) $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$

$5(7 \times 6 \times \cancel{5} \times 4 \times 3 \times 2 \times 1 + 1)$

$5(1008 + 1)$

5×1009

Since, it has more than two factors so it is composite.

Q7: Time taken by Sonia to complete 1 round = 18 min.

Time taken by Ravi to complete 1 round = 12 min.

Since, both start at the same point and at the same time to go in same direction.

So, the time after they meet again at the starting point = LCM(12, 18)

$18 = 2 \times 3^2$
$12 = 2^2 \times 3$
$LCM(18,12) = 2^2 \times 3^2 = 36$ min

Ex-1.2

1. $\sqrt{5}$

Let us assume the contrary that $\sqrt{5}$ is rational no.

$\therefore \sqrt{5} = \frac{p}{q}$ where p & q are coprime integers $q \neq 0$

Squaring both sides

$(\sqrt{5})^2 = \left(\frac{p}{q}\right)^2$

$5 = \frac{p^2}{q^2}$

$q^2 = \frac{p^2}{5}$ — (1)

5 divides p^2 so it will divide p also

Let $p = 5m$

$q^2 = \frac{25m^2}{5}$

$q^2 = 5m^2$

$m^2 = \frac{q^2}{5}$

It means it divides q^2 so it will divide q also.

Since 5 divides q & p both but p & q are coprime so there is contradiction. Our assumption is wrong. Hence $\sqrt{5}$ is irrational.

$\sqrt{3}$

Let us assume that $\sqrt{3}$ is rational no.

$\therefore \sqrt{3} = \frac{p}{q}$, where p & q are coprime integers $q \neq 0$

Squaring both sides

$\Rightarrow (\sqrt{3})^2 = \left(\frac{p}{q}\right)^2$

$\Rightarrow 3 = \frac{p^2}{q^2}$

$\Rightarrow q^2 = \frac{p^2}{3}$ — (1)

3 divides p^2 so it will divide p also.

Let $p = 3m$

$q^2 = \frac{9m^2}{3}$

$m^2 = \frac{q^2}{3}$

It means it divides q^2 so it will divide q also

Since 3 divides p & q both p & q are co-prime so there is contradiction. Our assumption is wrong. Hence $\sqrt{3}$ is irrational.

2. Prove that $3 + 2\sqrt{5}$ is irrational

Let us assume that $3 + 2\sqrt{5}$ is rational no.

$\therefore 3 + 2\sqrt{5} = \frac{p}{q}$ where p & q are coprime integers, $q \neq 0$

$\Rightarrow 2\sqrt{5} = \frac{p}{q} - 3$

$\Rightarrow 2\sqrt{5} = \frac{p - 3q}{q}$

$\Rightarrow \sqrt{5} = \frac{p - 3q}{2q}$

Since $p, q, 2, 3$ are integers so $\frac{p-3q}{2q}$ is rational

& $\sqrt{5}$ is irrational

$\therefore$ irrational no. = rational no.

which is not possible so our assumption is wrong

hence $3+2\sqrt{5}$ is irrational.

Q 3: Prove :-

(i) $\frac{1}{\sqrt{2}}$

Let us assume that $\frac{1}{\sqrt{2}}$ is rational no.

$\frac{1}{\sqrt{2}} = \frac{p}{q}$ are co-prime integers, $q \neq 0$

$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{p}{q}$

$\Rightarrow \frac{\sqrt{2}}{2} = \frac{p}{q}$

$\Rightarrow \sqrt{2} = \frac{2p}{q}$

Hence $2, p, q$ are integers so $\frac{2p}{q}$ is rational no. but $\sqrt{2}$ is irrational

So here irrational no. = rational no.

which is not possible. So our assumption is wrong hence $\frac{1}{\sqrt{2}}$ is irrational.

(ii) $7\sqrt{5}$

Let us assume that $7\sqrt{5}$ is rational no.

$7\sqrt{5} = \frac{p}{q}$, p & q are co-prime integers, $q \neq 0$

$\Rightarrow 7\sqrt{5} = \frac{p}{q}$

$\Rightarrow \sqrt{5} = \frac{p}{7q}$

Hence $p, q, 7$ are rational no. but $\sqrt{5}$ is irrational

Here, Irrational no. = Rational no.

which is not possible. So our assumption is wrong

hence $\frac{p}{7q}$ $7\sqrt{5}$ is irrational.

(iii) $6+\sqrt{2}$

Let us assume that $6+\sqrt{2}$ is rational no.

$6+\sqrt{2} = \frac{p}{q}$, p & q are co-prime integers, $q \neq 0$

$6+\sqrt{2} = \frac{p}{q}$

$\sqrt{2} = \frac{p}{q} - 6$

$\sqrt{2} = \frac{p-6q}{q}$

Hence $p, q, 6$ are rational no. but $\sqrt{2}$ is irrational no.

Here, Irrational no. = Rational no.

which is not possible. So our assumption is wrong

hence $6+\sqrt{2}$ is irrational.

Chapter-2
Polynomials

Ex- 2.2

Q2:

(i) $S = \frac{1}{4}$

$P = -1$

req. Polynomial $= x^2 - Sx + P$

$x^2 - \frac{1}{4}x - 1$ (Multiply by 4)

or $4x^2 - x - 4$

(ii) $\sqrt{2}, \frac{1}{3}$

$S = \sqrt{2}$

$P = \frac{1}{3}$

$= x^2 - Sx + P$

$= x^2 - \sqrt{2}x + \frac{1}{3}$

$= 3x^2 - 3\sqrt{2}x + 1$

(iii) $0, \sqrt{5}$

$S = 0$

$P = \sqrt{5}$

$= x^2 - Sx + P$

$= x^2 - 0x + \sqrt{5}$

$= x^2 + \sqrt{5}$

(iv) 1, 1

$S = 1$

$P = 1$

$= x^2 - Sx + P$

$= x^2 - 1x + 1$

$= x^2 - 1x + 1$

(v) $\frac{-1}{4}, \frac{1}{4}$ $S = \frac{-1}{4}$, $P = \frac{1}{4}$

$= x^2 - Sx + P = x^2 - \left(\frac{-1}{4}\right)x + \frac{1}{4}$

$= x^2 + \frac{1}{4}x + \frac{1}{4}$ (x4)

$= 4x^2 + x + 1$

(vi) 4, 1

$S = 4, P = 1$

$= x^2 - Sx + P = x^2 - 4x + 1$

$= x^2 - 4x + 1$

Q1:

(i) $x^2 - 2x - 8$

Here, $a = 1, b = -2, c = -8$

$x^2 - 2x - 8 = 0$

$x^2 - 4x + 2x - 8 = 0$

$x(x-4) + 2(x-4) = 0$

$(x+2)(x-4) = 0$

$(x-4)(x+2) = 0$

Either Or

$x - 4 = 0$ $x + 2 = 0$

$x = 4$ $x = -2$

Verification

Sum of zeroes $= -2 + 4 = 2 = \frac{-b}{a} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of zeroes $= 4(-2) = -8 = \frac{c}{a} = \frac{\text{constant}}{\text{coffecient of } x^2}$

(ii) $4s^2 - 4s + 1$

Here $a = 4, b = -4, c = 1$

$= 4s^2 - 4s + 1 = 0$

$= 4s^2 - 2s - 2s + 1 = 0$

$= 2s(2s-1) - 1(2s-1) = 0$

$= 2s(2s-1) - 1(2s-1) = 0$

$= (2s-1)(2s-1) = 0$

either: $2s - 1 = 0$, $2s = 1$, $s = \frac{1}{2}$

Or: $2s - 1 = 0$, $s = \frac{1}{2}$

Verification

Sum of zeroes $= \frac{1}{2} + \frac{1}{2} = \frac{1+1}{2} = \frac{2}{2} = 1 = \frac{-(-4)}{4} = 1 = \frac{\text{Coffecient of } x}{\text{coffecient of } x^2}$

Product of zeroes $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{c}{a} = \frac{\text{constant}}{\text{coffecient of } x^2}$

(iii) $6x^2 - 3 - 7x = 6x^2 - 7x - 3$

Here $a = 6, b = -7, c = -3$

$= 6x^2 - 7x - 3 = 0$

$= 6x^2 - 9x + 2x - 3 = 0$

$= 3x(2x-3) + 1(2x-3) = 0$

$= (2x-3)(3x+1) = 0$

either: $2x - 3 = 0$, $x = \frac{3}{2}$

Or: $3x + 1 = 0$, $x = \frac{-1}{3}$

Verification

Sum of Zeroes $= \frac{3}{2} + \left(\frac{-1}{3}\right) = \frac{9-2}{6} = \frac{7}{6} = \frac{-b}{a} = \frac{-(-7)}{6} = \frac{\text{Coffecient of } x}{\text{Coffecient of } x^2}$

Product of Zeroes $= \frac{3}{2} \times \left(\frac{-1}{3}\right) = \frac{-3}{6} = \frac{c}{a} = \frac{-3}{6} = \frac{\text{Constant}}{\text{coffecient of } x^2}$

(IV) $4u^2 + 8u$

$4u(u+2) = 0$

either: $4u = 0$, $u = 0$

Or: $u + 2 = 0$, $u = -2$

Verification

Sum of zeroes $= 0 - 2 = -2$

Product of zeroes $= 0 \times (-2) = 0$

(V) $t^2 - 15$

$t^2 - 15 = 0$

$(t)^2 - (\sqrt{15})^2 = 0$

$(t + \sqrt{15})(t - \sqrt{15}) = 0$

Either: $t + \sqrt{15} = 0$, $t = -\sqrt{15}$

Or: $t - \sqrt{15} = 0$, $t = \sqrt{15}$

Verification

Sum of zeroes $= -\sqrt{15} + \sqrt{15} = 0$

Product of zeroes $= -\sqrt{15} \times \sqrt{15} = -15$

(VI) $3x^2 - x - 4$, $a = 3, b = -1, c = -4$

$3x^2 - 4x + 3x - 4 = 0$

$x(3x-4) + 1(3x-4) = 0$

$(3x-4)(x+1) = 0$

either $\qquad$ Or

$3x - 4 = 0 \qquad x + 1 = 0$

$3x = 4 \qquad x = -1$

$x = \frac{4}{3}$

Verification

Sum of zeroes $= \frac{4}{3} + \frac{(-1)}{1} = \frac{4-3}{3} = \frac{1}{3} = \frac{-b}{a} = \frac{\text{coffecient of } x}{\text{coffecient of } x^2}$

Product of zeroes $= \frac{4}{3} \times (-1) = \frac{-4}{3} = \frac{c}{a} = \frac{\text{constant}}{\text{coffecient of } x^2}$

Chapter-3

Pair of Linear Equation in 2 variables

☆ The general form of pair of linear equation in 2 variable

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

☆ If the pair of linear equations is given by $a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$ then the following situation can arise :-

(i) when $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

In this pair of linear equation the lines are :- Intersecting, Unique Solution, Consistent

(ii) when $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

In this pair of linear equation the lines are :- inconsistent, No solution, Parallel lines

(iii) when $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

In this pair of linear equations the lines are :- Dependent, consistent, infinite many solutions, coincident lines.

Q2: (i) $5x-4y+8=0$

$7x+6y-9=0$

$\frac{a_1}{a_2}=\frac{5}{7}$, $\frac{b_1}{b_2}=\frac{-4}{\cancel{6}}\frac{-2}{3}$

Since $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$

So the pair of linear eq. is intersecting

(ii) $9x+3y+12=0$

$18x+6y+24=0$

$\frac{a_1}{a_2}=\frac{9}{18}=\frac{1}{2}$, $\frac{b_1}{b_2}=\frac{3}{6}=\frac{1}{2}$, $\frac{c_1}{c_2}=\frac{\cancel{12}\,1}{\cancel{24}\,2}$

Since $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

So the pair of linear eq. is coincident

(iii) $6x-3y+10=0$

$2x-y+9=0$

$\frac{a_1}{a_2}=\frac{6}{2}=\frac{3}{1}$, $\frac{b_1}{b_2}=\frac{-3}{-1}=\frac{3}{1}$, $\frac{c_1}{c_2}=\frac{10}{9}$

Since $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

So the pair of linear eq. is parallel

Q3: (i) $3x+2y=5$ $\quad = 3x+2y-5=0$

$2x-3y=7$ $\quad = 2x-3y-7=0$

$\frac{a_1}{a_2}=\frac{3}{2}$, $\frac{b_1}{b_2}=\frac{-2}{3}$

Since $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$

So the pair is consistent

(ii) $2x-3y-8=0$

$4x-6y-9=0$

$\frac{a_1}{a_2}=\frac{\cancel{2}}{4}=\frac{1}{2}$, $\frac{b_1}{b_2}=\frac{-3}{-6}=\frac{1}{2}$, $\frac{c_1}{c_2}=\frac{-8}{-9}$

Since $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

So the pair is inconsistent

(iii) $\frac{3}{2}x+\frac{5}{3}y-7=0$

$9x-10y-14=0$

$\frac{a_1}{a_2}=\frac{3}{2\times 9}=\frac{1}{6}$, $\frac{b_1}{b_2}=\frac{5}{3\times(-10)}=\frac{-1}{6}$

Since $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$

So the pair is consistent

(iv) $5x-3y-11=0$

$-10x+6y+22=0$

$\frac{a_1}{a_2}=\frac{-5}{10}=\frac{-1}{2}$, $\frac{b_1}{b_2}=\frac{-3}{6}=\frac{-1}{2}$, $\frac{c_1}{c_2}=\frac{-11}{22}=\frac{-1}{2}$

Since $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

So the pair is consistent

(v) $\frac{4}{3}x+2y-8=0$

$2x+3y-12=0$

$\frac{a_1}{a_2}=\frac{4}{3\times 2}=\frac{2}{3}$, $\frac{b_1}{b_2}=\frac{2}{3}$, $\frac{c_1}{c_2}=\frac{-8}{-12}$

Since $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

So the pair is inconsistent.

5. Half Perimeter of Rectangular Garden = 36m

So complete Perimeter = 2×36

$= 72$

Let breadth be x m

length be $(x+4)$ m

Perimeter of rectangle $= 2(l+b)$

$72 = 2(x+x+4)$

$\frac{72}{2} = 2x+4$

$36-4 = 2x \qquad 2x = 32$

$x = 16$

Breadth = 16m

length = 20m

6. $2x+3y-8=0$

(i) Intersecting lines

$4x+9y-10=0$

(ii) Parallel lines

$4x+6y-5=0$

(iii) Co-incident lines

$4x+6y-16=0$

Exercise 3.2

Q1: $x+y=14$ -①

$x-y=4$ -②

From eq.②

$x=4+y$ -③

Put eq.③ in eq.①

$4+y+y=14$

$4+2y=14 \quad = 2y-10$

$\therefore y=5$

Put $y=5$ in eq.③

$x=4+5 \quad x=9, y=5$

(ii) $s-t=3$ -①

$\frac{s}{3}+\frac{t}{2}=6$ -②

From eq.①

$s=3+t$ -③

Put eq.③ in eq.②

$\frac{3+t}{3}+\frac{t}{2}=6 \quad = \frac{6+2t+3t}{6}=6$

$6+5t=36 \quad = 5t=30$

$t=6$

Put $t=6$ in eq.③

$s=3+6=9$

$t=6, s=9$

(iii) $3x-y=3$ -①

$9x-3y=9$ -②

$3x=3+y$

$x=\frac{3+y}{3}$ -③

Put eq. 3 in eq. 1

$9x - 3y = 9$

$\cancel{9}^{3}\left(\frac{3+y}{\cancel{3}_1}\right) - 3y = 9$

$9 + \cancel{3y} - \cancel{3y} = 9$

$9 = 9$

Since $9 = 9$ so it has infinitely many solution

(IV) $0.2x + 0.3y = 1.3$

$0.4x + 0.5y = 2.3$

Multiply both sides by 10

$2x + 3y = 13$ — (1)

$4x + 5y = 23$ — (2)

From eq. (1)

$2x = 13 - 3y$ $\quad x = \frac{13-3y}{2}$ — (3)

Put eq. (3) in eq. (2)

$\cancel{4}^{2}\left(\frac{13-3y}{\cancel{2}_1}\right) + 5y = 23$

$26 - 6y + 5y = 23$

$26 - 6y = 23$

$-y = 23 - 26$ $\quad = -y = -3$

$y = 3$

Put $y = 3$ in eq. (3)

$x = \frac{13-3(3)}{2}$ $\quad x = \frac{13-9}{2}$

$x = 2$

(V) $\sqrt{2}x + \sqrt{3}y = 0$ — (1)

$\sqrt{3}x - \sqrt{8}y = 0$ — (2)

$\sqrt{2}x = -\sqrt{3}y$

$x = -\frac{\sqrt{3}y}{\sqrt{2}}$ — (3)

Put eq. (3) in eq. (2)

$\sqrt{3}\left(-\frac{\sqrt{3}}{\sqrt{2}}y\right) - \sqrt{8}y = 0$

$\frac{-3}{\sqrt{2}}y - \sqrt{8}y = 0$ $\quad = y\left(\frac{-3}{\sqrt{2}} - \sqrt{8}\right) = 0$

$y = 0$

Put $y = 0$ eq. (3)

$x = \frac{\sqrt{3}(0)}{\sqrt{2}}$ $\quad x = 0$

$x = 0,\ y = 0$

(VI) $\frac{3x}{2} - \frac{5y}{3} = -2$ — (1)

$\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$ — (2)

$\frac{x}{3} = \frac{13}{6} - \frac{y}{2}$ $\quad x = \frac{13}{6} - \frac{y}{2} \times 3$

OR

Multiply by 6 on both sides

$9x - 10y = -12$ — (1)

$2x + 3y = 13$ — (2)

From eq. (2)

$2x = 13 - 3y$ $\quad x = \frac{13-3y}{2}$ — (3)

Put eq. (3) in eq. (1)

$9\left(\frac{13-3y}{2}\right) - 10y = -12$

$\frac{117-27y}{2} - 10y = -12$ $\quad = \frac{117-27y-20y}{2} = -12$

$117 - 47y = -12 \times 2$ $\quad = 117 - 47y = -24$

$-47y = -24 - 117$

$y = 3$

Put $y=3$ in eq. ③
$x = \frac{13-9}{2} = \frac{4}{2}$

$x=2$

2. $2x+3y=11$ -①
$2x-4y=-24$ -②
$y=mx+3$
$2x=11-3y$ $\quad x=\frac{11-3y}{2}$ -③

Put eq. ③ in eq. ②
$\cancel{2}\left(\frac{11-3y}{\cancel{2}}\right)-4y=-24$

$11-7y=-24$
$-7y=-24-11$
$-7y=-35$ $\quad y=5$
Put $y=5$ in eq. ③
$x=\frac{11-15}{2}$ $\quad x=\frac{-4}{2}$

$x=-2$
$y=mx+3$
$5=m(-2)+3$
$5=-2m+3$
$+2=-2m$
$m=-1$

3. (i) Let 2 no. be x & y where $x>y$
ATQ
$x-y=26$ -①
$x=3y$ -②
Put eq. ② in eq. ①
$3y-y=26$

$y=13$
$x=3\times13$
$x=39,\ y=13$

(ii) Let x & y be 2 supplementary angles $x>y$
$x+y=180°$ -①
$x=y+18$ -②
Put eq. ② in eq. ①
$y+18+y=180$
$2y=180-18$
$2y=162$ $\quad y=\frac{162}{2}$

$y=81°$
Put $y=81$ in 2 $\quad x=81+18$ $\quad x=99$
So req. angles are $81°$ & $99°$

(iii) Let the cost of balls be ~~x~~ y and bats be x
$7x+6y=3800$ -①
$3x+5y=1750$ -②
From eq. ②
$3x=1750-5y$ $\quad x=\frac{1750-5y}{3}$ -③

Put eq. ③ in eq. ①
$7\left(\frac{1750-5y}{3}\right)+6y=3800$

$\frac{12250-35y}{3}+\frac{6y}{1}=3800$

$\frac{12250-35y+18y}{3}=3800$

$12250-17y=11400$
$-17y=11400-12250$
$y=\frac{\cancel{-}850}{\cancel{-}17}$ 50 $\quad y=50$

$1750 \times 7 = 12250$
$3800 \times 3 = 11400$

Put $y = 50$ in eq. (3)

$x = \frac{1750 - 5(5)}{3} = \frac{1750-25}{3}$

$x = 500$

Cost of one bat ₹500

Cost of one ball ₹50

(IV) Let fixed charge be ₹x & charge per km be ₹y

ATQ

$x + 10y = 105$ — (1)

$x + 15y = 155$ — (2)

$x = 105 - 10y$ (3)

Put eq.(3) in eq.(2)

$15y + 105 - 10y = 155$

$5y = 155 - 105 = 50$

$5y = 50$

$y = 10$

$x = 105 - 10(10)$

$x = 5$

So fixed charge is ₹5 and charge for per km ₹10

$x + 25y = 5 + 25(10)$

$= ₹255$

(Q) Let the numerator be x & denominator be y

ATQ

$\frac{x+2}{y+2} = \frac{9}{11}$

$\frac{x+3}{y+3} = \frac{5}{6}$

$= 11(x+2) = 9(y+2)$

$= 11x + 22 = 9y + 18$

$= 11x - 9y = 18 - 22$

$11x - 9y = -4$

$6(x+3) = 5(y+3)$

$6x + 18 = 5y + 15$

$6x - 5y = -3$

ATQ

$11x - 9y = -4$ — (1)

$6x - 5y = -3$ — (2)

From eq.(2)

$6x = -3 + 5y$ $\qquad x = \frac{-3+5y}{6}$ (3)

Put (3) in (1)

$11\left(\frac{-3+5y}{6}\right) - 9y = -4$ $\qquad = \frac{-33+55y}{6} - \frac{9y}{1} = -4$

$\frac{-33+55y-54y}{6} = -4$ $\qquad = -33 + y = -24$

$y = 9$ put $y = 9$ = $x = \frac{-3+45}{6} = 7$

So fraction is $\frac{7}{9}$

(V) Let the present age of Jacob be x years and the present age of Son be y years

5 year hence

age of Jacob $= (x+5)$ years

age of son $= (y+5)$ years

$x + 5 = 3(y+5)$

$x + 5 = 3y + 15$

$x - 3y = 10$ — (1)

5 years ago

Age of Jacob $= (x-5)$ years

Age of Son $= (y-5)$ years

$x - 7y = -30$ — (2)

From eq. 1

$x = 10 + 3y$ —(3)

Put eq. (3) in eq (2)

$10 + 3y - 7y = -30$

$-4y = -40$

$y = 10$

Put $y = 10$ in (3)

$x = 10 + 30$

$x = 40$

Age of Jacob = 40 yrs.

Age of Son = 10 yrs.

Exercise 3.3

Q1: Elimination

(i) $x + y = 5$ —(1)

$2x - 3y = 4$ —(2)

Multiply eq. (1) by 3

$3x + 3y = 15$

$2x - 3y = 4$

$5x = 19$

$x = \frac{19}{5}$

Put $x = \frac{19}{5}$ in eq. (1)

$\frac{19}{5} + y = 5$ $\quad y = 5 - \frac{19}{5}$

$y = \frac{5}{6}$

So $x = \frac{19}{5}$, $y = \frac{5}{6}$

(ii) $3x + 4y = 10$ —(1)

$2x - 2y = 2$ —(2)

Multiply eq. (2) by 2

$4x - 4y = 4$

$3x + 4y = 10$

$7x = 14$

$x = 2$ $\quad$ Put $x = 2$ in eq. (1)

$2(2) - 2y = 2$

$4 - 2y = 2$

$-2y = -2$

$y = 1, x = 2$

(iii) $3x - 5y = 4$ —(1)

$9x - 2y = 7$ —(2)

Multiply (2) by 3

$\cancel{9x} - 15y = 12$

$- \cancel{9x} - 2y = 7$

$-13y = 5$ $\quad y = -\frac{5}{13}$

Put $y = -\frac{5}{13}$ in (1)

$3x - 5\left(\frac{-5}{13}\right) = 4$ $\quad = 3x + \frac{25}{13} = 4$

$3x = 4 - \frac{25}{13}$ $\quad = 3x = \frac{52-25}{13}$

$3x = \frac{27}{13}$

$x = \frac{9}{13}$, $y = \frac{-5}{13}$

(iv) $\frac{x}{2} + \frac{2y}{3} = -1$ —①

$x - \frac{y}{3} = 3$ —②

Multiply ② by 2

$\frac{x}{2} + \frac{2y}{3} = -1 \quad = 2x - \frac{2y}{3} = 6$

$\frac{x}{2} + \frac{2y}{3} = -1$

$2x - \frac{2y}{3} = 6$

$\frac{x}{2} + 2x = 5$

$\frac{x + 4x}{2} = 5$

$5x = 10$

$x = 2$

Q3: (i) Let numerator be x & denominator be y, fraction $= \frac{x}{y}$

ATQ

$\frac{x+1}{y-1} = \frac{1}{1} \quad = x + 1 = y - 1$

$= x - y = -2$

$x - y = -2$ —①

$\frac{x}{y+1} = \frac{1}{2} \quad 2x = y + 1$

$2x - y = 1$

$2x - y = 1$ —②

Multiply eq.① by ②

$2x - 2y = -4$

$-2x + y = 1$

$-y = -5 \qquad y = 5$

Put $y = 5$ in ①

$x - 5 = -2$

$x = -2 + 5$

$x = 3, y = 5$

So fraction is $\frac{3}{5}$

(ii) Let the present age of Nuri be x and Sonu be y

5 years ago

Age of Nuri $= (x-5)$ yrs

Age of Sonu $= (y-5)$ yrs

ATQ

$x - 5 = 3(y - 5)$

$x - 5 = 3y - 15$

$x - 3y = -10$ —①

10 years later

Age of Nuri $= (x+10)$ yrs

Age of Sonu $= (y+10)$ yrs

ATQ

$x + 10 = 2(y + 10)$

$x + 10 = 2y + 20$

$x - 2y = 10$ —②

Solving ① & ②

$x - 3y = -10$

$-x + 2y = -10$

$-y = -20 \qquad y = 20$

Put $y = 20$ in eq. ①

$x - 60 = 10$

$x = 50$

Present age of Nuri - 50 years & Sonu = 20 years.

(iii) Let the digit at ones place be x & tens place be y

ATQ

$x+y=9$

Original No. $= 10y+x$

New No. $= 10x+y$

$9(10y+x) = 2(10x+y)$

$90y+9x = 20x+2y$

$88y = 11x$

$-11x+88y=0$

Divide by 11

$-x+8y=0$

$x+y=9$

$9y=9$

$y=1$

$x+y=9$

$x=8$

Req. No. $= 10\times1+8$

$= 18$

(iv) Let no. of ₹50 notes be x & ₹100 notes be y

ATQ

$x+y=25$

$x+2y=40$

$x+y=25$

$x+2y=40$

$y=15$

$x+y=25$

$x+15=25$

$x=10$

₹100 Notes $= 15$

₹50 Notes $= 10$

(v) Let the fixed charge for first 3 days be ₹x & the additional charge thereafter be ₹y for each day

ATQ

$x+4y=27$ —①

$x+2y=21$ —②

Solving

$x+4y=27$

$x+2y=21$

$2y=6$

$y=3$

Put $y=3$ in eq. ②

$x+6=21$

$x=15$

So charge for 3 day = ₹15 &

Charge thereafter be ₹3 for each day

Exercise 3.1

1. Let no. of boys be x & no. of girls be y

ATQ

$x+y=10$ —①

$y=4+x$ —②

from eq. ①

$x+y=10$

Put $x=0$, $y=10$

$y=0$, $x=10$

$x=5$, $y=5$

x	0	10	5
y	10	0	5

From eq. ②

$y=4+x$

$x=0$, $y=4$

$y=0$, $x=-4$

$x=1$, $y=5$

x	0	-4	1
y	4	0	5

2. Let the cost of 1 pencil be ₹x
1 pen be ₹y

ATQ

$5x+7y=50$ —(1)
$7x+5y=46$ —(2)

From eq.(1)

$7y=50-5x$
$y=\frac{50-5x}{7}$

x	10	3
y	0	5

From eq.(2)

$7x+5y=46$
$y=\frac{46-7x}{5}$

Put $x=8$, $y=-2$
$x=3$, $y=5$

x	8	3
y	-2	5

7. $x-y+1=0$ —(1)
$3x+2y-12=0$ —(2)

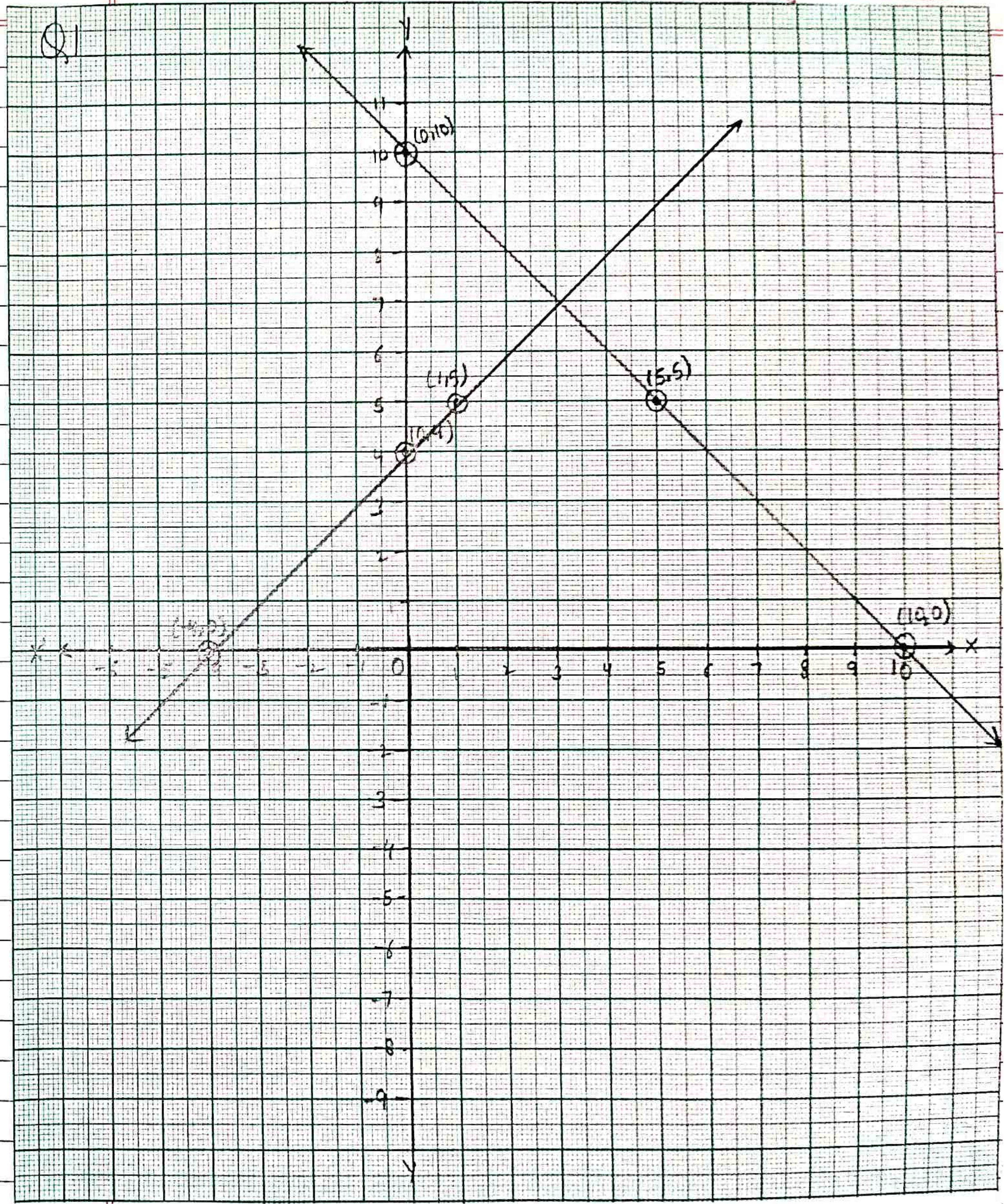
From (1)

$x=y-1$
$y=0$, $x=-1$
$x=0$, $y=1$
$x=1$, $y=2$

From (2)

$3x+2y=12$
$x=4$, $y=0$
$x=0$, $y=6$
$x=2$, $y=3$

Q1



2. Let the cost of 1 pencil be ₹x
1 pen be ₹y

ATQ

$5x + 7y = 50$ —(1)

$7x + 5y = 46$ —(2)

From eq.(1)

$7y = 50 - 5x$

$y = \frac{50 - 5x}{7}$

x	10	3
y	0	5

From eq.(2)

$7x + 5y = 46$

$y = \frac{46 - 7x}{5}$

Put $x = 8$, $y = -2$

$x = 3$, $y = 5$

x	8	3
y	-2	5

7. $x - y + 1 = 0$ —(1)

$3x + 2y - 12 = 0$ —(2)

From (1)

$x = y - 1$

$y = 0$, $x = -1$

$x = 0$, $y = 1$

$x = 1$, $y = 2$

From (2)

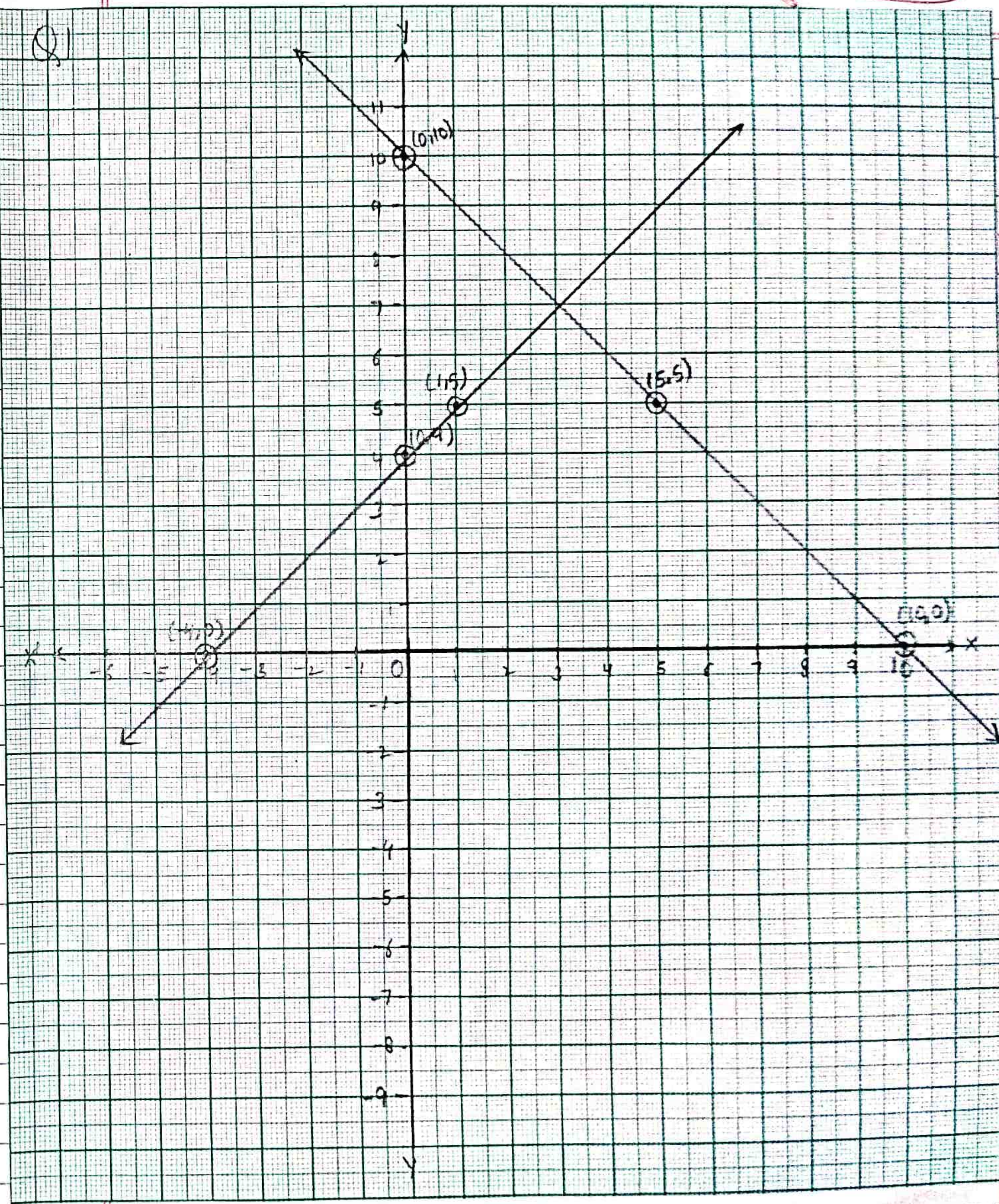
$3x + 2y = 12$

$x = 4$, $y = 0$

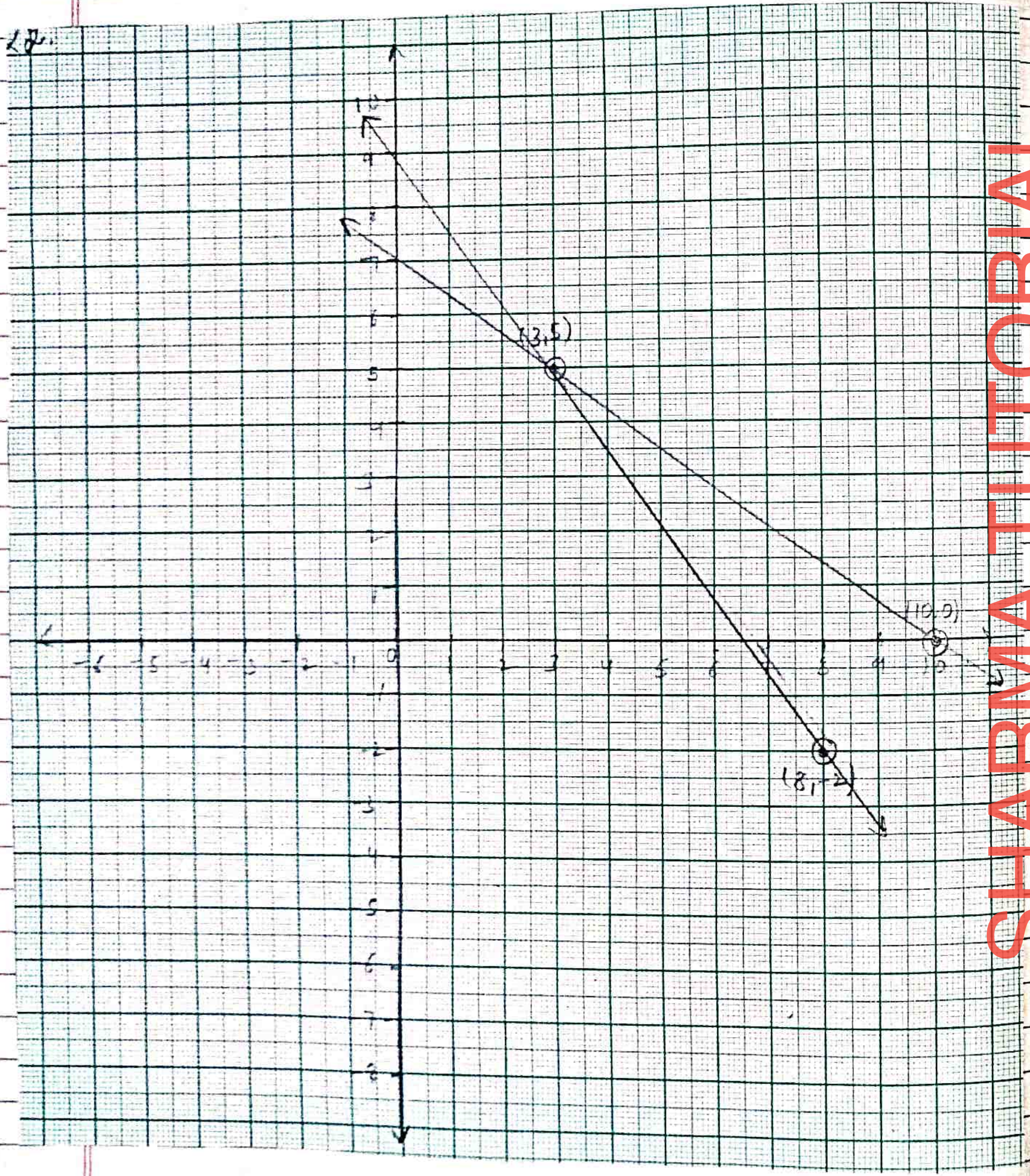
$x = 0$, $y = 6$

$x = 2$, $y = 3$

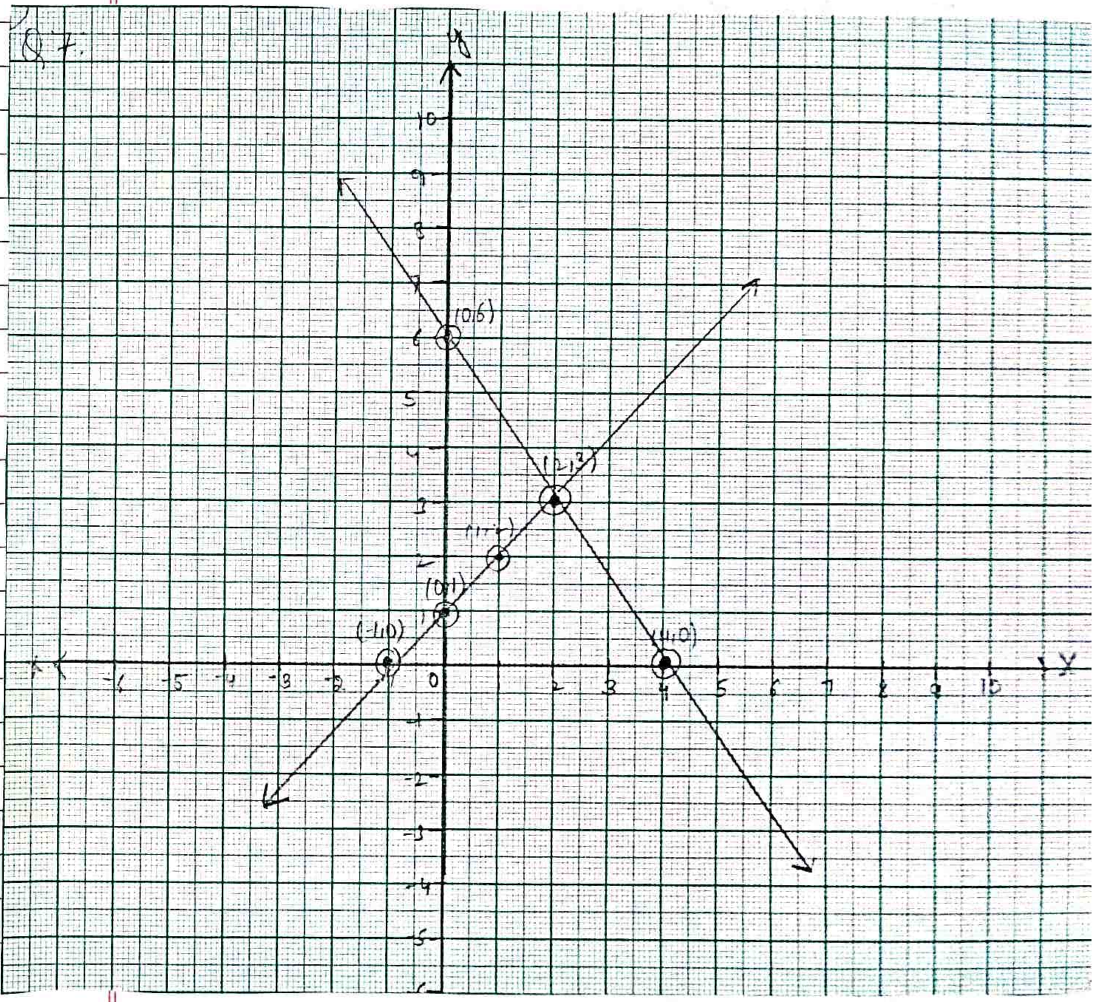
Q1



Q2:



Q7:



Chapter-4
Quadratic equations

General form of quadratic equation is $ax^2+bx+c=0$
$a \neq 0$, a, b & c are real no.s.

Exercise 4.1

Q1:

(i) $(x+1)^2 = 2(x-3)$
$x^2+1+2x = 2x-6$
$x^2+1+\cancel{2x}-\cancel{2x}+6=0$
$x^2+7=0$
$x^2+0x+7=0$
which is of the form $ax^2+bx+c=0$
So it is a quadratic eq.

(vi) $x^2+3x+1 = x^2+4-2x(2)$
$x^2+3x+1 = x^2+4-4x$
$\cancel{x^2}+3x+1-\cancel{x^2}-4+4x$
$7x-3=0$
No, because it is not in the form of $ax^2+bx+c=0$

(vii) $(x+2)^3 = 2x(x^2-1)$
$x^3+8+3x^2\times 2+3(x)(2)^2 = 2x^3-2x$
$x^3+8+6x^2+12x-2x^3+2x$
$-x^3+6x^2+14x+8=0$
No.

(ii) $x^2-2x = (-2)(3-x)$
$= x^2-2x = -2(3-x)$
$= x^2-2x-2x+6=0$
$x^2-4x+6=0$
Yes

(iii) $(x-2)(x+1) = (x-1)(x+3)$
$x^2+x-2x-2 = x^2+3x-x-3$
$\cancel{x^2}+x-2x-2-\cancel{x^2}-3x+x+3=0$
$-5x+2x+1=0$
$-3x+1=0$ ∴ No

(iv) $(x-3)(2x+1) = x(x+5)$
$2x^2+x-6x-3 = x^2+5x$
$2x^2+x-6x-3-x^2-5x=0$
$x^2-11x+x-3=0$
$x^2-10x-3=0$ ∴ Yes

(v) $(2x-1)(x-3) = (x+5)(x-1)$
$2x^2-3x-x+3 = x^2+x+5x+1=0$
$2x^2-4x+3 = x^2+6x+1=0$
$2x^2-4x+3-x^2-6x-1$
$x^2-10x+2=0$ ∴ Yes

(viii) $x^3-4x^2-x+1 = (x-2)^3$
$x^3-4x^2-x+1 = x^3-8-6x^2+12x$
$\cancel{x^3}-4x^2-x+1-\cancel{x^3}+8+6x^2-12x=0$
$2x^2-13x+9=0$

Q2:

(i) Let Breadth of rectangle be x m so length $= (2x+1)$ m

Area of rectangular plot $= 528$

$l \times b = 528$

$(2x+1)(x) = 528$

$2x^2 + x - 528 = 0$

So this is the req. Quadratic eq.

(ii) Let 2 consecutive positive ~~integer~~ integer be $x, x+1$

ATQ

$x(x+1) = 306$

$x^2 + x - 306 = 0$

So this is the req. Quadratic eq.

(iii) Let the age of Rohan be x yrs & age of mother be $(x+26)$ years

3 years Later

Age of Rohan - $(x+3)$ yrs

Age of Mother - $(x+29)$ yrs

$(x+3)(x+29) = 360$

$x^2 + 29x + 3x + 87 = 360$

$x^2 + 32x + 87 - 360 = 0$

$x^2 + 32x - 273 = 0$

So this is the req. Qud. eq.

(iv) Let the actual speed be x km/hr

$d = 480$ km

Actual time $= \frac{D}{S} = \frac{480}{x}$ hrs

New Speed $= (x-8)$ km/hr

$D = 480$ km

New Time $= \frac{D}{S} = \frac{480}{x-8}$ hrs

ATQ

$T_2 - T_1 = 3$ hrs

$\frac{480}{x-8} + \frac{480}{x} = 3$

$480\left(\frac{1}{x-8} \; \frac{-1}{x}\right) = 3$

$\frac{\cancel{x} - x + 8}{x(x-8)} = \frac{3}{480}$

$x^2 - 8x = 1280$

$x^2 - 8x - 1280$

It is the req. Quadratic eq.

Exercise 4.2

Q1: (i) $x^2 - 3x - 10 = 0$

$x^2 - 5x + 2x - 10 = 0$

$x(x-5) + 2(x-5) = 0$

$(x-5)(x+2) = 0$

Either $x - 5 = 0$, $x = 5$

Or $x + 2 = 0$, $x = -2$

(ii) $2x^2 + x - 6 = 0$

$2x^2 + 4x - 3x - 6 = 0$

$2x(x+2) - 3(x+2) = 0$

$(x+2)(2x-3) = 0$

Either $x + 2$, $x = -2$

Or $2x - 3 = 0$, $x = \frac{3}{2}$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0$

$x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = 0$

$(x + \sqrt{2})(\sqrt{2}x + 5) = 0$

Either	Or
$x = -\sqrt{2}$	$\sqrt{2}x + 5 = 0$
	$\sqrt{2}x = -5$
	$x = -\frac{5}{\sqrt{2}}$

(iv) $2x^2 - x + \frac{1}{8} = 0$

Multiply by 8

$16x^2 - 8x + 1 = 0$

$16x^2 - 4x - 4x + 1 = 0$

$4x(4x-1) - 1(4x-1) = 0$

$(4x-1)(4x-1) = 0$

Either	Or
$4x - 1 = 0$	$4x - 1 = 0$
$x = \frac{1}{4}$	$x = \frac{1}{4}$

(v) $100x^2 - 20x + 1 = 0$

$100x^2 - 10x - 10x + 1 = 0$

$10x(10x-1) - 1(10x-1) = 0$

$(10x-1)(10x-1) = 0$

Either	Or
$10x - 1 = 0$	$10x - 1 = 0$
$10x = 1$	$10x = 1$
$x = \frac{1}{10}$	$x = \frac{1}{10}$

Q2:

(i) $x^2 - 45x + 324 = 0$

$x^2 - 36x - 9x + 324 = 0$

$x(x-36) - 9(x-36) = 0$

$(x-9)(x-36) = 0$

Either	Or
$x = 9$	$x = 36$

2	324
2	162
3	81
3	27
3	9
3	3
	1

(ii) $x^2 - 55x + 750 = 0$

$x^2 - 25x - 30x + 750 = 0$

$x(x-25) - 30(x-25) = 0$

$(x-30)(x-25) = 0$

Either	Or
$x = 30$	$x = 25$

2	750
5	375
5	75
5	15
3	3
	1

Q3: Let one no. be x & other be $27 - x$

$x(27 - x) = 182$

$27x - x^2 = 182$

$-x^2 + 27x - 182 = 0$

$-x^2 + 14x + 13x - 182$

$-x^2 + 14x + 13x - 182 = 0$

$-x(x-14) + 13(x-14) = 0$

$(-x+13)(x-14) = 0$

Either	Or
$-x = -13$	$x = 14$
$x = 13$

2	182
13	91
7	7
	1

Or

$27 - x^2 = 182$

$182 + x^2 - 27 = 0$

Date ____
Page ____

When $x = 13$
First no. $= 13$
Second no. $= 14$

When $x = 14$
First no. $= 14$
Second no. $= 13$

4. Let 2 consecutive positive integers are $x, x+1$
ATQ
$x^2 + (x+1)^2 = 365$
$x^2 + x^2 + 1 + 2x = 365$
$2x^2 + 2x - 364$
Dividing both sides by 2
$x^2 + x - 182 = 0$
$x^2 + 14x - 13x - 182 = 0$
$x(x+14) - 13(x+14) = 0$
$(x+14)(x-13) = 0$
Either — Or
$x + 14 = 0$ — $x - 13 = 0$
$x = -14$ — $x = 13$
Rejected because it is a negative integer
$\therefore$ Req. integers are 14 & 13

5. Let the base of right led $\Delta = x$ cm
So, Altitude $= (x-7)$ cm
Hypotenuse $= 13$ cm
Using Pythagoras Theorem
$Base^2 + Alt^2 = Hyp^2$
$x^2 + (x-7)^2 = 13^2$
$x^2 + x^2 + 49 - 14x = 169$

classmate
Date ____
Page ____

$2x^2 - 14x - 120 = 0$
Dividing by 2
$x^2 - 7x - 60 = 0$
$x^2 - 12x + 5x - 60 = 0$
$\Rightarrow x(x-12) + 5(x-12) = 0$
$(x+5)(x-12) = 0$
Either — Or
$x = -5$ — $x = 12$
-5 is rejected because altitude can't be negative
$\therefore$ Base $= 12$ cm
Altitude $= 5$ cm

6. Let no. of articles produced be x
Cost of each article $= ₹(2x+3)$
Total cost $= ₹90$
ATQ
$x(2x+3) = 90$
$2x^2 + 3x - 90 = 0$
$2x^2 + 15x - 12x - 9 = 0$
$x(2x+15) - 6(2x+15) = 0$
$(2x+15)(x-6) = 0$
Either — Or
$2x + 15 = 0$ — $x - 6 = 0$
$2x = -15$ — $x = 6$
$x = -\frac{15}{2}$

$x = -\frac{15}{2}$ Rejected because no. of articles can't be -ve

No. of articles $= 6$
Cost $= 2 \times 6 + 3$
$= ₹15$

Q2: (i) $2x^2 + kx + 3 = 0$

$a = 2, b = k, c = 3$

Since it has 2 real & equal roots

$D = 0$

$D = b^2 - 4ac$

$b^2 - 4ac = 0$

$k^2 - 24 = 0$

$k^2 = 24$

$k = \pm\sqrt{24}$

$k \pm 2\sqrt{6}$

Hence, $+2\sqrt{6}, -2\sqrt{6}$

(ii) $kx(x-2) + 6 = 0$

$kx^2 - 2kx + 6 = 0$

$a = k, b = -2k, c = 6$

$\therefore D = 0$

$b^2 - 4ac = 0$

$(-2k)^2 - 4(k)(6) = 0$

$4k^2 - 24k = 0$

$4k(k-6) = 0$

$4k = 0$ $\qquad$ $k - 6 = 0$

$k = 0$ $\qquad$ $k = 6$

$k = 0$ Rejected because it does not satisfy the eq.

Q3: Let the Breadth be x m & length be $2x$ m

Area of rectangle = $800\,m^2$

$l \times b = 800$

$2x(x) = 800$

$2x^2 - 800$

$x^2 - 400 = 0$

$a = 1, b = 0, c = -400$

$D = b^2 - 4ac$

$= 0^2 - 4(1)(-400)$

$= 1600$

$D > 0$

So there are two real & distinct zeroes

$x = \frac{-b \pm \sqrt{D}}{2a}$

$x = \frac{-0 \pm \sqrt{1600}}{2}$

$= \frac{\pm 40}{2}$

$x = +20, -20$

$x = -20$ rejected because breadth can't be -ve.

$\therefore$ Breadth = 20m

length = 2×20

$= 40$ m

Q 4. Let the present age of 2 friends be x yrs & $(20-x)$ yrs

4 Years ago

Age of 2 friends will be $(x-4$ yrs$)$ & $(16-x)$ yrs

ATQ

$(x-4)(x-16) = 48$

$16x - x^2 - 64 + 4x = 48$

$-x^2 + 20x - 112 = 0$

$x^2 - 20x + 112 = 0$

$a = 1, b = -20, c = 112$

$D = b^2 - 4ac$

$= (-20)^2 - 4(1)(112)$

$= 400 - 448$

$= -48$

Since $D < 0$

So no real root exists for this equation

Q5: Let length be x m & breadth be y m

perimeter = 80m

$2(l+b) = 80$

$2(x+y) = 80$

$x+y = 40$

$y = 40-x$

Area of rectangle = $400m^2$

$l \times b = 400$

$xy = 400$

$x(40-x) = 400$

$x^2 - 40x + 400 = 0$

$a = 1, b = -40, c = 400$

$D = b^2 - 4ac$

$= (-40)^2 - 4(1)(400)$

$= 1600 - 1600$

$= 0$

$D = 0$

So there will be 2 real & equal roots

$x = \frac{-b}{2a}, \frac{-b}{2a}$

$= \frac{-(-40)}{2}, \frac{-(-40)}{2}$

$= 20, 20$

length & Breadth = 20 & 20m

Chapter-5
Arithmetic Progressions (AP)

Exercise 5.1

Q1: List of numbers obtained from the given situation are :-

15, 15+8, 15+8+8

15, 23, 31 ......

Here $a_2 - a_1 = 23-15$

$= 8$

$a_3 - a_2 = 31-23 = 8$

So the common difference between any two term of any no. is 4. So it is an AP.

(ii) Let the air in the beginning be 64 unit^3

Air removed in first step = $\frac{1}{4} = \frac{64}{4} = 16$

Air left in cylinder = $64-16 = 48$

Air left in the second step = $\frac{48}{4} = 12$

Air left = $48-12 = 36$

List of no. of AP-

64, 48, 36 - - - -

$a_2 - a_1 = 48-64 = -16$

$a_3 - a_2 = 36-48 = -12$

So the common difference between any two terms is not ~~same~~ an AP.

(iii) cost of digging 1metre = ₹150

Cost of digging first two meters = ₹200

Cost of digging first three meters = ₹250

150, 200, 250 .....

List of no.>

$a_2-a_1 = 200-150 = 50$

$a_3-a_2 = 250-200 = 50$

Since the common difference between any two terms of any no. is 50.
So it is an AP
150, 200, 250, 300 . . . . .

Q2: (i) $a=10$, $d=10$
First four terms are
$a, a+d, a+2d, a+3d$
10, 20, 30, 40

(ii) $a=-2$, $d=0$
First four terms are:-
$a, a+d, a+2d, a+3d$
$-2, -2, -2, -2$

(iii) $a=4$, $d=-3$
First four terms are -
$a, a+d, a+2d, a+3d$
$4, 1, -2, -5$

(IV) $a=-1$, $d=\frac{1}{2}$
First four terms are
$a, a+d, a+2d + a+3d$
$-1, -\frac{1}{2}, 0, \frac{1}{2}$

(V) $a=-1.25$, $d=-0.25$
First four terms are -
$a, a+d, a+2d, a+3d$
$-1.25, -1.5, -1.75, -2$

Q3: (i) 3, 1, −1, −3 . . . .
$a = 3$
$d = 1-3 = -2$

(ii) −5, −1, 3, 7 . . . . .
$a = -5$
$d = -1-(-5) = 4$

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}$ . . . . .
$a = \frac{1}{3}$

$d = \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$

(IV) 0.6, 1.7, 2.8, 3.9 . . . . .
$a = 0.6$
$d = 1.7-0.6$
$d = 1.1$

Q4: (i) 2, 4, 8, 16
$a_2 - a_1 = 4-2 = 2$
$a_3 - a_2 = 8-4 = 4$
Since difference is not same, so its not an AP.

(ii) $2, \frac{5}{2}, 3, \frac{7}{2}$ . . . . . .

$a_2 - a_1 = \frac{5}{2} - 2 = \frac{5-4}{2} = \frac{1}{2}$

$a_3 - a_2 = 3 - \frac{5}{2} = \frac{1}{2}$

Since the common difference is same, so the given no.s are in AP

3 More terms are :-

$\frac{7}{2} + \frac{1}{2} = 4$

$4 + \frac{1}{2} = \frac{9}{2}$

$\frac{9}{2} + \frac{1}{2} = 5$

(iii) $-1.2, -3.2, -5.2, -7.2 \ldots$

$a_2 - a_1 = -3.2 - (-1.2)$
$= -3.2 + 1.2$
$= -2.0$

Since the common difference is same, so the given no.s are in AP

$a_3 - a_2 = -5.2 - (-3.2)$
$= -5.2 + 3.2$
$= -2.0$

3 More terms are :-

$-7.2 + (-2) = -9.2$, $-9.2 + (-2) = -11.2$, $-11.2 + (-2) = -13.2$

(iv) $-10, -6, -2, 2 \ldots$

$a_2 - a_1 = -6 + (-10) = -16$

$a_3 - a_2 = -2 + (-6) = -8$

Since the common difference is not same so the given no.s are in AP.

(v) $3, 3+\sqrt{2}, 3+2\sqrt{3}, 3+3\sqrt{2} \ldots$

$a_2 - a_1 = 3 + \cancel{2}\sqrt{2} - (3)$
$= \cancel{3} + \sqrt{2} - \cancel{3} = \sqrt{2}$

$a_3 - a_2 = 3 + 2\sqrt{3} - (3 + \sqrt{2}$
$\cancel{3} + 2\sqrt{2} - \cancel{3} - \sqrt{2}$

Since the common difference is same. So the no.s are in AP

$3 + 3\sqrt{2} + \sqrt{2} = 3 + 4\sqrt{2}$

$3 + 4\sqrt{2} + \sqrt{2} = 3 + 5\sqrt{2}$

$3 + 5\sqrt{2} + \sqrt{2} = 3 + 6\sqrt{2}$

(vi) $0.2, 0.22, 0.222, 0.2222 \ldots$

$a_2 - a_1 = 0.22 - 0.2$
$= 0.02$

$a_3 - a_2 = 0.222 - 0.22$
$= 0.002$

Since the common difference is not same. So the no.s are not in AP.

(vii) $0, -4, -8, -12 \ldots$

$a_2 - a_1 = -4 - 0 = -4$

$a_3 - a_2 = -8 - (-4) = -4$

Since the common difference is same the given no.s are in AP

$-12 + (-4) = -16$

$-16 + (-4) = -20$

$-20 + (-4) = -24$

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \ldots$

$a_2 - a_1 = -\frac{1}{2} - (-\frac{1}{2}) = 0$

$a_3 - a_2 = -\frac{1}{2} - (-\frac{1}{2}) = 0$

Since the common difference is same so they are in AP

$-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}$

(IX) $1, 3, 9, 27 \dots$

$a_2 - a_1 = 3 - 1 = 2$

$a_3 - a_2 = 9 - 3 = 6$

Since the common difference is not same. So they are not an AP.

(X) $a, 2a, 3a, 4a \dots$

$a_2 - a_1 = 2a - a = a$

$a_3 - a_2 = 3a - 2a = a$

$5a, 6a, 7a$

(XI) $a, a^2, a^3, a^4 \dots$

$a_2 - a_1 = a^2 - a_1 = a^2 - a = a(a-1)$

$a_3 - a_2 = a^3 - a_2^2 = a^3 - a^2 \quad a^2(a-1)$

Since the common difference is not same so they are not in AP.

(XII) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32} \dots$

$\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2} \dots$

$a_2 - a_1 = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$

$a_3 - a_2 = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$

Since the common difference is not same so they are in AP.

3 More terms :-

$4\sqrt{2} + \sqrt{2} = 5\sqrt{2} = \sqrt{50}$

$5\sqrt{2} + \sqrt{2} = 6\sqrt{2} = \sqrt{72}$

$6\sqrt{2} + \sqrt{2} = 7\sqrt{2} = \sqrt{98}$

(XIII) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12} \dots$

$a_2 - a_1 = \sqrt{6} - \sqrt{3}$

$a_3 - a_2 = 3 - \sqrt{6}$

Since the common difference is not same so they are in AP.

(XIV) $1^2, 3^2, 5^2, 7^2 \dots$

$a_2 - a_1 = 3^2 - 1^2 = 9 - 1 = 8$

$a_3 - a_2 = 5^2 - 3^2 = 25 - 9 = 16$

Since they are not in AP.

(XV) $1^2, 5^2, 7^2, 73$

$a_2 - a_1 = 25 - 1 = 24$

$a_3 - a_2 = 49 - 25 = 24$

Since the common difference is same so it's an AP

$73 + 24 = 97$

$97 + 24 = 121$

$121 + 24 = 145$

Exercise 5.2

Q1: (i) $a = 7, d = 3, n = 8, a_n = ?$

$a_n = a + (n-1)d$

$a_n = 7 + (8-1)3$

$a_n = 7 + 21$

$a_n = 28$

Q2: (i) 30th term of AP: $10, 7, 4 \dots$

$a = 10, d = -3$

$a_{30} = a + 29d$

$a_{30} = 10 + 29(-3)$

$10 - 87$

$= -77$

(ii) 11th term of AP: $-3, -\frac{1}{2}, 2 \dots$

$a = -3, d = -\frac{1}{2} - (-3) = \frac{5}{2}$

Date ____
Page ____

$a_{11} = a + 10d$

$a_{11} = -3 + 10\left(\frac{5}{2}\right)$

$a_{11} = 28$

Q2: (ii) $a = -18$, $d = ?$, $n = 10$, $a_n = 0$

$a_n = a + (n-1)d$

$0 = -18 + (n-1)d$

$0 = -18 + 9d$

$18 = 9d$

$d = 2$

(iii) $a = ?$, $d = -3$, $n = 18$, $a_n = -5$

$a_n = a + (n-1)d$

$-5 = a + (18-1)-3$

$-5 = a + (17)(-3)$

$-5 = a - 51$

$-5 + 51 = a$

$46 = a$

$a = 46$

(iv) $a = -18.9$, $d = 2.5$, $n = ?$, $a_n = 3.6$

$a_n = a + (n-1)d$

$3.6 = -18.9 + (n-1)2.5$

$3.6 + 18.9 = (n-1)2.5$

$\frac{22.5}{2.5} = (n-1)$

$9 = n - 1$

$n = 10$

(v) $a = 3.5$, $d = 0$, $n = 105$, $a_n = ?$

$a_n = a + (n-1)d$

$a_n = 3.5 + (105-1)0$

$a_n = 3.5 + (104)0$

$a_n = 3.5 + 0$

$a_n = 3.5$

Q3: (i)

$a = 2$, $a_3 = 26$

$a_3 = a + 2d$

$2 + 2d = 26$

$2d = 24$

$d = 12$

$a_2 = a + d$

$a_2 = 2 + 12$

$a_2 = 14$

(ii) $a = ?$, $a_2 = 13$, $a_3 = ?$, $a_4 = 3$

$a_2 = a + d$

$a_4 = a + 3d$

$a + 3d = 3$

$-a + -d = -13$

$2d = -10$

$d = -5$

$a + d = 13$

$a - 5 = 13$

$a = 13 + 5 = 18$

$a_3 = a + 2d$

$a_3 = 18 + 2(-5)$

$= 8$

iii) $a=5,\ a_4=\frac{19}{2}$

$a+3d=\frac{19}{2}$

$5+3d=\frac{19}{2}$

$3d=\frac{19}{2}-5 \quad = 3d=\frac{19-10}{2}$

$3d=\frac{9}{2} \qquad d=\frac{3}{2}$

$a_2=a+d$

$=5+\frac{3}{2} = \frac{10+3}{2} = \frac{13}{2}$

$a_3=a+2d$

$5+\cancel{2}\left(\frac{3}{\cancel{2}}\right) = 8$

(IV)

$a=-4,\ a_6=6$

$a_6=a+5d=6$

$-4+5d=6$

$5d=10$

$d=2$

$a_2=a+d = -4+2=-2$

$a_3=a+2d = -4+(+4)=0$

$a_4=a+3d=-4+6=2$

$a_5=a+4d = -4+8=4$

(V) $a_2=38$

$a_6=-22$

$a_2=a+d=38$ —(1)

$a_6=a+5d=-22$ —(2)

$\cancel{a}+d=38$

$-\cancel{a}+-5d=-22$

$-4d=60$

$d=-15$

$a+d=38$

$a+(-15)=38$

$a=38+15=53$

$a_3=a+2d = 53+2(-15) = 23$

$a_4=a+3d = 53+3(-15)=8$

$a_5=a+4d = 53+4(-15)=-7$

Q4: AP: 3, 8, 13, 18 78

$a=3,\ d=5,\ a_n=78$

$a_n=a+(n-1)d$

$78=3+(n-1)5$

$75=(n-1)5$

$\frac{75}{5}=n-1$

$n=16$

Q5: 7, 13, 19 205

$a=7,\ d=6,\ a_n=205,\ n=?$

$a_n=a+(n-1)d$

$205=7+(n-1)6$

$205-7=(n-1)6$

$=\frac{198}{6}=n-1$

$=33+1=n$

$n=34$

(ii) $18, \frac{151}{2}, 13 \ldots\ldots -47$

$a = 18, \frac{31}{2}, 13 \ldots\ldots -47$

$a = 18, d = \frac{-5}{2}, a_n = -47$

$a_n = a+(n-1)d$

$-47 = 18+(n-1)\frac{-5}{2}$

$\frac{\cancel{65}^{13} \times 2}{\cancel{5}_1} = (n-1)$

$26 = (n-1)$

$n = 27$

Q6: $AP: 1\cancel{5}, 8, 5, 2 \ldots\ldots\ldots -150$

$a = 1\cancel{5}, d = -\cancel{7}3, a_n = -150$

$a_n = a+(n-1)d$

$-150 = 11+(n-1)(-3)$

$-150-11 = (n-1)(-3)$

$\frac{\cancel{-}161}{\cancel{-}3} = n-1$

$\frac{161}{3} + 1 = n$

$\frac{161+3}{3} = n$ $\qquad n = \frac{164}{3}$

Since n is not a ~~whole~~ natural no. so -150 is not ~~of~~ a term of AP.
N should be a natural no.

Q7: $a_{11} = 38$

$a+10d = 38$ —(1)

$a_{16} = 73$

$a+15d = 73$

$\cancel{a}+15d = 73$

$-\cancel{a}+-10d = -38$

$5d = 35$

$d = 7$

$a+10d = 38$

$a+10(7) = 38$

$a+70 = 38$

$a = -32$

$a_{31} = a+(n-1)d$

$a_{31} = -32+(31-1)7$

$= -32+(30)7$

$= 178$

8. $n = 50, a_3 = 12, a_{50} = 106$

$\cancel{a}+2d = 12$ —(1)

$-\cancel{a}+-49d = 106$ —(2)

$-47d = -94$

$d = \frac{\cancel{-94}\,2}{\cancel{-47}\,1}$

$d = 2$

$a+2d = 12$

$a+4 = 12$

$a = 8$

29th term

$a+28d$

$8+28(2)$

$= 64$

9. $a_3 = 4$

$a_9 = -8$

Let the n^{th} term be 0

$a_n = 0$

SHARMA TUTORIAL

classmate
Date
Page

$\cancel{a}+2d=4$
$\cancel{a}+8d=-8$
$-6d=12$
$d=-2$
$a+2d=4$
$a+2(-2)=4$
$a=8$
$a_n=a+(n-1)d$
$a_n=0$
$0=8+(n-1)(-2)$
$+\cancel{8}^{4}=(n-1)$
$+\cancel{2}_1$
$n=5$

10. $a_{17}-a_{10}=7$
$a+16d-(a+9d)=7$
$\cancel{a}+16d-\cancel{a}-9d=7$
$16d-9d=7$
$7d=7$
$d=1$

11. AP: 3, 15, 27, 39....
$a=3,\ d=15-3=12$
Let the req. term be n
$a_n=132+a_{54}$ — (1)
$a+(n-1)d=a_n$
$3+(n-1)12=132+a+53d$
$\cancel{3}+(n-1)12=132+\cancel{3}+53(12)$
$12n-12=132+636$
$12n=768+12$
$12n=780$
$n=\frac{780}{12}$ $\therefore n=65$

classmate
Date
Page

12. Let 'a' and be the first term and 'd' be the common difference of 1st AP and a' be the first term of 2nd AP.
ATQ
$a_{100}-a'_{100}=100$
$a+99d-(a'+99d)=100$
$a+99\cancel{d}-a'-99\cancel{d}=100$
$a-a'=100$ — (1)
$a_{1000}-a'_{1000}=a+999d-(a'+999d)=1000$
$a+999\cancel{d}-a'-999\cancel{d}=\cancel{100}$
$a-a'=100$ | From (1)

13. 3 digit no.s divisible by 7 are :- 105, 112, 119 994
$a=105,\ d=112-105=7,\ a_n=994,\ n=?$
$a_n=a+(n-1)d$
$994=105+(n-1)7$
$994-105=(n-1)7$
$\frac{\cancel{889}^{127}}{\cancel{7}_1}=(n-1)$
$n=128$

14. Multiples of 4 lying between 10 & 250 are :-
AP: 12, 16, 20. 248
$a=12,\ d=16-12=4,\ a_n=248$
$a_n=a+(n-1)d$
$248=12+(n-1)4$
$248-12=(n-1)4$
$236=(n-1)4$
$\frac{\cancel{236}^{59}}{\cancel{4}_1}=n-1$
$n=60$

classmate
Date ______
Page ______

15. AP: 63, 65, 67.......
AP: 3, 10, 17.........

$a = 63,\ d = 2$ $\qquad$ $a' = 3,\ d' = 7$

ATQ

$a + (n-1)d = a' + (n-1)d'$

$63 + (n-1)2 = 3 + (n-1)7$

$63 + 2n - 2 = 3 + 7n - 7$

$61 + 2n = -4 + 7n$

$61 + 4 = 7n - 2n$

$65 = 5n$

$n = 13$

So their 13^{th} term will be equal

16. $a_3 = 16$

$a_7 - a_5 = 12$

$a + 6d - (a + 4d) = 12$

$\cancel{a} + 6d - \cancel{a} - 4d = 12$

$2d = 12$

$d = 6$

$a_3 = 16$

$a + 2d = 16$

$a + 2(6) = 16$

$a = 16 - 12$

$a = 4$ $\qquad$ AP is $a,\ a+d,\ a+2d$

$\therefore 4, 10, 16.....$

17. AP = 3, 8, 13 253

$a = 3,\ d = 5$

On reversing the digits

~~253 258~~, 253, 248, 243 .... 13, 8, 3

$a = 253,\ d = -5,\ a_n = 3$

$a_{20} = a + 19d$

$= 253 + 19(-5)$

$a_{20} = 158$

classmate
Date ______
Page ______

18. $a_4 + a_8 = 24$ $\qquad$ $a_6 + a_{10} = 44$

$a + 3d + a + 7d = 24$ $\qquad$ $a + 5d + a + 9d = 44$

$2a + 10d = 24 \ (\div 2)$ $\qquad$ $2a + 14d = 44 \ (\div 2)$

$a + 5d = 12$ —(1) $\qquad$ $a + 7d = 22$ —(2)

$\cancel{a} + 5d = 12$

$-\cancel{a} + 7d = 22$

$-2d = -10$

$d = 5$ $\qquad$ Put $d = 5$ in (1)

$a + 5(5) = 12$

$a = 12 - 25$

$a = -13$

$a,\ a+d,\ a+2d....$

$-13,\ -13+(5),\ -13+2(5)$

~~-13, -18, -23~~ .....

$-13, -8, -3...$

19. Annual salary of Subba Rao = ₹5000

Increment = ₹200

$a = 5000,\ d = 200$

AP: 5000, 5200, 5400.......

$a = 5000,\ d = 200,\ a_n = 7000$

Let his income reaches to 7000 in nth year

$7000 = 5000 + (n-1)200$

$\dfrac{\cancel{2000}^{10}}{\cancel{200}_{1}} = (n-1)$

$n = 11$

In 11^{th} years his salary reaches ₹7000

1995 + 11

2006 is the year in which his salary increases ₹7000

20. Ramkali saving in first year $= ₹5$

Increased savings :- $₹1.75$

$5, 6.75 \ldots\ldots 20.75$

Here $a = 5$, $d = 1.75$

$a_n = 20.75$

$a_n = a + (n-1)d$

$20.75 = 5 + (n-1)1.75$

$\frac{15.75}{1.75} = n-1$

$n = 10$

Exercise 5.3

Q1:

(i) $2, 7, 12 \ldots\ldots$ to 10 terms

$a = 2, d = 5, n = 10$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\frac{\cancel{10}^{5}}{\cancel{2}_{1}}[2\times2 + (10-1)5]$

$5 \times [4 + 45)$

5×49

$= 245$

(ii) $-37, -33, -29 \ldots\ldots 12$ terms

$a = -37, d = -33-(-37) = 4, n = 12$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_n = \frac{12}{2}[2(-37) + (12-1)4]$

$6[-74 + 44]$

$6 \times -30 \quad = -180$

(iii) $0.6, 1.7, 2.8 \ldots\ldots 100$ terms

$a = 0.6, d = 1.7 - 0.6 = 1.1, n = 100$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_n = \frac{100}{2}[2(0.6) + (100-1)1.1]$

$S_n = 50[1.2 + 108.9)$

50×110.1

$= 5505$

$S_n = 5505$

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10} \ldots\ldots 10$ terms

$a = \frac{1}{15}, d = \frac{1}{12} - \frac{1}{15} \quad \frac{5-4}{60} = \frac{1}{60}, n = 10$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\frac{10}{2}\left[2\left(\frac{1}{15}\right) + (10-1)\frac{1}{60}\right]$

$5\left[\frac{2}{15} + \frac{9}{60}\right]$

$5\left[\frac{8+9}{60}\right]$

$\frac{\cancel{5} \times 17}{\cancel{60}_{12}}$

Q2: (i) $7 + 10\frac{1}{2} + 14 + \ldots\ldots + 84$

$a = 7$, $d = \frac{21}{2} - 7 = \frac{21-14}{2} = \frac{7}{2}$, $a_n = 84$

$a_n = a + (n-1)d$

$84 = 7 + (n-1)\frac{7}{2}$

$n-1 = 84-7$

$n-1 = \frac{77 \times 2}{7}$

$n-1 = 22$

$n = 23$

$S_n = \frac{n}{2}(a + a_n)$

$\frac{23}{2}(7+84) = \frac{2093}{2}$

$S_n = 1046.5$

(ii) $34 + 32 + 30 + \ldots\ldots + 10$

$a = 34$, $d = -2$, $a_n = 10$

$a_n = a + (n-1)d$

$10 = 34 + (n-1)-2$

$-24 = (n-1)(-2)$

$\frac{-24}{-2} = (n-1)$

$n = 13$

$S_n = \frac{n}{2}(a + a_n)$

$\frac{13}{2}(34 + 10)$

$= 286$

(iii) $-5 + (-8) + (-11) + \ldots\ldots (-230)$

$a = -5$, $d = -3$, $a_n = -230$

$a + (n-1)d = a_n$

$-5 + (n-1)-3 = -230$

$-5 + (n-1)(-3) = -230$

$(n-1)(-3) = -225$

$(n-1) = \frac{-225}{-3}$

$n-1 = 75$

$n = 76$

$S_n = \frac{n}{2}(a + a_n)$

$S_n = \frac{76}{2}(-5 + (-230))$

$\frac{76}{2}(-38)$

$S_n = -8930$

SHARMA TUTORIAL

Q3: (i) $a = 5, d = 3, a_n = 50, n = ?, S_n = ?$

$a_n = a + (n-1)d$

$50 = 5 + (n-1)3$

$\frac{\cancel{45}^{15}}{\cancel{3}_1} = n - 1$

$n = 16$

$S_n = \frac{n}{2}[a + a_n]$

$\frac{\cancel{16}^{8}}{\cancel{2}_1}[5 + 50]$

$= 440$

(ii) $a = 7, a_{13} = 35, d = ?, S_{13} = ?$

$a = 7, n = 13$

$a + 12d = 35$

$7 + 12d = 35$

$12d = 35 - 7$

$d = \frac{\cancel{28}^{7}}{\cancel{12}_{3}}$ $\qquad d = \frac{7}{3}$

$S_{13} = \frac{n}{2}(a + a_n)$

$= \frac{13}{2}(7 + 35)$

$\frac{13 \times \cancel{42}^{21}}{\cancel{2}_1}$

$= 273$

(iii) $a_{12} = 37, d = 3, a = ?, S_{12} = ?$

$a + 11d = 37$

$a + 33 = 37$

$a = 4$

$S_{12} = \frac{n}{2}(a + a_n)$

$\frac{12}{2}(4 + 37)$

6×41

$= 246$

(iv) $a_3 = 15, S_{10} = 125, d = ?, a_{10} = ?$

$a_3 = a + 2d$

$a + 2d = 15$

$a = 15 - 2d$ -(1)

$S_{10} = \frac{n}{2}[2a + (n-1)d]$

$\frac{10}{2}[2(15 - 2d) + (10 - 1)d] = 125$

$5[30 - 4d + 9d] = 125$

$5[30 + 5d] = 125$

~~$150 + 25d = 125$~~

$30 + 5d = 25$

$5d = -5$

$d = -1$

$a + 2d = 15$

$a = 15 + 2$

$a = 17$

$a_{10} = a + (n-1)d$

$a_{10} = 17 + (10-1)(-1)$

$17 - 9$

$= 8$

(v) $d = 5, S_9 = 75, a = ?, a_9 = ?$

$S_9 = \frac{n}{2}(2a + (n-1)d)$

$75 = \frac{9}{2}(2a+(9-1)5)$

$\frac{75\times2}{9} = 2a+40$

$\frac{150}{9} = 2a+40$

$\frac{150-40}{9} = 2a$

$\frac{150-360}{9} = 2a$

$\frac{-210}{9} = 2a$

$-\frac{\cancel{210}}{9\times\cancel{2}} = a$

$-\frac{\cancel{105}}{\cancel{9}\,3} = a$ $\qquad a = -\frac{35}{3}$

$a_9 = a+(n-1)d$

$-\frac{35}{3} + (9-1)5$

$\frac{-35}{3}+40$

$\frac{-35+120}{3}$

$=\frac{85}{3}$

VI) $a=2,\ d=8,\ S_n=90,\ n=?,\ a_n=?$

$S_n = \frac{n}{2}[2a+(n-1)d] = 90$

$\frac{n}{2}[4+8n-8] = 90$ $\qquad \frac{n(2)(-2+4n)}{2} = 90$

$\frac{n}{2}[-4+8n] = 90$ $\qquad -2n+4n^2 = 90$

$2n^2-n-45=0$

$2n^2-10n+9n-45=0$

$2n(n-5)+9(n-5)=0$

$(2n+9)(n-5)=0$

$n=-\frac{9}{2},\ n=5$

$n=-\frac{9}{2}$ Rejected

$a_5 = a+(n-1)d$

$= 2+32$

$= 34$

(VII) $a=8,\ a_n=62,\ S_n=210,\ n=?,\ d=?$

$a_n = a+(n-1)d$

$62 = 8+(n-1)d$

$54 = (n-1)d$ — (1)

$S_n = \frac{n}{2}(a+a_n)$

$\frac{n}{2}(8+62) = 210$

$\frac{n}{\cancel{2}\,1}\times \overset{35}{\cancel{70}} = 210$

$n=6$ — (2)

Put $n=6$ in (1)

$54 = (6-1)d$

$\frac{54}{5} = d$

$d = \frac{54}{5}$

$d = 10\frac{4}{5}$

(VIII) $a_n=4,\ d=2,\ S_n=-14,\ n=?\ a=?$

$a_n = a+(n-1)d$

$4 = a+(n-1)2$

$4 = a+2n-2$

$a = 6-2n$ — (1)

$S_n = -14$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_n = \frac{n}{2}[2(6-2n)+(n-1)2]$

$\frac{n}{2}[12-4n+2n-2] = -14$

$\frac{n}{2}[10-2n] = -14$

$10n-2n^2 = -28$

$n^2-5n+14=0$

$n(n-7)+2(n-7)=0$

$(n+2)(n-7)=0$

$n=-2, n=7$

$n=-2$ - Rejected

(IX) $a=3, n=8, S=192, d=?$

$S_8=192$

$\frac{n}{2}[2a+(n-1)d] = 192$

$\frac{\cancel{8}^4}{\cancel{2}_1}[2(3)+(8-1)d]=192$

$4[6+7d]=192$

$6+7d=48$

$7d=48-6$

$d=\frac{42}{7}$

$d=6$

(X) $l=28, S=144, n=9, a=?$

$S_9=144$

$\frac{9}{2}[a+l)=144$

$a+28=32$

$a=4$

Q4: AP: 9, 17, 25.....

$a=9, d=8, S_n=636$

Let the req. term be n

$S_n=\frac{n}{2}[2a+(n-1)d]$

$636=\frac{n}{2}[18+(n-1)8]$

$\frac{n}{2}(10+8n)=636$

$\frac{n}{\cancel{2}}\cancel{2}(5+4n)=636$

$5n+4n^2=636=0$

$4n^2+5n-636=0$

$4n^2+(-48n)+53n-636=0$

$4n^2-48n+53n-636=0$

$4n(n-12)+53(n-12)=0$

$(4n+53)(n-12)=0$

$n=12, n=\frac{-53}{4}$ - Rejected

$n=\frac{-53}{4}$ Rejected because 'n' can't be in fraction its should be a natural no.

Q5: $a=5, a_n=45, S_n=400, n=?, d=?$

$S_n=\frac{n}{2}(a+a_n)$

$400=\frac{n}{2}(5+45)$

$400=\frac{n}{\cancel{2}_1}(\cancel{50})^{25}$

$\frac{\cancel{400}^8}{\cancel{50}_1}=n$ ~~n=8~~ $\frac{400}{25}=n$

$n=16$

$a_{16}=45$

$a+(n-1)d=45$

$5+(16-1)d=45$

~~16d-d~~ $15d=40$

$d=\frac{8}{3}$

classmate Date ____ Page ____

Q6: $a=17, a_n=350, d=9$

$a_n=a+(n-1)d$

$350=17+(n-1)9$

$350-17=(n-1)9$ $\qquad 333=(n-1)9$

$\frac{\cancel{333}^{111}}{\cancel{9}_{3}}=(n-1)$

$37=(n-1)$

$n=38$

Q7: $d=7, a_{22}=149$

$a+21d=149$

$a+21(7)=149$

$a=2$

$S_{22}=\frac{22}{2}(2(2)+(22-1)7)$

$11(4+21(7))$

$11(4+147)$

11×151

$=1661$

Q8: $a_2=14$ $\qquad S_{51}=?$

$a_3=18$

$\cancel{a}+d=14$

$-\cancel{a}+-2d=-18$

$-d=-4$

$d=4$

$a+d=14$

$a=10$

$S_{51}=\frac{n}{2}[2a+(n-1)d]$

$\frac{51}{2}[2(10)+(51-1)4]$

$\frac{51}{2}(20+50\times4)$

$\frac{51}{2}\times220$

$=5610$

SHARMA TUTORIAL

classmate Date ____ Page ____

Q9: $S_7=49, S_{17}=289, S_n=?$

$S_7=\frac{7}{2}[2a+(n-1)d]=49$

$\frac{7}{2}[2a+6d]=49$

$2a+6d=\frac{\cancel{49}^{7}\times2}{\cancel{7}_{1}}$

$2a+6d=14 \quad (\div2)$

$a+3d=7$ —(1)

$S_{17}=289$

$S_{17}=\frac{17}{2}[2a+(17-1)d]=289$

$\frac{17}{2}[2a+16d]=289$

$2a+16d=\frac{\cancel{289}^{17}\times2}{\cancel{17}_{1}}$

$2a+16d=34 \quad (\div2)$

$a+8d=17$ —(2)

Solving (1) & (2)

$\cancel{a}+8d=17$

$-\cancel{a}+-3d=-7$

$5d=10$

$d=2$

Put $d=2$ in (1)

$a+3(2)=7$

$a=1$

$S_n=\frac{n}{2}[2a+(n-1)d]$

$=\frac{n}{2}[2+2n-2]$

$\frac{n}{\cancel{2}}(\cancel{2}n)$

$=n^2$

10. $a_1, a_2 \text{------} a_n$

(i) $a_n = 3 + 4n$

$a_1 = 3 + 4 = 7$

$a_2 = 3 + 4(2) = 11$

$a_3 = 3 + 4(3) = 15$

$a_4 = 3 + 4(4) = 19$

$a_2 - a_1 = 11 - 7$

$d = 4$

$S_{15} = ?$

$a = 7, n = 15, d = 4$

$S_{15} = \frac{n}{2}[2a + (n-1)d]$

$= \frac{15}{2}[2(7) + (15-1)4]$

$= \frac{15}{2}[14 + 56] = \frac{15}{2} \times 70$

$S_{15} = 525$

(ii) $a_n = 9 - 5n$

$a_1 = 9 - 5 = 4$

$a_2 = 9 - 10 = -1$

$a_3 = 9 - 15 = -6$

$a_4 = 9 - 20 = -11$

$a_2 - a_1 = -1 - (4)$

$d = -5$

$S_{15} = ?$

$a = 4, d = -5, n = 15$

$S_{15} = \frac{n}{12}[2a + (n-1)d]$

$\frac{15}{2}[8 + (-70)]$

$\frac{15}{2}(-62)$

$S_{15} = -465$

11. $S_n = 4n - n^2$

$S_1 = ?, S_2 = ?, a_2 = ?, a_3 = ?, a_{10} = ?, a_n = ?$

$S_n = 4n - n^2$

$S_1 = 4 - 1 = 3$

$S_2 = 4(2) - (2)^2 = 4$

$S_3 = 12 - 9 = 3$

$S_4 = 16 - 16 = 0$

We know $S_1 = a_1 = 3$

$a_2 = S_2 - S_1$

$= 4 - 3 = 1$

$a_3 = 3 - 4 = -1$

AP: 3, 1, -1....

$d = -2$

$a_{10} = a + 9d$

$= 3 + 9(-2)$

$= -15$

$a_n = a + (n-1)d$

$3 + 2 - 2n$

$= 5 - 2n$

12. AP: 6, 12, 18 240

$S_{40} = ?$

$a = 6, d = 6, a_n = 240, n = 40$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{40}{2}(6 + 240)$

20×246

$S_{40} = 4920$

13. 8, 16, 24, 32 120

$a = 8, d = 8, a_n = 120, S_{15} = ?$

$S_{15} = \frac{n}{2}(a + a_n) = \frac{15}{2}(8 + 120)$

$= \frac{15 \times 128}{2}$

$S_{15} = 960$

14. AP: 1, 3, 5, 7, 9 49

$a = 1, d = 2, a_n = 49$

$a_n = a + (n-1)d$

$49 = 1 + (n-1)(2)$

$49 = 1 + 2n - 2$

$50 = 2n, n = 25$

$S_n = \frac{n}{2}(a + a_n)$

$= \frac{25}{2}(1 + 49)$

$= 625$

15. Penality for 1^{st} day : ₹200
No. of dys he delayed the work = 30
AP : 200, 250, 300 . . . . . 30 terms

$a = 200,\ d = 50,\ n = 30$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\frac{30}{2}[2(200) + (30-1)50]$

$= 15[400 + 1450]$

$= 15 \times 1850$

$= 27750$

so req. penality is ₹ 27750

16. Let 7 cash prices be ₹a, ₹a-20, ₹a-40, ₹a-60, ₹a-80, ₹a-100, ₹a-120

so AP is :-
$a, a-20, a-40, a-60, a-80, a-100, a-120$
Here first term = a
$d = -20,\ n = 7,\ S_n = 700$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$700 = \frac{7}{2}[2a + a_n]$

$\frac{700 \times 2}{7} = [a + a - 120]$

$\frac{\overset{100}{\cancel{700}} \times 2}{\cancel{7}_1} = 2a - 120$

$200 + 120 = 2a$

$a = \frac{320}{2}$

$a = 160$

17. No. of sections in each class = 3
Total No. of section = 12
Total classes = 12
ATQ
Total trees planted = $(3\times1) + (3\times2) + (3\times3) + (3\times4) \ldots\ldots + (3\times12)$
$3(1 + 2 + 3 + 4, 5 \ldots\ldots + 12)$
Here $a = 1,\ d = 1,\ a_n = 12,\ n = 12$

$3S_{12}$

$3 \times \frac{n}{2}(a + a_n)$

$3 \times \frac{12}{2}(1 + 12)$

$3 \times 6 \times 13$

$= 234$

18. Radii of centre A = 0.5 cm, Circumference of semicircle = πr
$r_1 = 0.5$ cm, $r_2 = 1$ cm, $r_3 = 1.5$ cm, $r_4 = 2$ cm
$\pi r_1 + \pi r_2 + \pi r_3 + \pi r_4 \ldots\ldots\ldots + \pi r_{13}$
$\pi(r_1 + r_2 + r_3 \ldots\ldots\ldots + r_{13})$
$\pi(0.5 + 1 + 1.5 + 2 \ldots\ldots\ldots + 6.5)$
$a = 0.5,\ d = 0.5,\ n = 13,\ a_n = 6.5$

πS_{13}

$\pi \times \frac{13}{2}[0.5 + 6.5]$

$\frac{22}{7} \times \frac{13}{2} \times 7$

$= 143$ cm

19. Total no. of logs = 200
ATQ
log placed in different rows are
20, 19, 18 . . . . .

classmate

Date ____
Page ____

$a = 20,\ d = -1$

Let there are n rows

$S_n = 200$

$\frac{n}{2}[2a + (n-1)d] = S_n$

$\frac{n}{2}[2(20) + (n-1)(-1)] = 200$

$\frac{n}{2}[40 - n + 1] = 200$

$n(41 - n) = 400$

$-n^2 + 41n - 400 = 0$

$n^2 - 41n + 400 = 0$

$n^2 - 25n - 16n + 400 = 0$

$n(n-25) - 16(n-25) = 0$

$(n-16)(n-25) = 0$

$n = 16,\ n = 25$

$n = 25$ Rejected because rows can't exceed than 20.

So there are 16 rows.

$a_{16} = a + 15d$

$a_{16} = 20 + 15(-1)$

$20 - 15$

$a_{16} = 5$

So there are 5 logs in the top row.

20. Distance of 1st potato from the bucket = 5m

Since rest of the potatoes are 3-3 m apart, Hence total distance travelled to pick all 10 potatoes in metres

$= 2\times5 + 2(5+3) + 2(5+3+3) \ldots\ldots$ 10 terms

$10 + 16 + 22 \ldots\ldots$ 10 terms

$a = 10,\ d = 6,\ n = 10$

$S_n = \frac{n}{2}[2a + (n-1)d]$

SHARMA TUTORIAL

classmate

Date ____
Page ____

$\frac{10}{2}[20 + 9(6)]$

$5[20 + 54]$

$= 370\ m$

Exercise 5.4 (optional)

1. $121, 117, 113 \ldots\ldots$

$a = 121,\ d = -4$

Let its first negative term be n^{th}

$a_n < 0$

$a + (n-1)d < 0$

$121 + (n-1)(-4) < 0$

$121 - 4n + 4 < 0$

$125 < 4n$

$\frac{125}{4} < n$

$31\frac{1}{4} < n$

Since natural no. just greater than $31\frac{1}{4}$ is 32

So 32^{nd} is the first negative term of this AP.

2. $a_3 + a_7 = 6$

$a_3 \times a_7 = 8$

$S_{16} = ?$

$a_3 = a + 2d$

$a + 2d + a + 6d = 6$

$2a + 8d = 6$

$a + 4d = 3$ —(1)

$a = 3 - 4d$ —(2)

$a_3 \times a_7 = 8$

$(a + 2d)(a + 6d)$

using (1)

$(3-4d+2d)(3-4d+6d) = 8$

$(3-2d)(3+2d) = 8$

$3^2-(2d)^2 = 8$

$9-4d^2 = 8$

$-4d^2 = 8-9$

$+4d^2 = +1$

$4d^2 = 1$

$d^2 = \frac{1}{4}$ $\quad d = \pm\frac{1}{2}$

when $d = +\frac{1}{2}$

From (1)

$a = 3-4\left(\frac{1}{2}\right)$

$a = 1$

$S_{16} = \frac{16}{2}\left[2+15\times\frac{1}{2}\right]$

$8\left[2+\frac{15}{2}\right]$

$S_{16} = 76$

when $d = -\frac{1}{2}$

$a = 3-4\left(-\frac{1}{2}\right)$

$a = 5$

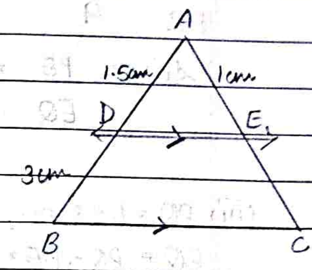
$S_{16} = \frac{16}{2}\left[2+15\times\left(-\frac{1}{2}\right)\right]$

$8\left[10-\frac{15}{2}\right]$

$S_{16} = 20$

Chapter-6
Triangles

Exercise 6.2



Q1: DE || BC

To find EC

since in ΔABC, DE || BC

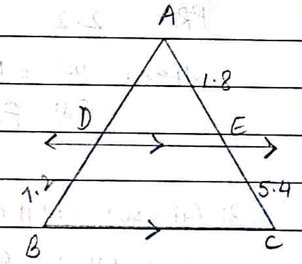
$\therefore$ By BPT

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{1.5}{3} = \frac{1}{EC}$$

$1.5EC = 3$

$EC = 2$ cm



(ii) since in ΔABC, DE || BC

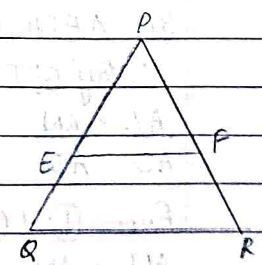
so by BPT

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{AD}{7.2} = \frac{1.8}{5.4}$$

$AD \times 5.4 = 7.2 \times 1.8$

$AD = 2.4$ cm



Q2: PE = 3.9, EQ = 3.6cm, PF = 3.6cm, FR = 2.4cm

$$\frac{PE}{EQ} = \frac{3.9}{3} = 1.3$$

$$\frac{PF}{FR} = \frac{3.6}{2.4} = \frac{3}{2}$$

Since $\frac{PE}{EQ} \neq \frac{PE}{FR}$ $\therefore$ By converse of BPT EF is not || to QR.

(ii) PE = 4cm, QE = 4.5cm, PF = 8cm, RF = 9cm

$\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9}$

$\frac{PF}{FR} = \frac{8}{9}$

since $\frac{PE}{EQ} = \frac{PF}{FR}$ so by converse of BPT
EF ∥ QR

(iii) PQ = 1.28cm, PR = 2.56cm, PE = 0.18cm, PF = 0.36cm

EQ = PQ − PE = 1.28 − 0.18 = 1.1 cm

FR = PR − PF = 2.56 − 0.36 = 2.2cm

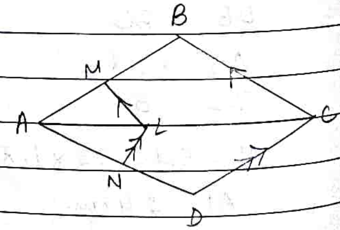
$\frac{PE}{EQ} = \frac{0.18}{1.1} = \frac{9}{55}$

$\frac{PF}{FR} = \frac{0.36}{2.2} = \frac{9}{55}$

since $\frac{PE}{EQ} = \frac{PF}{FR}$ so by converse of BPT
EF ∥ QR

Q3: Given :- LM ∥ CB, LN ∥ CD

To prove :- $\frac{AM}{AB} = \frac{AN}{AD}$



Proof :- In △ABC, ML ∥ BC

∴ By BPT

$\frac{AM}{AB} = \frac{AL}{AC}$ —(1)

In △ACD, NL ∥ CD

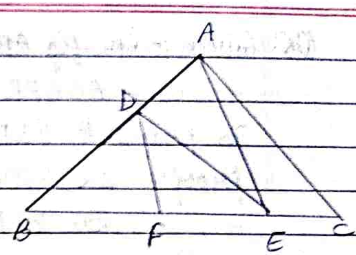
∴ By BPT

$\frac{AL}{AC} = \frac{AN}{AD}$ —(2)

From (1) & (2)

$\frac{AM}{AB} = \frac{AN}{AD}$ Hence Proved

Q4: Given :- DE ∥ AC
DF ∥ AE



To prove :- $\frac{BF}{FE} = \frac{BE}{EC}$

Proof :- In △ABC, DE ∥ AC

∴ By BPT

$\frac{BD}{DA} = \frac{BE}{EC}$ —(1)

In △ABE, DF ∥ AE

$\frac{BD}{DA} = \frac{BF}{FE}$ —(2)

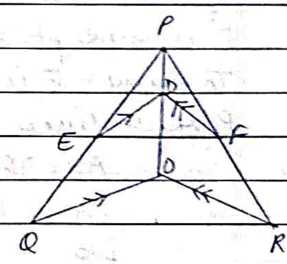
∴ From (1) & (2)

$\frac{BE}{EC} = \frac{BF}{FE}$

Hence Proved

Q5: Given :- DE ∥ OQ, DF ∥ OR

To prove :- EF ∥ QR



Proof :- In △POQ & △POR

In △POQ, DE ∥ OQ

∴ By BPT

$\frac{PD}{DO} = \frac{PE}{EQ}$ —(1)

In △POR, DF ∥ OR

∴ By BPT

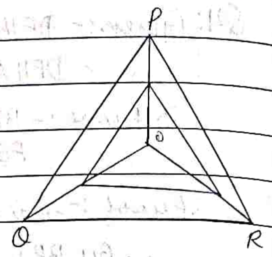
$\frac{PD}{DO} = \frac{PF}{FR}$ —(2)

From (1) & (2)

$\frac{PE}{EQ} = \frac{PF}{FR}$

∴ By converse of BPT
EF ∥ QR

Q6: Given :- In fig $AB \parallel PQ$
$AC \parallel PR$



To prove :- $BC \parallel QR$

proof :- In ΔPOQ, $AB \parallel PQ$

$\therefore$ By BPT

$\frac{OA}{AP} = \frac{OB}{BQ}$ — (1)

In ΔPOR, $AC \parallel PR$

$\frac{OC}{CR} = \frac{OA}{AP}$ — (2)

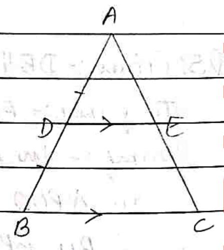
From (1) & (2)

$\frac{OB}{BQ} = \frac{OC}{CR}$

$\therefore$ By converse of BPT

$BC \parallel QR$

Q7: Given :- Let in fig In ΔABC, D is mid pt. of AB & $DE \parallel BC$



To prove :- E is mid pt. of AC

Proof :- Since D is mid pt. of AB

$AD = DB$

$\frac{DA}{DB} = 1$ — (1)

In ΔABC, $DE \parallel BC$ so by BPT

$\frac{AD}{DB} = \frac{AE}{EC}$

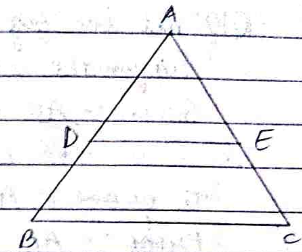
From (1)

$\frac{AE}{EC} = 1$

$AE = EC$

So E is mid pt. of AC

Q8: Given :- Let in fig In ΔABC, D & E are mid pts. of AB & AC respectively



To prove :- $DE \parallel BC$

Proof :- When D is mid pt. of ~~BC~~ AB

$AD = DB$

$\frac{AD}{DB} = 1$ — (1)

When E is mid pt. of AC

$AE = EC$

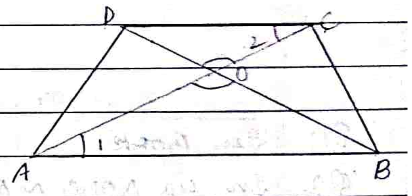
$\frac{AE}{EC} = 1$ — (2)

From (1) & (2)

$\frac{AD}{DB} = \frac{AE}{EC}$

$\therefore$ By converse of BPT $DE \parallel BC$

Q9: Given :- Let in fig ABCD is a trapezium, in which diagonal intersect at O.



To prove :- $\frac{AO}{BO} = \frac{CO}{DO}$

In ΔAOB & ΔCOD

$\angle 1 = \angle 2$ | A.I.A

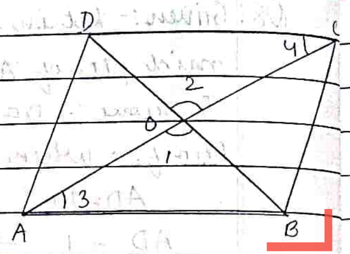
$\angle AOB = \angle COD$ | V.O.A

So by AA $\Delta AOB \sim \Delta COD$

$\frac{AO}{CO} = \frac{BO}{DO}$

$\frac{AO}{BO} = \frac{CO}{DO}$ | CPST

Q10: Let in fig a quad ABCD, whose diagonals intersect at O.



Given :- $\frac{AO}{BO} = \frac{CO}{DO}$

To prove :- ABCD is a trapezium

Proof :- $\frac{AO}{BO} = \frac{CO}{DO}$

or $\frac{AO}{CO} = \frac{BO}{DO}$ —(1)

In ΔAOB & ΔCOD

$\frac{AO}{CO} = \frac{BO}{DO}$ | From (1)

$\angle 1 = \angle 2$ | V.O.A

So $\Delta AOB \sim \Delta COD$ by SAS

Hence $\angle 3 = \angle 4$ | CPST

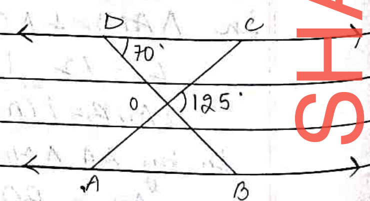
But these are A.I.A so $AB \parallel CD$

Hence ABCD is a trapezium.

Exercise 6.3

Q1: In Book

Q2: In fig $\Delta ODC \sim \Delta OBA$



$\angle BOC = 125°$, $\angle CDO = 70°$

To find :- $\angle DOC$, $\angle DCO$, $\angle OAB$

$\angle DOC + 125 = 180°$ | Linear Pair

$\angle DOC = 55°$

In ΔDOC by ASP

$70° + 55° + \angle DCO = 180°$

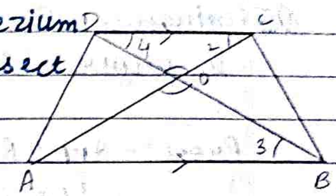
$\angle DCO = 55°$

$\angle C = \angle A$ | CPST

$\therefore \angle A = 55°$

$\angle OAB = 55°$

Q3: Let in fig. ABCD is a given trapezium in which $AB \parallel DC$ & diagonals intersect at O.



To Prove :- $\frac{OA}{OC} = \frac{OB}{OD}$

In ΔAOB & ΔCOD, $AB \parallel DC$

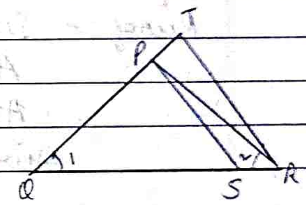
$\angle 1 = \angle 2$
$\angle 3 = \angle 4$ [A.I.A]

$\angle AOB = \angle COD$ | V.O.A

So by AAA $\Delta AOB \sim \Delta COD$

$\frac{OA}{OC} = \frac{OB}{OD}$ [CPST]

Q4: Given- In fig ΔPQS & ΔTQR



$\frac{QR}{QS} = \frac{QT}{PR}$ & $\angle 1 = \angle 2$

To prove :- $\Delta PQS \sim \Delta TQR$

Proof :- In ΔPQR

$\angle 1 = \angle 2$ | Given

So $PQ = PR$ - (1) | sides opp. to equal angles

$\frac{QR}{QS} = \frac{QT}{QR}$ - (2) | using (1)

In ΔPQS & ΔTQR

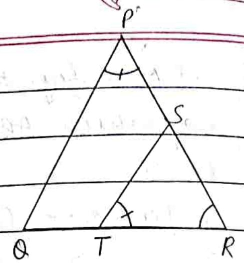
$\frac{QR}{QS} = \frac{QT}{QR}$ | Using (2)

$\angle 1 = \angle 2$ | Given

$\therefore$ By SAS $\Delta PQS \sim \Delta TQR$

Q5: Given :- $\angle P = \angle RTS$

To prove :- $\triangle RPQ \sim \triangle RTS$



Proof :- In $\triangle RPQ$ & $\triangle RTS$

$\angle P = \angle RTS$ | Given

$\angle PRQ = \angle TRS$ | Common

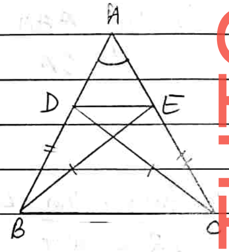
$\therefore$ By AA similarity Rule

$\triangle RPQ \sim \triangle RTS$

Hence Proved.

Q6: Given :- $\triangle ABE \cong \triangle ACD$

To prove :- $\triangle ADE \sim \triangle ABC$



Proof :- Since $\triangle ABE \cong \triangle ACD$

$AD = AE$ —(1)
$AB = AC$ —(2) [CPCT]

Dividing (1) & (2)

$\frac{AD}{AB} = \frac{AE}{AC}$ —(3)

In $\triangle ADE$ & $\triangle ABC$

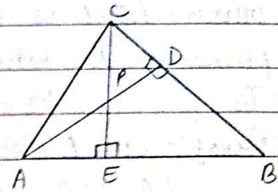
$\frac{AD}{AB} = \frac{AE}{AC}$ | From (3)

$\angle A = \angle A$ | Common

$\triangle ADE \sim \triangle ABC$ by SAS

Q7: Given :- $AD \perp BC$

$AE \perp AB$



To Prove :- (i) $\triangle AEP \sim \triangle CDP$

(ii) $\triangle ABD \sim \triangle CBE$

(iii) $\triangle AEP \sim \triangle ADB$

(iv) $\triangle PDC \sim \triangle BEC$

Proof :- (i) In $\triangle AEP$ & $\triangle CDP$

$\angle AEP = \angle CDP$ | $90°$ each

$\angle APE = \angle CPD$ | V.O.A

$\therefore$ By AA $\triangle AEP \sim \triangle CDP$

(ii) In $\triangle ABD$ & $\triangle CBE$

$\angle B = \angle B$ | Common

$\angle ADB = \angle CEB$ | $90°$ each

By AA, $\triangle ABD \sim \triangle CBE$

(iii) In $\triangle AEP$ & $\triangle ADB$

$\angle AEP = \angle BEC$ | $90°$ each

$\angle A = \angle A$ | Common

By AA $\triangle AEP \sim \triangle ADB$

(iv) In $\triangle PDC$ & $\triangle BEC$

$\angle C = \angle C$ | Common

$\angle PDC = \angle BEC$ | $90°$ each

$\therefore$ By AA $\triangle PDC \sim \triangle BEC$

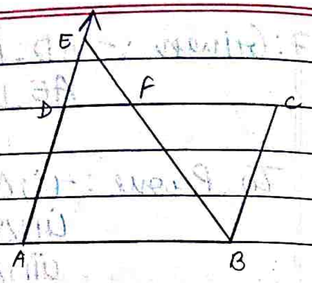
Q8: Given- ABCD is a ||gm, E is a point on AD which is produced.

To prove- $\triangle ABE \sim \triangle CFB$

Proof:- In $\triangle ABE$ & $\triangle CFB$

$\angle A = \angle C$ | opp. ∠s of ||gm

$\angle AEB = \angle EBC$ | A.I.A

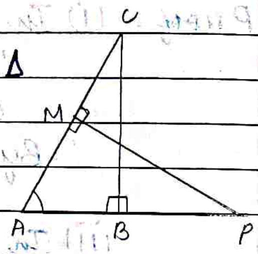


So by AA, $\triangle ABE \sim \triangle CFB$

Q9: Given:- ABC & AMP are two right ∠ed Δ at B & M

To prove:-(i) $\triangle ABC \sim \triangle AMP$

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$



Proof:- In $\triangle ABC$ & $\triangle AMP$

$\angle A = \angle A$ | Common

$\angle ABC = \angle AMP$ | 90° each

So by AA $\triangle ABC \sim \triangle AMP$

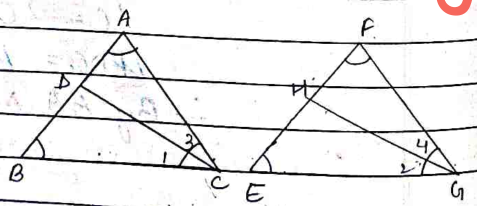
(ii) $\frac{CA}{PA} = \frac{BC}{MP}$ | CPST

Q10: Given:- CD & GH are bisectors of $\angle ACB$ & $\angle EGF$. $\triangle ABC \sim \triangle FEG$

To prove:- $\frac{CD}{GH} = \frac{AC}{FG}$

(ii) $\triangle DCB \sim \triangle HGE$

(iii) $\triangle DCA \sim \triangle HGF$



Proof:- $\angle ACB = \angle FGE$

$\frac{1}{2}\angle ACB = \frac{1}{2}\angle FGE$

$\therefore \angle 1 = \angle 2$

$\angle 3 = \angle 4$

(iii) In $\triangle DCA$ & $\triangle HGF$

$\angle A = \angle F$ | CPST

$\angle 3 = \angle 4$ | Proved Above

$\therefore$ so by AA $\triangle DCA \sim \triangle HGF$

(i) Since $\triangle DCA \sim \triangle HGF$

So $\frac{CD}{GH} = \frac{AC}{FG}$ | CPST

(ii) In $\triangle DCB$ & $\triangle HGE$

$\angle B = \angle E$ | CPST

$\angle 1 = \angle 2$ | Proved Above

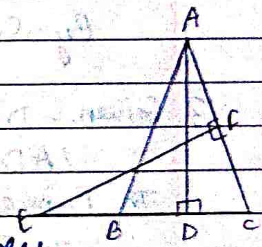
So by AA $\triangle DCB \sim \triangle HGE$

Q11: Given:- ABC is an isoceless Δ

AB = AC

AD ⊥ BC, EF ⊥ AC

To prove:- $\triangle ABD \sim \triangle ECF$



Proof:- $\angle B = \angle C$ | ∠s. opp. to equal sides

In $\triangle ABD$ & $\triangle ECF$

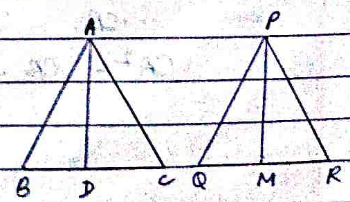
$\angle B = \angle C$ | Proved Above

$\angle ADB = \angle EFG$ | 90° each

So by AA $\triangle ABD \sim \triangle ECF$

Q12: Given :- In $\triangle ABC$ & $\triangle PQR$

$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{BC}{QR}$



AD & PM are medians

Proof:- $\frac{AB}{PQ} = \frac{AD}{PM} = \frac{BC}{QR}$ | Given

$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{2BD}{2QM}$ — (1)

In ΔADB & ΔPQM

$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{BD}{QM}$ | From (1)

$\therefore$ By SSS $\Delta ADB \sim \Delta PQM$

So $\angle B = \angle Q$ | CPST

In ΔABC & ΔPQR

$\frac{AB}{PQ} = \frac{BC}{QR}$ | Given

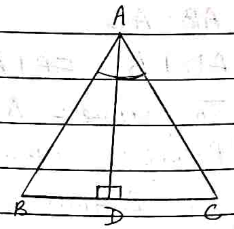
$\angle B = \angle Q$ | Proved Above

By SAS, $\Delta ABC \sim \Delta PQR$

13. Given:- D is mid pt. on BC of ΔABC

$\angle ADC = \angle BAC$

To Prove:- $CA^2 = CB.CD$



In ΔCDA & ΔCAB

$\angle C = \angle C$ | Common

$\angle CDA = \angle BAC$ | Given

$\therefore$ By AA, $\Delta CDA \sim \Delta CAB$

Now, By CPST

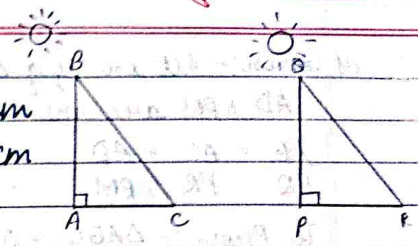
$\frac{CA}{CB} = \frac{CD}{CA}$

$CA^2 = CB \cdot CD$

15. Let in fig, 2 Poles, AB & PQ

AB = 6 cm | Cast shadow, AC = 4 cm

Pole PQ cast a shadow PR = 28 cm



In ΔABC & ΔPQR

$\angle P = \angle A$ | 90° each

$\angle C = \angle R$ | Angular Elevation of Sun

$\therefore$ By AA, $\Delta ABC \sim \Delta PQR$

$\frac{AB}{PQ} = \frac{AC}{PR}$ | CPST

$\frac{6}{PQ} = \frac{4}{28}$

$4PQ = 28 \times 6$

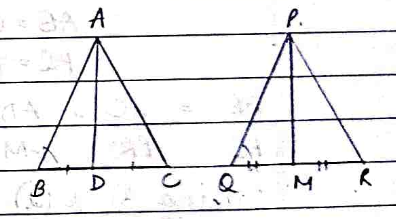
$PQ = \frac{28 \times 6}{4}$

$PQ = 42$ cm

16. Given :- In fig $\Delta ABC \sim \Delta PQR$

AD & PM are medians

To Prove:- $\frac{AB}{PQ} = \frac{AD}{PM}$



Proof:- Since, $\Delta ABC \sim \Delta PQR$

$\frac{AB}{PQ} = \frac{BC}{QR}$ | CPST

$\frac{AB}{PQ} = \frac{2BD}{2QM}$ | AD & PM are medians

In ΔABD & ΔPQM

$\frac{AB}{PQ} = \frac{BD}{QM}$ | From (1)

$\angle B = \angle Q$ | CPST

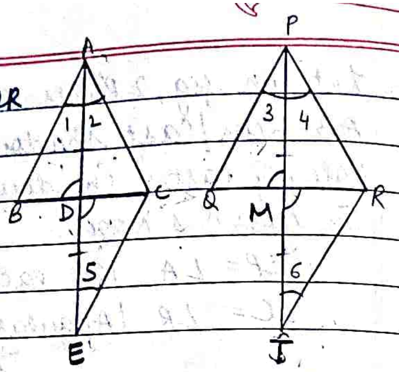
$\therefore$ By SAS $\Delta ABD \sim \Delta PQM$

$\frac{AB}{PQ} = \frac{AD}{PM}$

14. Given :- Let in fig ΔABC & ΔPQR

AD & PM are medians

$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$



To Prove :- $\Delta ABC \sim \Delta PQR$

Construction :- Extend AD & PM to points E & T respectively such that AD = DE & PM = MT

Proof: In ΔADB & ΔEDC

AD = DE | By const.

$\angle ADB = \angle EDC$ | V.O.A

BD = CD | AD is median

$\therefore$ By SAS $\Delta ADB \cong \Delta EDC$

Similarly we can prove $\Delta PMQ \cong \Delta TMR$ by SAS

$\angle 1 = \angle 5$ — (1)

$\angle 6 = \angle 3$ — (2) [CPCT]

AB = CE — (3)

PQ = TR (4) [CPCT]

$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$

Using (3) & (4)

$\frac{CE}{TR} = \frac{AC}{PR} = \frac{2AD}{2PM}$

$\frac{CE}{TR} = \frac{AC}{PR} = \frac{AD}{PM}$

$\frac{CE}{TR} = \frac{AC}{PR} = \frac{AE}{PT}$

In ΔAEC & ΔPTR

$\angle 5 = \angle 6$ (5) | CPST

Using (1), (2) (5)

$\angle 1 = \angle 5$

$\angle 5 = \angle 6$

$\angle 6 = 3$

So

$\angle 1 = \angle 3$ — (6)

$\angle 2 = \angle 4$ — (7) [CPST]

Adding (6) & (7)

$\angle 1 + \angle 2 = \angle 3 + \angle 4$

$\angle BAC = \angle QPR$

In ΔABC & ΔPQR

$\frac{AB}{PQ} = \frac{AC}{PR}$

$\therefore$ By SAS $\Delta ABC \sim \Delta PQR$

Chapter - 7
Coordinate Geometry

$A(x_1, y_1)$ & $B(x_2, y_2)$ is $AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Distance Formula $= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Exercise 7.1

Q1: (i) $(2,3), (4,1)$

Let $A(2,3)$, $B(4,1)$

Using distance formula

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(4-2)^2+(1-3)^2}$

$= \sqrt{(2)^2+(-2)^2}$

$= \sqrt{4+4}$

$= \sqrt{8} = 2\sqrt{2}$ units

(ii) $(-5,7)(-1,3)$

$D = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(-5-(-1))^2) + (7-3)^2}$

$= \sqrt{16+16}$

$= \sqrt{32}$

$= 4\sqrt{2}$ units

(iii) $(a,b), (-a,-b)$

Let $A(a,b)$, $B(-a,-b)$

$D = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(-a-a)^2+(-b-b)^2}$

$= \sqrt{4a^2+4b^2}$

$= 4\sqrt{4(a^2+b^2)}$

$= 2\sqrt{(a^2+b^2)}$ units

Q3: Let $A(1,5)$, $B(2,3)$, $C(-2,-11)$

Using distance formula :- $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$AB = \sqrt{(2-1)^2+(3-5)^2}$

$= \sqrt{1^2+(-2)^2}$

$= \sqrt{5}$ units

$BC = \sqrt{[2-(-2)]^2+[-3-(-11)]^2}$

$= \sqrt{(2+2)^2+(14)^2}$

$= \sqrt{16+196}$

$= \sqrt{212} = 2\sqrt{53}$ units

$AC = \sqrt{(-2-1)^2+(-11-5)^2}$

$= \sqrt{9+256}$

$= \sqrt{265}$

$= \sqrt{5\times 53}$ units

Since $AB + BC \neq AC$

$AB + AC \neq BC$

$BC + AC \neq AB$

So they are not collinear points

Q4: Let $A(5,-2)$, $B(6,4)$, $C(7,-2)$

$D = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$AB = \sqrt{(6-5)^2+[4-(-2)]^2}$

$= \sqrt{1^2+36}$

$= \sqrt{37}$ units

$BC = \sqrt{(6-7)^2+[4-(-2)]^2}$

$\sqrt{1^2+6^2}$

$= \sqrt{37}$ units

$AC = \sqrt{(7-5)^2+[2-(-2)]^2}$

$= \sqrt{4}$, 2units

classmate
Date
Page

They are vertices of isosceles Δ, since the measure of the two sides of this Δ is same, so these are the vertices of isosceles Δ.

Q5: Let A(3,4), B(6,7), C(9,4), D(6,1)

$D = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$AB = \sqrt{(6-3)^2+(7-4)^2}$

$= \sqrt{3^2+3^2} = \sqrt{18}$

$= 3\sqrt{2}$ units

$BC = \sqrt{(9-6)^2+(4-7)^2} = \sqrt{18} = 3\sqrt{2}$ units

$CD = \sqrt{(9-6)^2+(4-1)^2}$

$= \sqrt{3^2+3^2} = \sqrt{18}$

$= 3\sqrt{2}$ units

$AD = \sqrt{(3-6)^2+(4-1)^2} = \sqrt{3^2+3^2}$

$= \sqrt{18} = 3\sqrt{2}$ units

$AC = \sqrt{(9-3)^2+(4-4)^2}$

$= \sqrt{36} = 6$ units

$BD = \sqrt{(6-6)^2+(7-1)^2}$

$= \sqrt{36} = 6$ units

Since all the sides are same & all the diagonals are also equal so it's a square.
Champa is right, it's a square.

Q6: (i) Let A(1,-2), B(1,0), C(-1,2), D(-3,0)

$D = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$AB = \sqrt{(-1-(-1))^2+(-2-0)^2} = \sqrt{(-2)^2+(-2)^2} = \sqrt{8} = 2\sqrt{2}$ units

$BC = \sqrt{[1-(-1)]^2+(-2+0)^2} = \sqrt{8} = 2\sqrt{2}$ units

$CD = \sqrt{[1-(-3)]^2+[2-0]^2} = \sqrt{8} = 2\sqrt{2}$ units

$DA = \sqrt{[1-(-3)]^2+(-2-0)^2} = \sqrt{8} = 2\sqrt{2}$ units

classmate
Date
Page

$AC = \sqrt{[-1-(-1)]^2+[-2-(+2)]^2}$

$= \sqrt{0+(-2-2)^2}$

$\sqrt{16} = 4$ units

$BD = \sqrt{[1-(-3)]^2+(0-0)^2}$

$= \sqrt{4^2}$

$= \sqrt{16} = 4$ units

Since all the sides and diagonals are equal so it's a square.

(ii) Let A(-3,5), B(3,1), C(0,3), D(-1,-4)

$D = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$AB = \sqrt{[-3-3]^2+[5-1]^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$ units

$BC = \sqrt{(3-0)^2+(1-3)^2} = \sqrt{9+4} = \sqrt{13}$ units

$CD = \sqrt{[0-(-1)^2]+(3+(-4))^2} = \sqrt{1+49} = \sqrt{50}$ units

$AD = \sqrt{(-3-(-1))^2+(5-(-4))^2} = 2\sqrt{13}$ units

(iii) A(4,5), B(7,6), C(4,3), D(1,2)

$AB = \sqrt{(7-4)^2+(6-5)^2} = \sqrt{10}$ units

$BC = \sqrt{(7-4)^2+(6-3)^2} = \sqrt{18}$ units

$CD = \sqrt{(4-1)^2+(3-2)^2} = \sqrt{10}$ units

$AD = \sqrt{(4-1)^2+(5-2)^2} = \sqrt{18}$ units

Since all the oppisite sides are equal so its diagonals must be found.

$AC = \sqrt{(4-4)^2+(5-3)^2} = \sqrt{4} = 2$ units

$BD = \sqrt{(7-1)^2+(6-2)^2} = \sqrt{52} = 2\sqrt{13}$ units

Since all opp. sides are equal but diagonals are not equal so it's a llgm

Q7 Let A(2,-5), B(-2,9)
Let req. pt. on x axis be P(x,0)
ATQ
AP = BP

$= \sqrt{(x-2)^2 + [0-(-5)]^2} = \sqrt{(x+2)^2 + (-9)^2}$

Squaring both sides

$(x-2)^2 + (+5)^2 = (x+2)^2 + (-9)^2$

$\cancel{x^2} + \cancel{4} - 4x + 25 = \cancel{x^2} + \cancel{4} + 4x + 81$

$-4x - 4x = 81 - 25$

$= x = -7$

So req. Pt $= (-7, 0)$

Q8: $P(2,-3), Q(10,y)$

ATQ

$PQ = 10$

$\sqrt{(10-2)^2 + (y+3)^2} = 10$

Squaring both sides

$8^2 + (y+3)^2 = 10^2$

$64 + y^2 + 9 + 6y = 100$

$y^2 + 6y - 27 = 0$

$y^2 + 9y - 3y - 27 = 0$

$y(y+9) - 3(y+9) = 0$

$(y-3)(y+9)$

$y = 3, y = -9$

Q9: $Q(0,1), P(5,-3), R(x,6), x = ?, QR, PR = ?$

ATQ

$QP = QR$

$\sqrt{(5-0)^2 + (-3-(1))^2} = \sqrt{x^2 + (6-1)^2}$

$= \sqrt{5^2 + (-4)^2} = \sqrt{x^2 + 5^2}$

Squaring both sides

$25 + 16 = x^2 + 25$

$x = \pm 4$

$Q(0,1), P(5,-3), R(\pm 4, 6)$

When $x = +4$

$QR = \sqrt{(0-4)^2 + (1-6)^2}$

$= \sqrt{-4^2 + (-5)^2}$

$= \sqrt{41}$ units

$PR = \sqrt{(5-4)^2 + (-3-6)^2}$

$= \sqrt{1 + 81}$

$= \sqrt{82}$ units

When $x = -4$

$QR = \sqrt{(0+4)^2 + (1-6)^2}$

$= \sqrt{16 + 25}$

$= \sqrt{41}$ units

$PR = \sqrt{(5+4)^2 + (-3-6)^2}$

$= \sqrt{81 + 81}$

$= 9\sqrt{2}$ units

Q10: $P(x,y), A(3,6), B(-3,4)$

Since P is equidistant from AB

$PA = PB$

$\sqrt{(x-3)^2 + (y-6)^2} = \sqrt{(x+3)^2 + (y-4)^2}$

Squaring both sides

$\cancel{x^2} + \cancel{9} - 6x + y^2 + 36 - 12y = \cancel{x^2} + \cancel{9} + 6x + y^2 + 16 - 8y$

$45 - 6x - 12y = 25 + 6x - 8y$

$-6x + 45 - 12y - 25 - 6x + 8y$

$= -12x - 4y + 20 = 0 \quad (\div -4)$

$= 3x + y - 5 = 0$

SECTION FORMULA

$A(x_1, y_1) \xrightarrow{m_1} P(x, y) \xrightarrow{m_2} B(x_2, y_2)$

$P(x,y) = P\left(\dfrac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \dfrac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$

MID POINT FORMULA

$\dfrac{x_1 + x_2}{m_1 + m_2}, \dfrac{y_1 + y_2}{m_1 + m_2} = \dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}$

Exercise 7.2

1. $A(-1,7)$ —2— $P(x,y)$ —3— $B(4,-3)$

Let $A(-1,7), B(4,-3), P(x,y)$

$m_1 = 2,\ m_2 = 3$

Using section formula

$P(x,y) = P\left(\dfrac{m_1x_2 + m_2x_1}{m_1+m_2}, \dfrac{m_1y_2 + m_2y_1}{m_1+m_2}\right)$

$= P\left[\dfrac{2\times4 + 3(-1)}{2+3}, \dfrac{2(-3) + 3\times7}{2+3}\right]$

$= P\left[\dfrac{8-3}{5}, \dfrac{-6+21}{5}\right]$

$P(1,3)$

So req. pt. $P(1,3)$

2. $A(4,-1)$ — P — Q — $B(-2,-3)$

For Pt. P

$m_1 = 1,\ m_2 = 1+1 = 2$

Using section formula

$P(x,y) = P\left[\dfrac{m_1x_2 + m_2x_1}{m_1+m_2}, \dfrac{m_1y_2 + m_2y_1}{m_1+m_2}\right]$

$= P\left[\dfrac{-2+8}{3}, \dfrac{-3-2}{3}\right]$

$P\left[2, \dfrac{-5}{3}\right]$

For Pt. Q, $m_1 = 2,\ m_2 = 1$

$Q\left[\dfrac{2(-2)+4}{2+1}, \dfrac{2(-3)+1(-1)}{2+1}\right]$

$Q\left[\dfrac{-4+4}{3}, \dfrac{-6-1}{3}\right]$ $Q = \left(0, \dfrac{-7}{3}\right), P\left(2, \dfrac{-5}{3}\right)$

3. Niharika $(2,25)$, Preet $(8,20)$, Rashmi = ?

Using Mid Pt. Formula $= \dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}$

Rashmi $= \left(\dfrac{2+8}{2}, \dfrac{45}{2}\right) = R(5, 22.5)$

Distance Formula $= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(8-5)^2 + (20-22.5)^2}$ $= \sqrt{(8-2)^2 + (25-20)^2}$

$= \sqrt{9 + 6.25}$ $= \sqrt{36+25}$

$= \sqrt{15.25}$ $\sqrt{61}$ units $= \sqrt{61}$ units

4. A —k— P — B

Let $A(-3,10), B(6,-8), P(-1,6)$

Let req. Ratio be $k:1$

Using section formula

$P(-1,6) = P\left[\dfrac{6k-3}{k+1}, \dfrac{-8k+10}{k+1}\right]$

$\dfrac{6k-3}{k+1} = \dfrac{-1}{1}$

$6k - 3 = -1(k+1)$

$6k - 3 = -k - 1$

$6k + k = -1 + 3$

$7k = 2$

$k = \dfrac{2}{7}$ $(\times 7)$

$\dfrac{2}{7} : 1$ / $2:7$

$\dfrac{-8k+10}{k+1} = \dfrac{6}{1}$

$-8k + 10 = 6k + 6$

$10 - 6 = 8k + 6k$

$4 = 14k$

$k = \dfrac{2}{7}$ $(\times 7)$

$\dfrac{2}{7} : 1$ / $2:7$

5. Let $A(-1,5), B(-4,5), P(x,0)$

Let req. pt. of x axis be $P(x,0)$ which divides AB in ratio $k:1$

$P(x,0) = P\left(\dfrac{m_1x_2 + m_2x_1}{m_1+m_2}, \dfrac{m_1y_2 + m_2y_1}{m_1+m_2}\right)$

$P(x,0) = P\left(\dfrac{-4k+1}{k+1}, \dfrac{5k-5}{k+1}\right)$

Comparing both sides

$\frac{5k-5}{k+1} = 0$ $\qquad$ $\frac{-4k+1}{k+1} = x$

$5k-5 = 0$ $\qquad$ Put $k=1$

$5k = 5$ $\qquad$ $\frac{-4+1}{2} = x$

$k = 1$

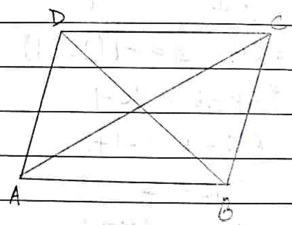
$x = \frac{-3}{2}$

So the req. pt. $= \left(\frac{-3}{2}, 0\right)$ ratio is 1:1

6. Let $A(1,2)$, $B(4,y)$, $C(x,6)$, $D(3,5)$

We know diagonals of ||gm bisect each other

$\therefore$ Mid pt. of AC = Mid pt. of BD



$\left(\frac{1+x}{2}, \frac{2+6}{2}\right) = \left(\frac{4+3}{2}, \frac{5+y}{2}\right)$

$\left(\frac{1+x}{2}, 4\right) = \left(\frac{7}{2}, \frac{5+y}{2}\right)$

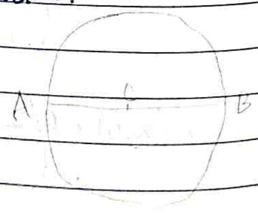
Comparing both sides

$\frac{1+x}{\cancel{2}} = \frac{7}{\cancel{2}}$ $\qquad$ $\frac{5+y}{2} = 4$

$x = 6$ $\qquad$ $y = 3$

7. Let $A(x,y)$, $B(1,4)$, $C(2,-3)$

We know center is the mid pt. of diameter.



Using Mid pt. formula :-

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

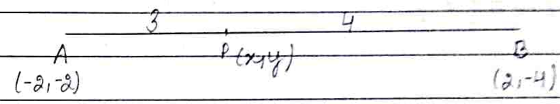
$\frac{x+1}{2} = 2$ $\qquad$ $\frac{y+4}{2} = -3$

$x+1 = 4$ $\qquad$ $y+4 = -6$

$x = 3$ $\qquad$ $y = -10$

So $A(3,-10)$

8.



$AP = \frac{3}{7} AB$

$\frac{AP}{AB} = \frac{3}{7}$ $\qquad$ $= \frac{AB}{AP} - 1 = \frac{7-3}{3}$

$\frac{AB - AP}{AP} = \frac{7-3}{3}$

$\frac{PB}{AP} = \frac{4}{3}$

$\frac{AP}{PB} = 3:4$

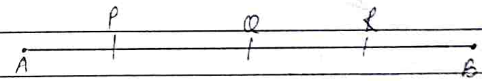
$AP : PB = 3:4$

Using section formula

$P\left(\frac{6-8}{3+4}, \frac{-12-8}{3+4}\right)$

$P\left(\frac{-2}{7}, \frac{-20}{7}\right)$

9.



$A(-2,2)$, $B(2,8)$, Let pt. P, Q, R divide AB is four equal parts as shown in fig.

Since Q is mid pt. of AB, using mid pt. formula

$Q\left(\frac{-2+2}{2}, \frac{2+8}{2}\right)$

$Q(0,5)$

Since P is mid pt. of AQ

$P\left(\frac{-2+0}{2}, \frac{2+5}{2}\right)$

$P\left(-1, \frac{2}{7}\right)$

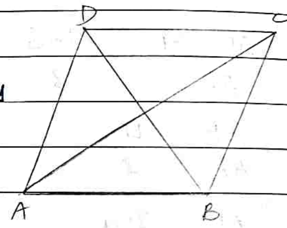
Since R is the mid. pt. of BQ

$R\left(\frac{0+2}{2}\right), \left(\frac{5+8}{2}\right)$

$R\left(1, \frac{13}{2}\right)$

Req. Pt. are $\left(-1, \frac{7}{2}\right), (0,5), \left(1, \frac{13}{2}\right)$

10. Let A (3,0), B(4,5), C(-1,4), D(-2,-1) are the vertices of given rhombus



Using distance formula

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$AC = \sqrt{(-1-3)^2+(4-0)^2}$

$= \sqrt{16+16}$

$= \sqrt{32} = 4\sqrt{2}$ units

$BD = \sqrt{(4+2)^2+(5+1)^2}$

$= \sqrt{36+36} = \sqrt{72}$

$= 6\sqrt{2}$ units

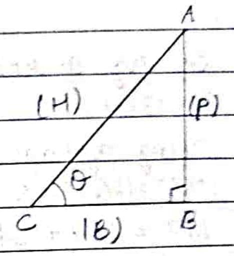
Area of rhombus $= \frac{1}{2} \times d_1 \times d_2$

$= \frac{1}{2} \times 4\sqrt{2}, 6\sqrt{2}$

$= 24 \text{ units}^2$

Chapter-8
Introduction to Trignometry.

There are six trignometric ratios (T-Ratios) :-



$\sin\theta = \frac{P}{H}$

$\frac{P}{H} \quad \frac{B}{H} \quad \frac{P}{B}$

$\cos\theta = \frac{B}{H}$

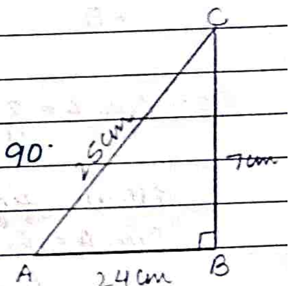
$\tan\theta = \frac{P}{B}$

$\cot\theta = \frac{BP}{P}$

$\sec\theta = \frac{H}{B}$

$\text{cosec}\,\theta = \frac{H}{P}$

Exercise 8.1



Q1:(i) sin A, cos A

(ii) sin C, cos C

Let in fig a rt.led $\triangle ABC$ is which $\angle B = 90°$

AB = 24 cm, BC = 7 cm, AC = 25 cm

Using Pythagoras Theorem

$(AC)^2 = (AB)^2 + (BC)^2$

$AC^2 = 576+49$

$AC^2 = 625$

$AC = 25$ cm

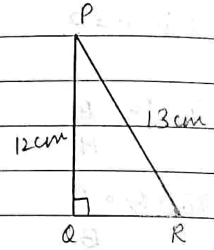
(i) $\sin A = \frac{BC}{AC} = \frac{7}{25}$

(ii) $\cos A = \frac{AB}{AC} = \frac{24}{25}$

(ii) $\sin C = \frac{AB}{AC} = \frac{24}{25}$

$\cos C = \frac{BC}{AC} = \frac{7}{25}$

Q2: In fig a rt. led ΔPQR, $\angle Q = 90°$, $PQ = 12$ cm



PR = 13 cm.

Using pythagoras Theorem

$(PR)^2 = (PQ)^2 + (QR)^2$

$169 = 144 + QR^2$

$QR^2 = 25$

$QR = 5$ cm

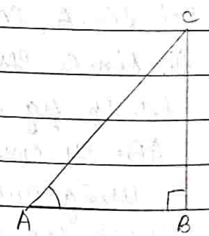
Tan P - Cot R

$\frac{QR}{QP} - \frac{QR}{QP}$

$\frac{5}{12} - \frac{5}{12}$

$= 0$

Q3: $\sin A = \frac{3}{4}$, Cos A & Tan A = ?



Let in fig a rt. led ΔABC, $\angle B = 90°$

$\sin A = \frac{BC}{AC} = \frac{3}{4}$

Let $BC = 3k$, $AC = 4k$

Using pythagoras Theorem

$(AB)^2 + (BC)^2 = (AC)^2$

$AB^2 + (3k)^2 = (4k)^2$

$AB^2 = 7k^2$

$AB = \sqrt{7}k$

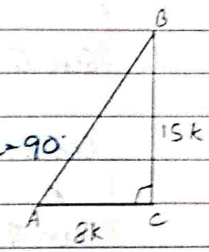
(i) $\cos A = \frac{B}{H} = \frac{AB}{AC} = \frac{\sqrt{7}\cancel{k}}{4\cancel{k}} = \frac{\sqrt{7}}{4}$

S C T

$\frac{P}{H}$ $\frac{B}{H}$ $\frac{P}{B}$

$\tan A = \frac{P}{B} = \frac{BC}{AB} = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}$

Q4: $15 \cot A = 8$



Find → sin A & Sec A

Let in fig a rt. led ΔABC, in which $\angle C = 90°$

$15 \cot A = 8$

$\cot A = \frac{8}{15}$

$\cot A = \frac{AB}{BC} = \frac{8}{15}$

Let $AC = 8k$, $BC = 15k$

Using phagoras thm.

$AB^2 = BC^2 + AC^2$

$= AB^2 = 8k^2 + 15k^2$

$= AB^2 = 64k^2 + 225k^2$

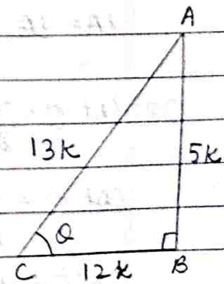
$= AB^2 = 289k^2$

$AB = 17k$

$\sin A = \frac{BC}{AB} = \frac{15}{17}$

$\sec A = \frac{AB}{AC} = \frac{17}{8}$

Q5: $\sec \theta = \frac{13}{12}$



Find → All T Ratios

$\sec \theta = \frac{13}{12} = \frac{AC}{BC}$

$AC = 13k$, $BC = 12k$

Date ____
Page ____

Using pythagoras Theorem.

$AC^2 = BC^2 + AB^2$

$169k^2 - 144k^2 = AB^2$

$25k^2 = AB^2$

$AB = 5k$

$\sin Q = \frac{P}{H} = \frac{BA}{AC} = \frac{5}{13}$

$\cos Q = \frac{B}{H} = \frac{BC}{AC} = \frac{12}{13}$

$\tan Q = \frac{P}{B} = \frac{AB}{BC} = \frac{5}{13}$

$\cot Q = \frac{B}{P} = \frac{BC}{AB} = \frac{13}{5}$

$\sec Q = \frac{H}{B} = \frac{AC}{BC} = \frac{13}{12}$

$\operatorname{cosec} Q = \frac{H}{P} = \frac{AC}{AB} = \frac{13}{5}$

Q6: $\cos A = \cos B$

Let in fig ΔABC, $\angle C = 90°$

$\cos A = \cos B$

$\frac{AC}{AB} = \frac{BC}{AB}$

$AC = BC$

$\angle A = \angle B$ | $\angle$s opp. to equal sides

Q7: $\cot Q = \frac{7}{8}$

$\cot Q = \frac{BC}{AB} = \frac{7k}{8k}$

$AC^2 = AB^2 + BC^2$

$AC^2 = 64k^2 + 49k^2$

$AC = \sqrt{113}\,k$

$\sin Q = \frac{P}{H} = \frac{8}{\sqrt{113}}$

$\cos Q = \frac{B}{H} = \frac{7}{\sqrt{113}}$

(i) $\frac{(1+\sin Q)(1-\sin Q)}{(1+\cos Q)(1-\cos Q)} = \frac{1-\sin^2 Q}{1-\cos^2 Q}$

$= \frac{\left(1+\frac{8k}{\sqrt{113}k}\right)\left(1-\frac{8k}{\sqrt{113}k}\right)^2}{\left(1+\frac{7k}{\sqrt{113}k}\right)\left(1-\frac{7k}{\sqrt{113}k}\right)^2} = \frac{\frac{1-64}{113}}{\frac{1-49}{113}}$

$= \frac{113-64}{113}$

$= \frac{49}{64}$

(ii) $\cot^2 = \frac{49}{64}$

$= \left(\frac{7}{8}\right)^2 = \frac{49}{64}$

Q8: $3\cot A = 4$

$\cot A = \frac{4}{3}$

Let in fig a rt. led ΔABC, $\angle B = 90°$

$\cot A = \frac{AB}{BC} = \frac{4}{3}$

Let $AB = 4k$, $BC = 3k$

Using Pythagoras Theorem

$(AB)^2 + (BC)^2 = AC^2$

$16k^2 + 9k^2 = AC^2$

$AC = 5k$

$\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$

LHS, RHS

$\frac{[1-(\frac{3}{4})]}{[1+(\frac{3}{4})^2]}$ $\quad$ $\left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$= \frac{1-\frac{9}{16}}{1+\frac{9}{16}}$ $\quad$ $= \frac{16}{25} - \frac{9}{25}$

$= \frac{7}{25}$

$= \frac{16-9}{16}$

$\frac{16+9}{16}$ $\quad$ LHS = RHS

$\frac{7}{16} \times \frac{16}{25}$ $\quad$ $\frac{7}{25} = \frac{7}{25}$

$= \frac{7}{25}$

Q9: $\tan A = \frac{1}{\sqrt{3}}$

(i) $(\sin A)(\cos A) + (\sin C + \cos C)$

$\frac{BC}{AB} = \frac{1}{\sqrt{3}}$ $\quad$ $BC = 1k, AB = \sqrt{3}k$

$AC^2 = AB^2 + BC^2$

$AC^2 = (\sqrt{3}k)^2 + 1^2$

$AC^2 = 4k^2$

$AC = 2k$

$\frac{BC}{AC} \times \frac{BC}{AC} + \frac{AB}{AC} \times \frac{AB}{AC}$

$\left(\frac{k}{2k}\right)^2 + \left(\frac{\sqrt{3}k}{2k}\right)^2$

$\frac{1}{4} + \frac{3}{4} = \frac{4}{4}$

$= 1$

(ii) $\cos A \cos C - \sin A \sin C$

$\frac{AB}{AB} \times \frac{BC}{AC} - \frac{BC}{AC} \times \frac{AB}{AC}$

$\left(\frac{\sqrt{3}}{2} \times \frac{1}{2}\right) - \left(\frac{1}{2} \times \frac{\sqrt{3}}{2}\right)$

$= 0$

Q10: $PR + QR = 25$ cm

$PQ = 5$ cm

Find → $\sin P, \cos P, \tan P$

Let $PR = x$ cm

$QR = (25-x)$ cm

Pythagoras Theorem

$x^2 = 5^2 + (25-x)^2$

$\cancel{x^2} = 25 + 625 + \cancel{x^2} - 50x$

$25 + 625 - 50x$

$650 - 50x$

$-650 = -50x$

$x = 13$

$PR = 13$ cm, $QR = 12$ cm

$\sin P = \frac{P}{H} = \frac{12}{13}$

$\cos P = \frac{B}{H} = \frac{5}{13}$

$\tan P = \frac{P}{B} = \frac{12}{5}$

TRIGNOMETRIC RATIOS

Trignometric Ratios at some specific angles.

θ	$0°$	$30°$	$45°$	$60°$	$90°$
Sin	$\sqrt{\frac{0}{4}}=0$	$\sqrt{\frac{1}{4}}=\frac{1}{2}$	$\sqrt{\frac{2}{4}}=\frac{1}{\sqrt{2}}$	$\sqrt{\frac{3}{4}}\ \frac{\sqrt{3}}{2}$	$\sqrt{\frac{4}{4}}\ 1$
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	N.D
Cot	N.D	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
Sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	N.D
Cosec	N.D	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Exercise 8.2

1.(i) $\sin 60° + \cos 30° + \sin 30° + \cos 60°$

$$\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$$

$$\frac{3}{4} + \frac{1}{4} = \frac{4}{4}$$

$$= 1$$

(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

$$2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 2$$

(iii) $\dfrac{\cos 45°}{\sec 30° + \operatorname{cosec} 30°}$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2} = \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}+2\sqrt{6}} = \frac{\sqrt{3}}{2(\sqrt{2}+\sqrt{6})}$$

$$= \frac{\sqrt{3}}{2(\sqrt{6}+\sqrt{2})} \times \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}$$

$$= \frac{\sqrt{18}-\sqrt{6}}{2(6-2)}$$

$$= \frac{3\sqrt{2}-\sqrt{6}}{8}$$

(iv) $\dfrac{\sin 30° + \tan 45° - \operatorname{cosec} 60°}{\sec 30° + \cos 60° + \cot 45°}$

$= \dfrac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1} = \dfrac{\frac{\sqrt{3} + 2\sqrt{3} - 4}{2\sqrt{3}}}{\frac{4 + \sqrt{3} + 2\sqrt{3}}{2\sqrt{3}}}$

$= \dfrac{3 \times \sqrt{3} - 4}{3\sqrt{3} + 4} \times \dfrac{3\sqrt{3} - 4}{3\sqrt{3} - 4} = \dfrac{27 + 16 - 24\sqrt{3}}{27 - 16}$

$= \dfrac{43 - 24\sqrt{3}}{11}$

(v) $\dfrac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$

$= \dfrac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - 1}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \dfrac{\frac{5}{4} + \frac{16}{3} - 1}{\frac{1}{4} + \frac{3}{4}}$

$= \dfrac{15 + 64 - 12}{12} = \dfrac{67}{12}$

Q2: (i) $\dfrac{2\tan^2 30°}{1 + \tan^2 30°}$

$= \dfrac{2\left(\frac{1}{\sqrt{3}}\right)^2}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} = \dfrac{\frac{2}{\sqrt{3}}}{\frac{1+1}{3}} = \dfrac{2}{\sqrt{3}} \times \dfrac{3}{4}$

$= \dfrac{\sqrt{3}}{2} = \sin 60°$

(ii) $\dfrac{1 - \tan^2 45°}{1 + \tan^2 45°}$

$= \dfrac{1 - (1)^2}{1 + (1)^2} = \dfrac{1 - 1}{2} = \dfrac{0}{2}$

$= 0$

(iii) $\sin 2A = 2\sin A$

$= 0$

(iv) $\dfrac{2\tan 30°}{1 - \tan^2 30°}$

$= \dfrac{2\left(\frac{1}{\sqrt{3}}\right)}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \dfrac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}}$

$= \dfrac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \dfrac{\cancel{2}}{\sqrt{3}} \times \dfrac{3}{\cancel{2}}$

$= \sqrt{3}$

$= \tan 60°$

3. $\tan(A + B) = \sqrt{3}$

$\tan(A - B) = \dfrac{1}{\sqrt{3}}$

$\tan(A + B) = \tan 60°$

Comparing both sides

$A + B = 60° \quad - (1)$

$\tan(A - B) = \tan 30°$

Comparing both sides

$A - B = 30° \quad - (2)$

Solving (1) & (2)

$2A = 90°$

$A = 45°$

Put $A = 45°$ in eq. (1)

$B = 60 - 45°$

$B = 15°$

Exercise 8.3

Q3: $9\sec^2 A - 9\tan^2 A$

$9(\sec^2 A - \tan^2 A)$

9×1

$= 9$

(ii) $(1+\tan\theta+\sec\theta)(1+\cot\theta-\operatorname{cosec}\theta)$

$= \left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)$

$= \left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)\left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)$

$= \frac{(\sin\theta+\cos\theta)^2-1^2}{\sin\theta\cos\theta}$

$= \frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta}$

$\frac{\cancel{1}+2\cancel{\sin\theta\cos\theta}\cancel{-1}}{\cancel{\sin\theta\cos\theta}}$

$= 2$

(iii) $(\sec A+\tan A)(1-\sin A)$

$\left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)(1-\sin A)$

$\left(\frac{1+\sin A}{\cos A}\right)(1-\sin A)$

$\frac{1^2-\sin^2 A}{\cos A}$

$= \frac{\cos^2 A}{\cos A}$

$= \cos A$

(iv) $\frac{1+\tan^2 A}{1+\cot^2 A}$

$= \frac{1+\tan^2 A}{1+\frac{1}{\tan^2 A}}$

$= \frac{1+\tan^2 A}{\frac{\tan^2 A+1}{\tan^2 A}} = \tan^2 A$

Q4(i) $(\operatorname{cosec}\theta-\cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

$LHS = (\operatorname{cosec}\theta-\cot\theta)^2$

$= \left(\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}\right)^2$

$= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$

$= \frac{(1-\cos\theta)^2}{\sin^2\theta} = \frac{(1-\cos\theta)^2}{1-\cos^2\theta}$

$= \frac{(1-\cos\theta)^2}{(1+\cos\theta)(1-\cos\theta)}$

$= \frac{1-\cos\theta}{1+\cos\theta}$ Ans

(ii) $\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A} = 2\sec A$

$LHS = \frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}$

$= \frac{\cos^2 A+(1+\sin A)^2}{\cos A(1+\sin A)}$

$= \frac{\cos^2 A+\sin^2 A+1+2\sin A}{\cos A(1+\sin A)}$

$$= \frac{1+1+2\sin A}{\cos A(1+\sin A)}$$

$$= \frac{(2+2\sin A)}{\cos A(1+\sin A)}$$

$$= \frac{2\cancel{(1+\sin A)}}{\cos A\cancel{(1+\sin A)}} = \frac{2}{\cos A} \qquad \left| \frac{1}{\cos A} = \sec A \right.$$

$$= 2\sec A$$

Hence Proved.

(iii) $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1+\sec\theta+\operatorname{cosec}\theta$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{1-\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{1-\sin\theta}{\cos\theta}} = \frac{\sin\theta}{\cos\theta} \times \frac{\sin\theta}{\sin\theta-\cos\theta} + \frac{\cos\theta}{\sin\theta} \times \frac{\cos\theta}{\cos\theta-\sin\theta}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$= \frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)} \qquad \left| a^3-b^3 = (a-b)(a^2+b^2+ab) \right.$$

$$= \frac{\cancel{(\sin\theta-\cos\theta)}(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\sin\theta\cos\theta\cancel{(\sin\theta-\cos\theta)}}$$

$$= \frac{(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\sin\theta\cos\theta} \qquad \left| \because \sin^2\theta+\cos^2\theta = 1 \right.$$

$$= \frac{1+\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta} + \frac{\sin\theta\cos\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta} \times \frac{1}{\cos\theta} + \frac{\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$= \operatorname{cosec}\theta\sec\theta + 1$$

(iv) $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

LHS $= \frac{1+\sec A}{\sec}$

$$= \frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}$$

$$= \frac{\frac{\cos A+1}{\cos A}}{\frac{1}{\cos A}}$$

$$= \cos A + 1$$

RHS $= \frac{\sin^2 A}{1-\cos A}$

$$= \frac{1-\cos^2 A}{1-\cos A}$$

$$= \frac{(1+\cos A)\cancel{(1-\cos A)}}{\cancel{(1-\cos A)}}$$

$$= 1+\cos A$$

LHS = RHS

Hence verified

(v) $\frac{\cos A-\sin A+1}{\cos A+\sin A-1} = \operatorname{cosec} A + \cot A$

LHS $= \frac{\cos A-\sin A+1}{\cos A+\sin A-1}$

Dividing Num & Deno. by $\sin A$

$$= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \frac{\operatorname{cosec} A + \cot A - 1}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec} A + \cot A) - [\operatorname{cosec}^2 A - \cot^2 A]}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec} A + \cot A) - (\operatorname{cosec} A + \cot A)(\operatorname{cosec} A - \cot A)}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec} A + \cot A)\cancel{(1 - \operatorname{cosec} A + \cot A)}}{\cancel{(\cot A + 1 - \operatorname{cosec} A)}}$$

$$= \operatorname{cosec} A + \cot A$$

LHS = RHS

Hence verified

(VI) $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

$$LHS = \sqrt{\frac{1+\sin A}{1-\sin A}}$$

$$= \sqrt{\frac{1+\sin A \times 1+\sin A}{1-\sin A \times 1+\sin A}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}$$

$$= \frac{1+\sin A}{\cos A}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$= \sec A + \tan A$$

LHS = RHS

Hence Proved.

(VII) $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

$$LHS = \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$$

$$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$$

$$= \frac{\tan\theta(1-2\sin^2\theta)}{(2\cos^2\theta - 1)}$$

$$= \frac{\tan\theta[1-2(1-\cos^2\theta)]}{2\cos^2\theta - 1}$$

$$= \frac{\tan\theta\cancel{(2\cos^2\theta - 1)}}{\cancel{(2\cos^2\theta - 1)}}$$

$$= \tan\theta$$

LHS = RHS

Hence Proved

(VIII) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

LHS = $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$ | Using $(a+b)^2$

$= \sin^2 A + \operatorname{cosec}^2 A + 2\sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2\cos A \sec A$

$= \sin^2 A + \cos^2 A + 2 + 2 + \operatorname{cosec}^2 A + \sec^2 A$ | $\sin A = \frac{1}{\operatorname{cosec} A}$

$= 1 + 2 + 2 + \operatorname{cosec}^2 A + \sec^2 A$

$= 5 + (1 + \cot^2 A) + (1 + \tan^2 A)$ | $\cos A = \frac{1}{\sec A}$

$= 5 + 2 + \cot^2 A + \tan^2 A$ | $\operatorname{cosec}^2 A = 1 + \cot^2 A$

$= 7 + \cot^2 A + \tan^2 A$ | $\sec^2 A = 1 + \tan^2 A$

= RHS

LHS = RHS

Hence Proved.

(IX) $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \dfrac{1}{\tan A + \cot A}$

LHS $= (\operatorname{cosec} A - \sin A)(\sec A - \cos A)$

$= \left(\dfrac{1}{\sin A} - \sin A\right)\left(\dfrac{1}{\cos A} - \cos A\right)$

$= \left(\dfrac{1-\sin^2 A}{\sin A}\right)\left(\dfrac{1-\cos^2 A}{\cos A}\right)$

$= \dfrac{\cos^2 A}{\sin A} \times \dfrac{\sin^2 A}{\cos A}$

$= \sin A \cos A \quad - (1)$

RHS $= \dfrac{1}{\tan A + \cot A}$

$= \dfrac{1}{\dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A}}$

$= \dfrac{1}{\dfrac{\sin^2 A + \cos^2 A}{\sin A \cos A}}$

$= \dfrac{1}{\dfrac{1}{\sin A \cos A}} = \sin A \cos A$

LHS = RHS

Hence Proved

(X) $\left(\dfrac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\dfrac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

$= \dfrac{1+\tan^2 A}{1+\cot^2 A}$

$= \dfrac{1+\tan^2 A}{1+\dfrac{1}{\tan^2 A}}$

$= \dfrac{1+\tan^2 A}{\tan^2 A + 1}$

$= \tan^2 A \quad - (1)$

$\left(\dfrac{1-\tan A}{1-\cot A}\right)^2$

$= \left(\dfrac{1-\tan A}{1-\dfrac{1}{\tan A}}\right)^2$

$= \left(\dfrac{1-\tan A}{\dfrac{\tan A - 1}{\tan A}}\right)^2$

$= \left[\dfrac{-(\tan A - 1)}{\dfrac{\tan A - 1}{\tan A}}\right]^2$

$= (-\tan A)^2$

$= \tan^2 A \quad - (2)$

From (1) & (2)

Hence Proved.

Chapter-13
Statistics

$$\text{Mode} = l + \left[\frac{F_1 - F_0}{2F_1 - F_0 - F_2}\right] h$$

Exercise 13.2

Q1:

Age (yrs)	5-15	15-25	25-35	35-45	45-55	55-65
No. of Patients (f)	6	11	21	23	14	5
			F_0	F_1	F_2	

Model class = 35-45

Here $l = 35$, $f_1 = 23$, $F_2 = 14$, $F_0 = 21$, $h = 10$

$$\text{Mode} = l + \left[\frac{F_1 - F_0}{2F_1 - F_0 - F_2}\right] \times h$$

$$35 + \left[\frac{23-21}{46-21-14}\right] 10$$

$$35 + \left[\frac{20}{11}\right]$$

$$35 + 1.818$$

$$36.82 \text{ yrs (approx)}$$

By Mistake

Q2:

Daily wages (₹)	500-520	520-540	540-560	560-580	580-600
No. of workers	12	14	8	6	10
	F_0	F_1	F_2		

Model class = 520-540

Here $l = 520$, $F_1 = 14$, $F_2 = 8$, $F_0 = 12$, $h = 20$

$$\text{Mode} = l + \left[\frac{F_1 - F_0}{2F_1 - F_0 - F_2}\right] h$$

$$= 520 + \left[\frac{14-12}{2(14)-12-8}\right]20$$

$$= 520 + \left[\frac{2}{28-20}\right]20 = 520 + \left[\frac{40}{8}\right]$$

$$= 525$$

Q2:

Lifetimes (in hours)	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	10	35	52	61	38	29
			f_0	f_1	f_2	

Modal class = 60 − 80

$l = 60, f_0 = 52, f_1 = 61, f_2 = 38, h = 20$

$$\text{Mode} = l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

$$= 60 + \left[\frac{61-52}{2(61)-52-38}\right] \times h$$

$$= 60 + \left[\frac{9}{122-52-38}\right] \times 20$$

$$= 60 + \left[\frac{180}{32}\right] \frac{90}{16} \frac{45}{8}$$

$$= 60 + \frac{45}{8}$$

$$= 60 + 5.625$$

$= 65.625$ hours

Q3:

Expenditure (in ₹)	Number of Families	
1000-1500	24	f_0
1500-2000	40	f_1
2000-2500	33	f_2
2500-3000	28	
3000-3500	30	
3500-4000	22	
4000-4500	16	
4500-5000	7	

Modal class = 1500 − 2000

$l = 1500, f_0 = 24, f_1 = 40, f_2 = 33, h = 500$

$$\text{Mode} = l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] h$$

$$= 1500 + \left[\frac{40-24}{2(40)-24-33}\right] \times 500$$

$$= 1500 + \left[\frac{16}{80-57}\right] \times 500$$

$$= 1500 + \left[\frac{16}{23}\right] \times 500$$

$$= 1500 + \left[\frac{8000}{23}\right]$$

$$= 1847.83$$

Q4:

No. of students	No. of states/UT	
15-20	3	
20-25	8	
25-30	9	f_0
30-35	10	f_1
35-40	3	f_2
40-45	0	
45-50	0	
50-55	2	

Modal class = 30 − 35

$f_1 = 10, f_2 = 3, f_0 = 9, l = 30, h = 5$

$$l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] h$$

$$= 30 + \left[\frac{10-9}{20-9-3}\right] 5$$

$$= 30 + \left[\frac{5}{8}\right]$$

$= 30 + \left[\frac{5}{8}\right]$

$= 30.625$

0.625
8)5
-0
50
-48
20
-16
40

Q5:

Runs scored	No. of Batsmen	
3000-4000	4	f_0
4000-5000	18	f_1
5000-6000	9	f_2
6000-7000	7	
7000-8000	6	
8000-9000	3	
9000-10,000	1	
10,000-11,000	1	

Modal class = 4000-5000

$f_1 = 18,\ f_2 = 9,\ f_0 = 4,\ l = 4000,\ h = 1000$

36
13
23

$= l + \left[\frac{f_1 - f_0}{2f_1 - f_2 - f_0}\right] h$

$= 4000 + \left[\frac{18-4}{36-9-4}\right] 1000$

$= 4000 + \left[\frac{14000}{23}\right]$

$= 4000 + 608.69$

$= 4608.69$

$= 4608.7$ (approx)

Q6:

No. of Runs	f_i
1-10	7
10-20	14
20-30	13
30-40	12

No. of runs	f_i
40-50	20
50-60	11
60-70	15
70-80	8

$l = 40$, Modal class = 40-50, $f_1 = 20,\ f_0 = 12,\ f_2 = 11,\ h = 10$

$= l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h \quad = 40 + \left[\frac{20-12}{2(20)-12-11}\right] \times 10$

$= 40 + \left[\frac{80}{17}\right]$

$= 40 + 4.7$

$= 44.7$

Exercise 13.1

(i) $\frac{\Sigma f_i x_i}{\Sigma f_i}$ (direct method) (ii) $a + \frac{\Sigma f_i d_i}{\Sigma f_i}$ (Assumed mean method)

(iii) $a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) h$ (step deviation method)

Q1:

No. of plants	x_i	No. of houses (f_i)	$f_i x_i$
0-2	1	1	1
2-4	3	2	6
4-6	5	1	5
6-8	7	5	35
8-10	9	6	54
10-12	11	6	22
12-14	13	2	39
		$\Sigma f_i = 20$	$\Sigma f_i x_i = 162$

Mean $= \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{162}{20}$

$= 8.1$ plants

Q2:

Daily wages	x_i	f_i	$d_i(x_i-a)$	f_id_i
500 - 520	510	12	-40	-480
520 - 540	530	14	-20	-280
540 - 560	550	8	0	0
560 - 580	570	6	20	120
580 - 600	590	10	40	400
		$\Sigma f_i = 50$		$\Sigma f_id_i = -240$

$$\text{Mean} = a + \frac{\Sigma f_id_i}{\Sigma f_i}$$

$$a = 550 \quad = 550 + \frac{(-240)}{50}$$

$$= ₹542.2$$

Q4:

No. of Heartbeat	x_i	f_i	$d_i(x_i-a)$	f_id_i
65-68	66.5	2	-9	-18
68-71	69.5	4	-6	-24
71-74	72.5	3	-3	-9
74-77	75.5	8	0	0
77-80	78.5	7	3	21
80-83	81.5	4	6	24
83-86	84.5	2	9	18
		$\Sigma f_i = 30$		$\Sigma f_id_i = 12$

$$\text{Mean} = a + \frac{\Sigma f_id_i}{\Sigma f_i}$$

$$= 75.5 + \frac{12}{30} = 75.5 + 0.4$$

$$= 75.9$$

Q3:

Daily Allowance	f_i	x_i	f_ix_i
11-13	7	12	84
13-15	6	14	84
15-17	9	16	144
17-19	13	18	234
19-21	f	20	20f
21-23	5	22	110
23-25	4	24	96
	$\Sigma f_i = 44+f$		$\Sigma f_ix_i = 752+20f$

$$\text{Mean} = \frac{\Sigma f_ix_i}{\Sigma f_i} = \frac{752+20f}{44+f} = 18$$

$$792 + 18f = 752 + 20f$$

$$792 - 752 = 20f - 18f$$

$$40 = 2f$$

$$f = 20$$

Q5:

No. of Mangoes	f_i	x_i	f_ix_i
49.5 - 52.5	15	51	765
52.5 - 55.5	110	54	5940
55.5 - 58.5	135	57	7695
58.5 - 61.5	115	60	6900
61.5 - 64.5	25	63	1575
	$\Sigma f_i = 400$		$\Sigma f_ix_i = 22875$

$$\text{Mean} = \frac{\Sigma f_ix_i}{\Sigma f_i} = \frac{22875}{400}$$

$$= 57.187$$

$$= 57.19 \text{ (approx)}$$

Q6:

Daily expenditure	f_i	x_i	f_ix_i
100 - 150	4	125	500
150 - 200	5	175	875
200 - 250	12	225	2700
250 - 300	2	275	550
300 - 350	2	325	650
	$\Sigma f_i = 25$		$\Sigma f_ix_i = 5275$

$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{5275}{25}$

$= 211$

Q7:

Concentration of SO_2	f_i	x_i	$f_i x_i$
0.00 - 0.04	4	0.02	0.08
0.04 - 0.08	9	0.06	0.54
0.08 - 0.12	9	0.10	0.90
0.12 - 0.16	2	0.14	0.28
0.16 - 0.20	4	0.18	0.72
0.20 - 0.24	2	0.22	0.44
	$\Sigma f_i = 30$		$\Sigma f_i x_i = 2.96$

$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{2.96}{30}$

$= 0.0986$

$= 0.099$ (approx)

Q8:

No. of days	f_i	x_i	$f_i x_i$
0 - 6	11	3	33
6 - 10	10	8	80
10 - 14	7	12	84
14 - 20	4	17	68
20 - 28	4	24	96
28 - 38	3	33	99
38 - 40	1	39	39
	$\Sigma f_i = 40$		$\Sigma f_i x_i = 499$

$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{499}{40}$

$= 12.475$ days

$= 12.48$ (approx)

Q9:

Literacy Rate (%)	f_i	x_i	d_i	u_i	$f_i u_i$
45 - 55	3	50	-20	-2	-6
55 - 65	10	60	-10	-1	-10
65 - 75	11	70	0	0	0
75 - 85	8	80	10	1	8
85 - 95	3	90	20	0	6
	$\Sigma f_i = 35$				$\Sigma f_i u_i = -2$

$\text{Mean} = a + \left[\frac{\Sigma f_i u_i}{\Sigma f_i}\right] \times h = 70 + \left[\frac{-2}{35}\right] 10$

$= 70 - \frac{4}{7} = 70 - 0.57$

$= 69.43\%$

$\text{Median} = l + \left[\frac{n/2 - cf}{f}\right] \times h$

Exercise 13.3

1.

Monthly consumption	f	cf
65 - 85	4	4
85 - 105	5	9
105 - 125	13	22
125 - 145	20	42
145 - 165	14	56
165 - 185	8	64
185 - 205	4	68
	N = 68	

$\text{Median} = l + \left[\frac{n/2 - cf}{f}\right] \times h = 125 + \left[\frac{34 - 22}{20}\right] 20$

Median class = 125 - 145 $\quad = 125 + 20$

$l = 125,\ f = 20,\ h = 20,\ cf = 22 \quad = 137$ Units

2.

CI	F	Cf
0-10	5	5
10-20	x	$5+x$
20-30	20	$25+x$
30-40	15	$40+x$
40-50	y	$40+x+y$
50-60	5	$45+x+y$
N = 60		

Median = 28.5

Median class :- 20-30

$l = 20, f = 20, Cf = 5+x$

$45+x+y = 60$

$x+y = 15$ —(1)

$l + \left[\frac{n/2 - Cf}{f}\right] \times h$

$20 + \left[\frac{30-5-x}{20}\right] \times 10 = 28.5$

$\frac{25-x}{20} = 8.5$

$25 - x = 17$

$-x = 17 - 25$

$x = 8, y = 7$

$x + y = 15$

$8 + y = 15$

$y = 7$

3.

Age (in yrs)	Cf	f
15-20	2	2
20-25	6	4
25-30	24	18
30-35	45	21
35-40	78	33
40-45	89	11
45-50	92	3
50-55	98	6
55-60	100	2
		N = 100

Median class = 35-40

$l = 35, n/2 = 50, f = 33, Cf = 45$

Median = $l + \left[\frac{n/2 - Cf}{f}\right] h$

$35 + \left[\frac{50-45}{33}\right] 5$

$= 35 + \left[\frac{25}{33}\right]$

$= 35 + 0.75$

$= 35.75$

Q4:

Length	F	Cf
117.5-126.5	3	3
126.5-135.5	5	8
135.5-144.5	9	17
144.5-153.5	12	27
153.5-162.5	5	34
162.5-171.5	4	38
171.5-180.5	2	40

Median class :- 144.5-153.5

$l = 144.5, f = 12, Cf = 17, h = 9$

$l + \left[\frac{n/2 - Cf}{f}\right] \times h$

$144.5 + \left[\frac{20-17}{12}\right] \times 9$

$= 144.5 + \left[\frac{9}{4}\right]$

$= 146.7$ (approx)

Q5:

Lifetime	F	Cf
1500-2000	14	14
2000-2500	56	70
2500-3000	60	130
3000-3500	86	216
3500-4000	74	290
4000-4500	62	352
4500-5000	48	400
	N = 400	

Median class = 3000-3500

$l = 3000, f = 86, Cf = 130, h = 500$

Median = $l + \left[\frac{n/2 - Cf}{f}\right] \times h$

$= 3000 + \left[\frac{200-130}{86}\right] \times 500$

$= 3000 + \left[\frac{35000}{86}\right]$

$= 3000 + \left[\frac{17500}{43}\right]$

$= 3000 + 407$

$= 3407$

Q6:

No. of letters	F	Cf
1-4	6	6
4-7	30	36
7-10	40	76
10-13	16	92
13-16	4	96
16-19	4	100
	N = 100	

Median class = 7-10

$l = 7, Cf = 36, h = 3, f = 40$

Median = $l + \left[\frac{n/2 - Cf}{f}\right] \times h$

$7 + \left[\frac{50-36}{40}\right] 3$

$= 7 + \left[\frac{14}{40}\right] 3$

$= 7 + \frac{21}{20}$

$= 7 + 1.05$

$= 8.05$

Q7:

Weight	F	Cf
40-45	2	2
45-50	3	5
50-55	8	13
55-60	6	19
60-65	6	25
65-70	3	28
70-75	2	30

N = 30

Median Class = ~~60-65~~ 55-60

$l = 55$, $f = 6$, $cf = 13$, $h = 5$, $n = 30$

$$\text{Median} = l + \left[\frac{n/2 - cf}{f}\right] \times h$$

$$= 55 + \left[\frac{15-13}{6}\right] \times 5$$

$$= 55 + \left[\frac{2}{6}\right] \times 5$$

$$= 55 + \left[\frac{5}{3}\right]$$

$$= 55 + 1.66$$

$$= 56.66$$

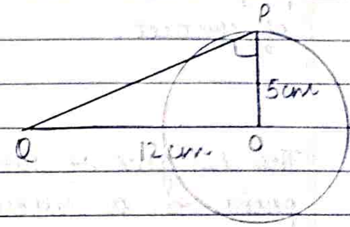
$$= 56.7 \text{ (approx)}$$

Chapter-10

Circles

Exercise 10.1

3. Let in fig PQ is a tangent



OP = 5 cm (Radius)

OQ = 12 cm

We know tangent is perpendicular to radius at the pt. of contact

$\therefore \angle OPQ = 90°$

In rt.-led $\triangle OPQ$, by pythagoras thm.

$OP^2 + PQ^2 = OQ^2$

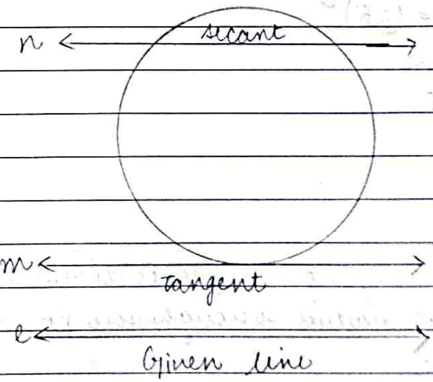
$5^2 + 12^2 = PQ^2$

$25 + 144 = PQ^2$

$119 = PQ^2$

$PQ = \sqrt{119}$ cm

4.



Here 'l' is given line

'n' is secant

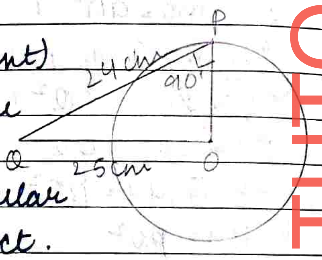
'm' is tangent

The tangent at any point of a circle is prependicular to the radius through the point of contact.

The lengths of tangent drawn from an external point to a circle are equal.

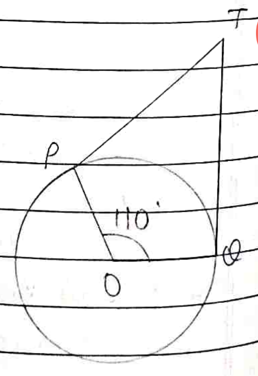
Exercise 10.2

1. Given - Let in fig OP=24cm (a tangent) OQ=25cm. Join OP where O is the centre of circle.



we know tangent is prendendicular to the circle at the pt. of contact.
So ∠OPQ = 90°
In rt. led ΔOPQ, by pythagoras thm.
$OP^2 + PQ^2 = OQ^2$
$OP^2 + (24)^2 = (25)^2$
$OP^2 + 576 = 625$
$OP^2 = 625 - 576$
$OP^2 = 49$
$OP = 7cm$

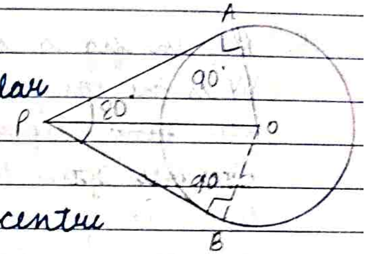
2. we know tangent is prependicular to the ~~circle~~ radius through the pt. of contact.
In fig TP & TQ are tangent from pt. T



∠POQ = 110°
∠OPT = ∠OQT = 90° each

In quad. OPQT using ASP
∠OPT + ∠OQT + ∠POQ + ∠PTQ = 360°
90° + 90° + 110° + ∠PTQ = 360°
290 + ∠PTQ = 360°
∠PTQ = 360° − 290°
∠PTQ = 70°

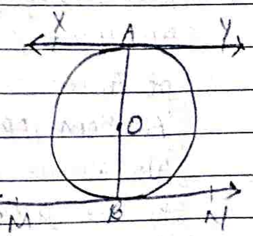
3. we know tangent is prependicular to the radius at the pt. of contact.



PA & PB are tangents O is the centre
∠APB = 80°
In ΔPOA & ΔPOB
PO = PO | common
AO = BO | Radii
∠PAO = ∠PBO = | 90° each
So by RHS ΔPOA ≅ ΔPOB
∠APO = ∠BPO = $\frac{80}{2}$ = 40°

In ΔPAO by ASP
= ∠PAO + ∠APO + ∠AOP = 180°
90° + 40° + ∠AOP = 180°
∠AOP = 180 − 130°
∠AOP = 50°

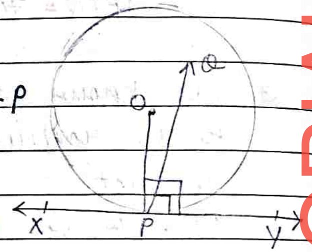
4. Let in fig a circle with centre O.
AB is the diameter of circle. Tangents XY & MN are drawn throught pt. A & B respectively.
To Prove XY ∥ MN



Proof:- we know tangent is prependicular to the

radius at the pt. of contact.
So ∠YAB = ∠MBA | 90° each
But these are alt. int. angles
So XY || MN
Hence Proved

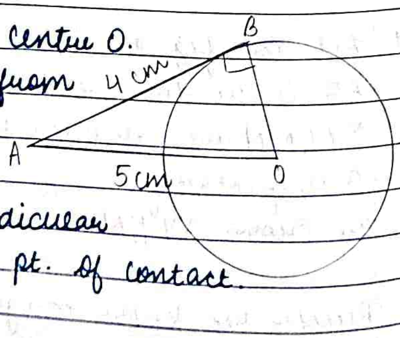
5. Let in fig a circle with centre O.
XY is the tangent on the circle at pt. P
Let ~~tano~~ Perpendicular at pt. P
does not pass through the centre
but it passes through other pt. Q
as shown in figure.



∴ ∠QPY = 90° —①
we know tangent is perpendicular to the radius
at the pt. of contact.
So ∠OPY = 90° —②
From ① & ②
∠QPY = ∠OPY | 90° each
But is possible only when OP & QP are coincident.

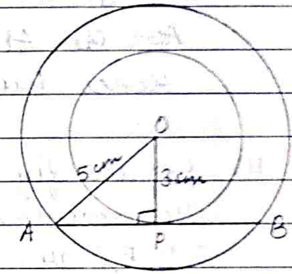
If these are coincident than P passes through the the centre.
Hence Proved.

6. Let in fig. a circle with centre O.
AB = 4cm, is the tangent from pt. A.
AO = 5cm, AB = 4cm,
we know tangent is perpendicular
to the radius through the pt. of contact.
∠OBA = 90



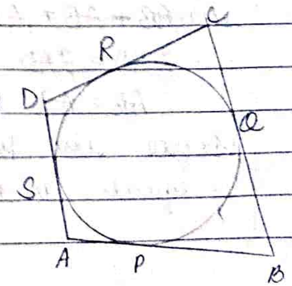
In rt. ∠d △OAB, using pythagoras thm.
$(AO)^2 = (OB)^2 + (AB)^2$
$5^2 = (OB)^2 + (4)^2$
$25 - 16 = OB^2$
$9 = OB^2$
OB = 3cm
Hence radius is 3cm

7. Let in fig. two concentric circles
of Radii, 3 cm & 5 cm
AB is the chord for bigger circle &
tangent for smaller circle.
P is the pt. of contact.
Join OP, OA.



we know ~~per~~ tangent is perpendicular to the radius
at the pt. of contact.
so ∠OPA = 90°, OA = 5cm, OP = 3cm
In △OPA, by pythagoras thm.
$(OA)^2 = (OP)^2 + AP^2$
$5^2 = 3^2 + AP^2$
$25 - 9 = AP^2$
$AP^2 = 16$
AP = 4cm
Similarly we can find PB = 4cm
Hence AB = 4+4cm
AB = 8cm
So req. length is 8cm

8. In fig ABCD quad. is circumscribing a circle.
To prove :- AB + CD = AD + BC



Proof:- we know tangents from external points to a circle are equal.

$AP = AS$ —(1)

$PB = BQ$ —(2)

$CR = CQ$ —(3)

$DR = DS$ —(4)

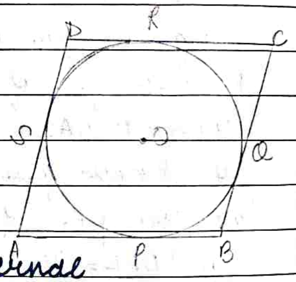
Adding above eq.s

$AP + PB + CR + DR = AS + BQ + CQ + DS$

$AB + CD = AD + BC$

Hence Proved.

11. Let in fig ABCD is a ||gm which circumscribes a circle.



P, Q, R, S are the pt. of contact

To prove:- ABCD is a rhombus.

Proof:- we know tangent from external pts. to a circle are equal.

$AP = AS$ —(1)

$PB = BQ$ —(2)

$CR = CQ$ — (3)

$DR = DS$ —(4)

Adding above eq.s

$AP + PB + CR + DR = AS + BQ + CQ + DS$

$AB + CD = AD + BC$

Since opp. sides of ||gm are equal.

so $AB + AB = AD + AB$

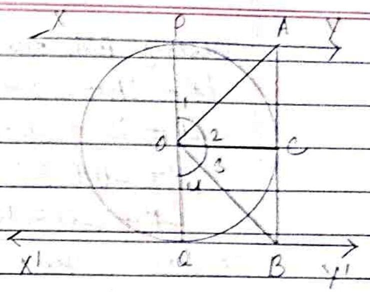
$2AB = 2AD$

$AB = AD$

Since in this ||gm one pair of adjecent sides is equal so it is a rhombus.

Hence Proved.

9. In fig $XY \parallel X'Y'$ are two tangents on circle with centre O.



AB is other tangent at pt. C.

To prove:- $\angle AOB = 90°$

In fig Join OC, since $XY \parallel X'Y'$ so PQ is a diameter, because we know tangents at the end pts. of diameter are ||.

In ΔAPO & ΔACO

$AP = AC$ | tangents from external pts.

$AO = AO$ | common

$OC = OP$ | Radii

$\therefore$ SSS $\Delta APO \cong \Delta ACO$

Hence $\angle 1 = \angle 2$ | CPCT —(1)

similarly we can prove $\Delta BOC \cong \Delta BOQ$ by SSS

$\therefore \angle 3 = \angle 4$ | CPCT —(2)

$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180°$ | Sum of angles on straight line.

using (1) & (2)

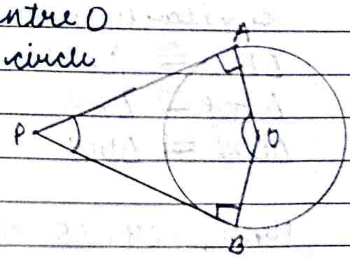
$2\angle 2 + 2\angle 3 = 180°$

$2(\angle 2 + \angle 3) = 180°$

$\angle 2 + \angle 3 = 90°$

Hence $\angle AOB = 90°$

10. Let in fig. a circle with centre O PA & PB are two tangents on the circle on external point P.



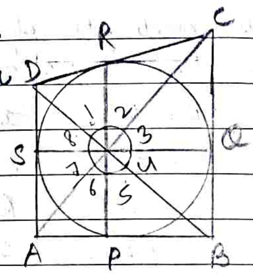
Join AO & BO

To Prove:- $\angle P + \angle AOB = 180°$

We know tangent is perpendicular to the radius through the point of contact.

Using ASP in a quad. PAOB
$\angle P + \angle A + \angle O + \angle B = 360°$
$\angle P + 90° + \angle O + 90° = 360°$
$\angle P + \angle O + 180° = 360°$
$\angle P + \angle O = 180°$
Hence 2 two angles are supplementary.

13. Given- Let in fig ABCD is a quad. which circumscribes a circle with centre O.
P, Q, R, S are the pts. of contact



To Prove :- $\angle DOC + \angle AOB = 180°$
$\angle COB + \angle AOD = 180°$

Proof:- Join the centre O to all the pts. of contact.
In ΔCRO & ΔCQO
CR = CQ | Tangents from external pt.
CO = CO | common
RO = QO | Radii

$\therefore$ SSS $\Delta CRO \cong \Delta CQO$
$\therefore \angle 2 = \angle 3$ —(1) | CPCT

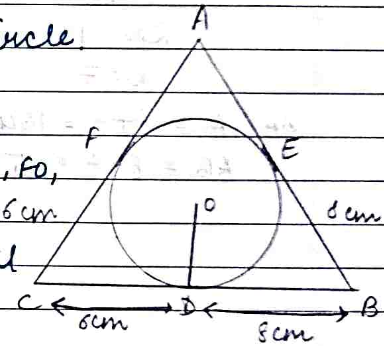
Similarly we can prove By SSS
$\Delta BOQ \cong \Delta BOP$
$\Delta AOP \cong \Delta AOS$
$\Delta DOS \cong \Delta DOR$

Hence, $\angle 4 = \angle 5$ —(2)
$\angle 6 = \angle 7$ —(3) [CPCT]
$\angle 8 = \angle 1$ —(4)

Since sum of angles around a pt. is 360°
$\therefore \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$
$\angle 1 + \angle 2 + \angle 2 + \angle 5 + \angle 5 + \angle 6 + \angle 6 + \angle 1 = 360°$
$2\angle 1 + 2\angle 2 + 2\angle 5 + 2\angle 6 = 360°$
$2(\angle 1 + \angle 2 + \angle 5 + \angle 6) = 360°$
$(\angle 1 + \angle 2 + \angle 5 + \angle 6) = 180°$

Hence $\angle DOC + \angle AOB = 180°$
Similarly we can prove
$\angle COB + \angle AOD = 180°$
Hence these are supplementary
Hence Proved.

12. In fig ΔABC circumscribes a circle
Radius of circle = 4cm
CD = 6cm, BD = 8cm
D, E, F are pts. of contact. Join EO, FO, CO, AO, BO. Since Tangents from external pts. of the circle are equal
So CD = CF = 6cm, BD = BE = 8cm
Let AE = AF = x cm



In ΔABC
$a = 14$ cm, $b = (6+x)$ cm, $c = (8+x)$ cm
$S = \dfrac{14+6+x+8+x}{2}$

$S = \dfrac{28+2x}{2}$ $\quad S = 14+x$

Using Heron's Formula:- $\sqrt{S(S-a)(S-b)(S-c)}$
$= \sqrt{(14+x)(\cancel{14}+x-\cancel{14})(14+\cancel{x}-6-\cancel{x})(14+\cancel{x}-8-\cancel{x})}$
$= \sqrt{(14+x)(x)(8)(6)}$
$= \sqrt{(14+x)(x)(2\times2\times2\times2\times3)}$
$= 4\sqrt{3(14+x)}$ —(1)

Area of ΔABC = Ar. ΔCOB + Ar ΔAOC + Ar. ΔAOB

$$= \frac{1}{2} \times 14 \times \cancel{4}^{2} + \frac{1}{\cancel{2}} \times (6+x)\cancel{4}^{2} + \frac{1}{\cancel{2}} \times (8+x)\cancel{4}^{2}$$

$$= 28 + 12 + 2x + 16 + 2x$$

$$= 56 + 4x$$

Ar. ΔABC = $4(14+x)$ — ②

equating ① & ② because they are the area of same Δ

$$\cancel{4}\sqrt{3x(14+x)} = \cancel{4}(14+x)$$

squaring both sides

$$3x\cancel{(14+x)} = (14+x)^{\cancel{2}}$$

$$3x = 14 + x$$

$$3x - x = 14$$

$$2x = 14$$

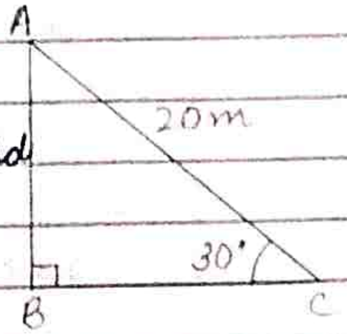
$$x = 7$$

So $AC = 6 + 7 = 13$ cm

$AB = 8 + 7 = 15$ cm

Chapter - 9
Some Applications of Trignometry

Exercise 9.1



1. In fig AB is a pole and AC = 20 m long rope which is tied at pt. A and to ground at pt. C. $\angle C = 30°$

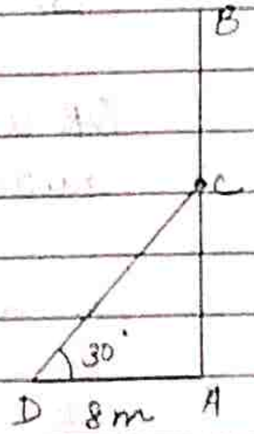
In rt. Led ΔABC

$$\frac{AB}{AC} = \sin 30°$$

$$\frac{AB}{20} = \frac{1}{2}$$

$AB = 10\,m$

So height of the pole is 10m



2. Let in fig AB is a tree broken at pt. C and the top of three touches the ground at pt. D, AD = 8m

$\angle D = 30°$

In rt. Led ΔABD

$$\frac{AC}{AD} = \tan 30°$$

$$\frac{AC}{8} = \frac{1}{\sqrt{3}} \qquad \left| \tan 30° = \frac{1}{\sqrt{3}} \right.$$

$$AC = \frac{8}{\sqrt{3}} \quad -(1)$$

Again,

$$\frac{CD}{AD} = \sec 30°$$

$$\frac{CD}{8} = \frac{2}{\sqrt{3}}$$

$CD = \frac{16}{\sqrt{3}}$ —(2)

Total height of tree = AC + CD

$= \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}}$ | using (1) & (2)

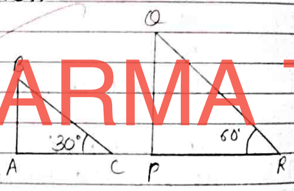
$= \frac{24}{\sqrt{3}}$

$= \frac{24 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$

$= \frac{24\sqrt{3}}{3}$

so total height of tree is $8\sqrt{3}$ m

3. Let in fig ABC is the slide for younger children. AB = 1.5m, $\angle C = 30°$



QR is the slide for ~~younger~~ elder children, PQ = 3m, $\angle R = 60°$

In rt. ∠ed ΔABC

$\frac{BC}{AB} = \text{cosec } 30°$

$\frac{BC}{1.5} = 2$ | $\text{cosec } 30° = 2$

BC = 3m

In rt ∠ed ΔPQR

$\frac{QR}{PQ} = \text{cosec } 60°$

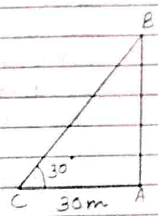
$\frac{QR}{3} = \frac{2}{\sqrt{3}}$ | $\text{cosec } 60° = \frac{2}{\sqrt{3}}$

$QR = \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$QR = \frac{6\sqrt{3}}{3}$ m $= 2\sqrt{3}$ m

So slide for children below 5 yrs is 3m and for elder children it is $2\sqrt{3}$ m

4. Let in fig AB is a tower, C is the pt. of observation. AC = 30m, $\angle C = 30°$



In rt ∠ed ΔABC

$\frac{AB}{AC} = \tan 30°$

$\frac{AB}{30} = \frac{1}{\sqrt{3}}$

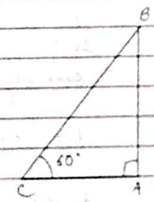
$AB = \frac{30}{\sqrt{3}}$

$AB = \frac{30 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{30\sqrt{3}}{3}$

$AB = 10\sqrt{3}$ m

So height of the tower is $10\sqrt{3}$ m

5. Let in fig kite is at pt. B, AB = 60m, $\angle C = 60°$



In rt ∠ed ΔABC

$\frac{BC}{AB} = \text{cosec } 60°$

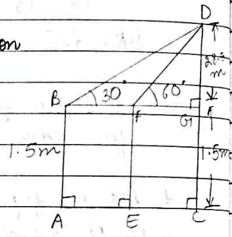
$\frac{BC}{60} = \frac{2}{\sqrt{3}}$ | $\text{cosec } 60° = \frac{2}{\sqrt{3}}$

$BC = \frac{120}{\sqrt{3}} = \frac{120 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$

$BC = \frac{\cancel{120}^{40} \times \sqrt{3}}{\cancel{3}_{1}}$ $BC = 40\sqrt{3}$ m

$\therefore$ So length of string is $40\sqrt{3}$ m

6. Let in fig AB is the initial position of the boy who is 1.5m tall. After some time the boy's position is EF. CD = 30 m tall building.



$DG = 30 - 1.5 = 28.5m$, $\angle DBG = 30°$, $\angle DFG = 60°$, $AB = FE = CG = 1.5m$

In rt. led ΔBDG

$\frac{BG}{DG} = \cot 30°$

$\frac{BG}{28.5} = \sqrt{3}$ $\quad | \cot 30 = \sqrt{3}$

$BG = 28.5\sqrt{3}\ m$ — ①

In rt. led ΔDGF

$\frac{FG}{DG} = \cot 60°$

$\frac{FG}{DG} = \frac{1}{\sqrt{3}}$ $\quad | \cot 60 = \frac{1}{\sqrt{3}}$

$\frac{FG}{28.5} = \frac{1}{\sqrt{3}}$

$FG = \frac{28.5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

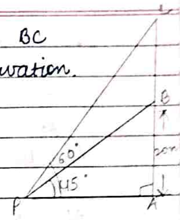
$FG = 9.5\sqrt{3}$ — ②

Distance travelled (BF) = BG − FG

$= 28.5\sqrt{3} - 9.5\sqrt{3}$

$= 19\sqrt{3}\ m$

7. Let in fig AB is 20m high building, BC is transmission tower. P is the pt. of observation. $\angle CPA = 60°$, $\angle BPA = 45°$.



In rt. led ΔABP

$\frac{AP}{AB} = \cot 45$

$\frac{AP}{20} = 1$ $\quad | \cot 45 = 1$

$AP = 20m$ — ①

In rt. led ΔPAC

$\frac{AC}{AP} = \tan 60°$

$\frac{AB + BC}{20} = \sqrt{3}$ $\quad |$ using ①

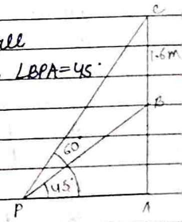
$20 + BC = 20\sqrt{3}$

$BC = 20\sqrt{3} - 20$

$BC = 20(\sqrt{3} - 1)\ m$

So height of transmission tower is $20(\sqrt{3}-1)$ m

8. Let in fig AB is a pedestal, BC is 1.6 m tall statue. P is the pt. of observation. $\angle CPA = 60°$, $\angle BPA = 45°$



In rt. led ΔABP

$\frac{AP}{AB} = \cot 45°$

$\frac{AP}{AB} = 1$

$AP = AB$ — ①

In rt. led ΔACP

$\frac{AP}{AC} = \cot 60$

$\frac{AP}{AB + BC} = \frac{1}{\sqrt{3}}$

$\frac{AP}{AB + 1.6} = \frac{1}{\sqrt{3}}$ $\quad | AP = AB$

$AB\sqrt{3} = AB + 1.6$

$AB\sqrt{3} - AB = 1.6$

$AB(\sqrt{3} - 1) = 1.6$

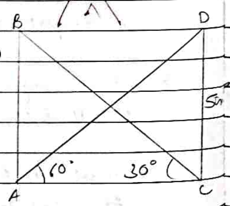
$AB = \frac{1.6 \times (\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$ $\quad AB = \frac{1.6(\sqrt{3}+1)}{2}$

$AB = 0.8(\sqrt{3}+1)$ m

So the height of tower is $0.8(\sqrt{3}+1)$ m

V. Good

9. Let in fig AB is a building and CD is a tower of 50m height.



$\angle BCA = 30°$, $\angle DAC = 60°$

In rt. led ΔACD

$\frac{AC}{CD} = \frac{B}{P} = \cot 60°$

$\frac{AC}{50} = \frac{1}{\sqrt{3}}$ $\quad | \cot 60° = \frac{1}{\sqrt{3}}$

$AC = \frac{50}{\sqrt{3}}$ m —(1)

In rt. led ΔABC

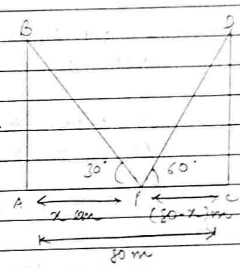
$\frac{AB}{AC} = \tan 30°$

$AB = AC \tan 30°$

$AB = \frac{50}{3}$, $AB = 16\frac{2}{3}$ m

So height of tower is $16\frac{2}{3}$ m

10. Let in fig AB & CD are two poles of equal height. Let AC is the width of road which is 80m. Let P is the pt. of observation. Let $AP = x$ m



$PC = (80 - x)$ m, $\angle BPA = 30°$, $\angle DPC = 60°$

In rt. led ΔABP

$\frac{AB}{AP} = \tan 30°$ $\quad | \tan 30° = \frac{1}{\sqrt{3}}$

$AB = \frac{x}{\sqrt{3}}$ m —(1)

In rt. led ΔDCP

$\frac{CD}{CP} = \tan 60°$

$\frac{CD}{80 - x} = \sqrt{3}$

$CD = (80 - x)(\sqrt{3})$ —(2)

Since the poles are of the same height equating (1) & (2)

$\frac{x}{\sqrt{3}} = (80 - x)(\sqrt{3})$

$x = (80 - x)3$

$x = 240 - 3x$

$x + 3x = 240$

$4x = 240$

$x = 60$

So $AP = 60$m, $PC = 20$m

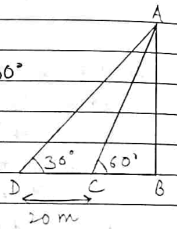
Put $x = 60$ in eq (1)

$CD = (80-60)(\sqrt{3})$

$CD = 20\sqrt{3}\ m$

So height of pole is $20\sqrt{3}\ m$ and distances of pt. of observation from 2 poles are $60\ m$ & $20\ m$.

11. In fig AB is a TV tower, BC is the width of canal, $CD = 20m$, $\angle ACD = 60^\circ$, $\angle D = 30^\circ$



In rt. led $\triangle ABC$

$\frac{AB}{BC} = \tan 60^\circ$

$AB = BC\sqrt{3}$ — ① $\quad | \tan 60^\circ = \sqrt{3}$

~~AB~~

In rt. led $\triangle ABD$

$\frac{AB}{BD} = \tan 30^\circ$

$\frac{AB}{20+BC} = \frac{1}{\sqrt{3}} \quad | \tan 30^\circ = \frac{1}{\sqrt{3}}$

$AB = \frac{20+BC}{\sqrt{3}}$ — ②

equating ① & ②

$BC\sqrt{3} = \frac{20+BC}{\sqrt{3}}$

$3BC = 20 + BC$

$3BC - BC = 20$

$2BC = 20$

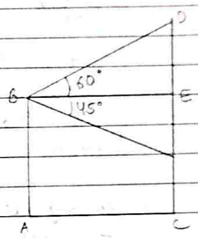
$BC = 10\ m$

Put $BC = 10$ in eq. ①

$AB = 10\sqrt{3}\ m$

12. Let in fig $AB = 7m$, high building. Pt B is the pt. of observation. CD is a tower, $\angle DBE = 60^\circ$

$\angle EDC = 45^\circ$



$AB = CE = 7m$

In rt. led $\triangle BEC$

$\frac{BE}{CE} = \cot 45$

$BE = 7m$ — ①

In rt. led $\triangle DEB$

$\frac{DE}{BE} = \tan 60^\circ$

$\frac{DE}{7} = \sqrt{3} \quad |$ using ①

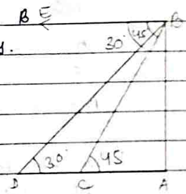
$DE = 7\sqrt{3}\ m$

Hight of tower $= DE + CE$

$7\sqrt{3} + 7$

$= 7(\sqrt{3}+1)\ m$

13. Let in fig AB is 75 m high light house. B is the pt. of observation. C & D are two ships.



$\angle EBD = \angle D = 30^\circ$ [A-I-A]

$\angle EBC = \angle C = 45^\circ$

In rt. led $\triangle ABC$

$\frac{AC}{AB} = \frac{B}{P} = \cot 45^\circ$

$\frac{AC}{75} = 1 \quad | \cot 45^\circ = 1$

$AC = 75\ m$ — ①

In rt. led $\triangle ABD$

$\frac{AD}{} = \cot 30^\circ$

$\frac{AC+CD}{75} = \cot 30°$ $\quad | \cot 30 = \sqrt{3}$

$\frac{75+CD}{75} = \sqrt{3}$

$75 + CD = 75\sqrt{3}$

$CD = 75\sqrt{3} - 75$

$CD = 75(\sqrt{3}-1)\ m$

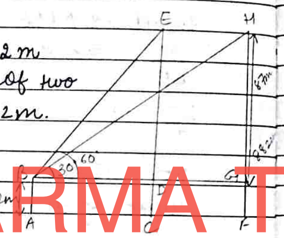
Hence req. distance is $75(\sqrt{3}-1)\ m$

14. In fig let AB is the position of 1.2 m tall girl, E & H are the positions of two balloons in two cases. $EC = HF = 88.2 m$.

$AB = CD = GF = 1.2 m$

$ED = HG = 88.2 - 1.2 = 87 m.$

$\angle EBD = 60°, \angle HBD = 30°$



In rt. ∠d $\triangle BHG$

$\frac{BG}{HG} = \cot 30°$

$\frac{BG}{87} = \sqrt{3}$ $\quad | \cot 30 = \sqrt{3}$

$BG = 87\sqrt{3}\ m$ — (1)

In rt. ∠d $\triangle BED$

$\frac{BD}{ED} = \cot 60°$

$\frac{BD}{87} = \frac{1}{\sqrt{3}}$

$BD = \frac{87\sqrt{3}}{\sqrt{3} \times \sqrt{3}}$

$BD = 29\sqrt{3}$ — (2)

Distance travelled by balloon (DG) = BG − BD

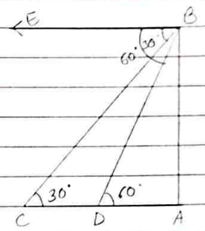
$= 87\sqrt{3} - 29\sqrt{3}$

$= 58\sqrt{3}\ m$

15. Let in fig AB is a tower pt. B is the pt. of observation. C & D are the position of car in two cases.

$\angle EBC = \angle C = 30°$

$\angle EBD = \angle D = 60°$ [A.I.A.]



In rt. ∠d $\triangle ABC$

$\frac{AB}{AD} = \tan 60°$

$\frac{AB}{AD} = \sqrt{3}$

$AB = AD\sqrt{3}$ — (1)

In rt. ∠d $\triangle ABC$

$\frac{AB}{AC} = \tan 30°$

$\frac{AB}{AD+CD} = \frac{1}{\sqrt{3}}$ $\quad | \tan 30° = \frac{1}{\sqrt{3}}$ $\quad AB = \frac{AD+CD}{\sqrt{3}}$ — (2)

equating (1) & (2)

$AD\sqrt{3} = \frac{AD+CD}{\sqrt{3}}$

$3AD - AD = CD$

$2AD = CD$

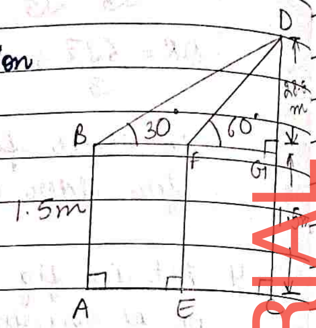
$AD = \frac{1}{2} CD$

Time taken to travel CD = 6 sec

Time taken to travel 1/2 CD i.e. = 1/2 × 6 = 3 sec

so req. time is 3 sec.

$\therefore$ So length of string is $40\sqrt{3}$ m



6. Let in fig AB is the initial position of the boy who is 1.5m tall. After some time the boy's position is EF. CD = 30 m tall building, DG = 30 − 1.5 = 28.5m, $\angle DBG = 30°$, $\angle DFG = 60°$, AB = FE = CG = 1.5m

In rt. led $\triangle BDG$

$\frac{BG}{DG} = \cot 30°$

$\frac{BG}{28.5} = \sqrt{3}$ | cot 30 = $\sqrt{3}$

$BG = 28.5\sqrt{3}$ m — (1)

In rt. led $\triangle DGF$

$\frac{FG}{DG} = \cot 60°$

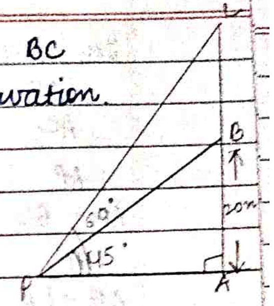
$\frac{FG}{DG} = \frac{1}{\sqrt{3}}$ | cot 60 = $\frac{1}{\sqrt{3}}$

$\frac{FG}{28.5} = \frac{1}{\sqrt{3}}$

$FG = \frac{28.5 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$

$FG = 9.5\sqrt{3}$ — (2)

Distance travelled (BF) = BG − FG
$28.5\sqrt{3} - 9.5\sqrt{3}$
$= 19\sqrt{3}$ m



7. Let in fig AB is 20m high building, BC is transmission tower. P is the pt. of observation. $\angle CPA = 60°$, $\angle BPA = 45°$.

In rt. led $\triangle ABP$

$\frac{AP}{AB} = \cot 45$

$\frac{AP}{20} = 1$ | cot 45 = 1

AP = 20m — (1)

In rt. led $\triangle PAC$

$\frac{AC}{AP} = \tan 60°$

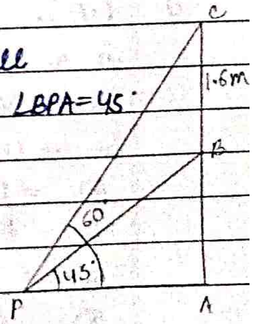
$\frac{AB + BC}{20} = \sqrt{3}$ | Using (1)

$20 + BC = 20\sqrt{3}$

$BC = 20\sqrt{3} - 20$

$BC = 20(\sqrt{3} - 1)$ m

So height of transmission tower is $20(\sqrt{3}-1)$m



8. Let in fig AB is a pedestal, BC is 1.6 m tall statue. P is the pt. of observation. $\angle CPA = 60°$, $\angle BPA = 45°$

In rt. led $\triangle ABP$

$\frac{AP}{AB} = \cot 45°$

$\frac{AP}{AB} = 1$

AP = AB. — (1)

In rt. led ΔACP

$\frac{AP}{AC} = \cot 60$

$\frac{AP}{AB + BC} = \frac{1}{\sqrt{3}}$

$\frac{AP}{AB + 1.6} = \frac{1}{\sqrt{3}}$ | $AP = AB$

$AB\sqrt{3} = AB + 1.6$

$AB\sqrt{3} - AB = 1.6$

$AB(\sqrt{3}-1) = 1.6$

$AB = \frac{1.6 \times (\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$ $AB = \frac{1.6(\sqrt{3}+1)}{2}$

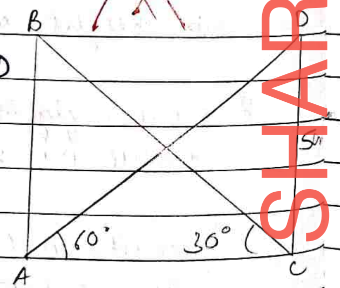
$AB = 0.8(\sqrt{3}+1)m$

So the height of tower is $0.8(\sqrt{3}+1)m$

V. Good

7/10/24

9. Let in fig AB is a building and CD is a tower of 50m height.



$\angle BCA = 30°$, $\angle DAC = 60°$

In rt. led ΔACD

$\frac{AC}{CD} = \frac{B}{P} = \cot 60°$

$\frac{AC}{50} = \frac{1}{\sqrt{3}}$ | $\cot 60° = \frac{1}{\sqrt{3}}$

$AC = \frac{50}{\sqrt{3}} m$ — (1)

In rt. led ΔABC

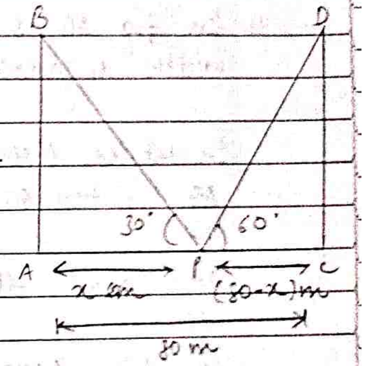
$\frac{AB}{AC} = \tan 30°$

$AB = AC \tan 30°$

$AB = \frac{50}{3}$, $AB = 16\frac{2}{3} m$

So height of tower is $16\frac{2}{3} m$

10. Let in fig AB & CD are two poles of equal height. Let AC is the width of road which is 80m. Let P is the pt. of observation. Let $AP = xm$



$PC = (80-x)m$, $\angle BPA = 30°$, $\angle DPC = 60°$

In rt. led ΔABP

$\frac{AB}{AP} = \tan 30°$ | $\tan 30° = \frac{1}{\sqrt{3}}$

$AB = \frac{x}{\sqrt{3}} m$ — (1)

In rt. led ΔDCP

$\frac{CD}{CP} = \tan 60°$

$\frac{CD}{80-x} = \sqrt{3}$

$CD = (80-x)(\sqrt{3})$ — (2)

Since the poles are of the same height equating (1) & (2)

$\frac{x}{\sqrt{3}} = (80-x)(\sqrt{3})$

$x = (80-x)3$

$x = 240 - 3x$

$x + 3x = 240$

$4x = 240$

$x = 60$

So $AP = 60m$, $PC = 20m$

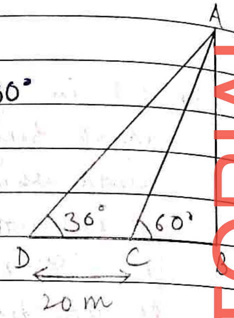
Put $x = 60$ in eq (1)

$CD = (80-60)(\sqrt{3})$

$CD = 20\sqrt{3}$ m

So height of pole is $20\sqrt{3}$ m and distances of pt. of observation from 2 poles are 60 m & 20 m.

11. In fig AB is a TV tower, BC is the width of canal, CD = 20 m, $\angle ACB = 60°$, $\angle D = 30°$



In rt. led $\triangle ABC$

$\frac{AB}{BC} = \tan 60°$

$AB = BC\sqrt{3}$ —(1) | $\tan 60° = \sqrt{3}$

In rt led $\triangle ABD$

$\frac{AB}{BD} = \tan 30°$

$\frac{AB}{20+BC} = \frac{1}{\sqrt{3}}$ | $\tan 30° = \frac{1}{\sqrt{3}}$

$AB = \frac{20+BC}{\sqrt{3}}$ —(2)

equating (1) & (2)

$BC\sqrt{3} = \frac{20+BC}{\sqrt{3}}$

$3BC = 20 + BC$

$3BC - BC = 20$

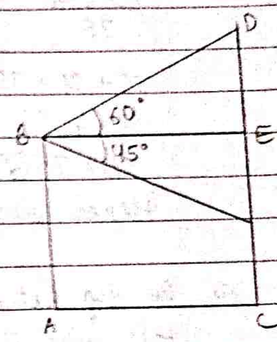
$2BC = 20$

$BC = 10$ m

Put BC = 10 in eq. (1)

$AB = 10\sqrt{3}$ m

12. Let in fig AB = 7 m, high building. Pt B is the pt. of observation. CD is a tower, $\angle DBE = 60°$, $\angle EDC = 45°$.



$AB = CE = 7$ m

In rt. led $\triangle BEC$

$\frac{BE}{CE} = \cot 45$

$BE = 7$ m —(1)

In rt. led $\triangle DEB$

$\frac{DE}{BE} = \tan 60°$

$\frac{DE}{7} = \sqrt{3}$ | using (1)

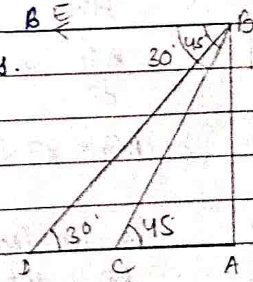
$DE = 7\sqrt{3}$ m

Height of tower = DE + CE

$7\sqrt{3} + 7$

$= 7(\sqrt{3}+1)$ m

13. Let in fig AB is 75 m high light house. B is the pt. of observation. C & D are two ships.



$\angle EBD = \angle D = 30°$ [A·I·A]

$\angle EBC = \angle C = 45°$

In rt. led $\triangle ABC$

$\frac{AC}{AB} = \frac{B}{P} = \cot 45°$

$\frac{AC}{75} = 1$ | $\cot 45° = 1$

$AC = 75$ m —(1)

In rt. led $\triangle ABD$

$\frac{AD}{AB} = \cot 30°$

$\frac{AC+CD}{75} = \cot 30°$ | $\cot 30 = \sqrt{3}$

$\frac{75+CD}{75} = \sqrt{3}$

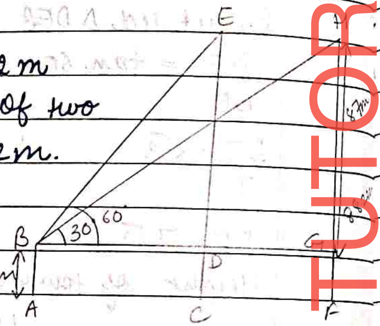
$75 + CD = 75\sqrt{3}$

$CD = 75\sqrt{3} - 75$

$CD = 75(\sqrt{3}-1)$ m

Hence req. distance is $75(\sqrt{3}-1)$ m

14. In fig let AB is the position of 1.2 m tall girl, E & H are the positions of two balloons in two cases. EC = HF = 88.2 m.



$AB = CD = GF = 1.2$ m

$ED = HG = 88.2 - 1.2 = 87$ m.

$\angle EBD = 60°$, $\angle HBD = 60°$

In rt. led $\triangle BHG$

$\frac{BG}{HG} = \cot 30°$

$\frac{BG}{87} = \sqrt{3}$ | $\cot 30 = \sqrt{3}$

$BG = 87\sqrt{3}$ m —(1)

In rt. led $\triangle BED$

$\frac{BD}{ED} = \cot 60°$

$\frac{BD}{87} = \frac{1}{\sqrt{3}}$

$BD = \frac{87\sqrt{3}}{\sqrt{3} \times \sqrt{3}}$

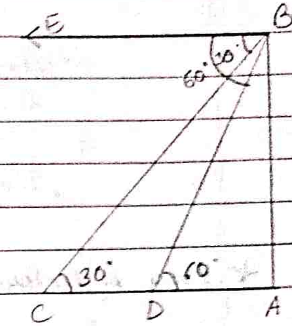
$BD = 29\sqrt{3}$ —(2)

Distance travelled by balloon (DG) = BG − BD

$= 87\sqrt{3} - 29\sqrt{3}$

$= 58\sqrt{3}$ m

15. Let in fig AB is a tower pt. B is the pt. of observation. C & D are the position of car in two cases.



$\angle EBC = \angle C = 30°$ [A.I.A.]

$\angle EBD = \angle D = 60°$

In rt. led $\triangle ABC$

$\frac{AB}{AD} = \tan 60°$

$\frac{AB}{AD} = \sqrt{3}$

$AB = AD\sqrt{3}$ —(1)

In rt. led $\triangle ABC$

$\frac{AB}{AC} = \tan 30°$

$\frac{AB}{AD+CD} = \frac{1}{\sqrt{3}}$ | $\tan 30° = \frac{1}{\sqrt{3}}$

$AB = \frac{AD+CD}{\sqrt{3}}$ —(2)

equating (1) & (2)

$AD\sqrt{3} = \frac{AD+CD}{\sqrt{3}}$

$3AD - AD = CD$

$2AD = CD$

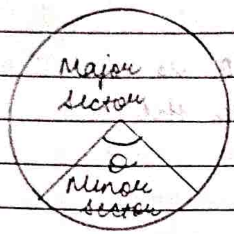
$AD = \frac{1}{2} CD$

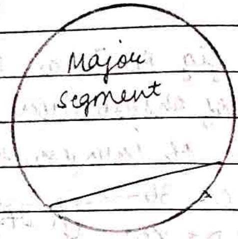
Time taken to travel CD = 6 sec

Time taken to travel 1/2 CD i.e = 1/2 × 6 = 3 sec

so req. time is 3 sec.

Chapter-11
Areas related to circles





★ Area of sector $= \frac{\theta}{360^\circ}\pi r^2$

★ Length of Arc $= \frac{\theta}{360^\circ} \times 2\pi r$

★ Area of segment $= \frac{\theta}{360^\circ}\pi r^2 - \frac{1}{2} r^2 \sin\theta$

★ When $\theta > 90^\circ$

Area of segment $= \frac{\theta}{360}\pi r^2 - r^2 \sin\frac{\theta}{2}\cos\frac{\theta}{2}$

Exercise 11.1

1. $r = 6$ cm, $\theta = 60^\circ$

Area of sector $= \frac{\theta}{360}\pi r^2$

$= \frac{\cancel{60}}{\cancel{360}_{60}} \times \frac{22}{7} \times 6 \times 6$

$= \frac{132}{7}$

$= 18\frac{6}{7}$ cm^2

2. Circumference of circle = 22 cm

$2\pi r = 22$

$r = \frac{22}{2\pi}$

$r = \frac{22 \times 7}{2 \times 22} = \frac{7}{2}$ cm

Area of quadrant :- $\frac{1}{4}\pi r^2$

$\frac{1}{4} \times \frac{\cancel{22}^{11}}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{2}_1} \times \frac{7}{2}$

$\frac{77}{8} = 9\frac{5}{8}$ cm^2

3. Length of minute hand $(r) = 14$ cm

Angle subtended by minute hand in 60 min $= 360^\circ$

Angle subtended by minute hand in 1 min $= 360/60 = 6^\circ$

(Q) Angle subtended by minute hand in 5 min $= 6 \times 5 = 30^\circ$

Area swept in 5 minutes

$\frac{\theta}{360}\pi r^2$

$\frac{30}{360} \times \frac{22}{7} \times 14 \times 14$

$= \frac{154}{3} = 5\frac{1}{3}$ cm^2

4. $r = 10$ cm, $\theta = 90^\circ$, $\pi = 3.14$

(i) Minor segment (ii) Major segment

Ar. of minor segment $= \frac{\theta}{360^\circ}\pi r^2 - \frac{1}{2} r^2 \sin\theta$

$$= r^2\left(\frac{\theta\pi}{360} - \frac{1}{2}\sin\theta\right)$$

$$= 10\times10\left(\frac{90}{360}\times3.14 - \frac{1}{2}\sin 90\right)$$

$$= 100\left(0.785 - \frac{1}{2}\right)$$

$$= 100(0.785 - 0.5)$$

$$= 100(0.285)$$

$$= 28.5\ cm^2$$

ii) For Major sector

$\theta = 360 - 90$

$\theta = 270^\circ$

Ar. of major sector

$$\frac{\theta}{360}\pi r^2$$

$$\frac{270}{360}\times3.14\times10\times10$$

$$= 235.5\ cm^2$$

5. $r = 21\ cm$, $\theta = 60^\circ$

i) Length of arc $= \frac{\theta}{360}\times 2\pi r$

$$\frac{\overset{1}{\cancel{60}}}{\underset{\cancel{6}\ \cancel{7}}{\cancel{360}}}\times \cancel{2}\times\frac{22}{\cancel{7}_1}\times\cancel{21}\ \cancel{3}\ 1$$

$$= 22\ cm$$

ii) Area of sector

$$\frac{\theta}{360}\times\pi r^2$$

$$\frac{60}{360}\times\frac{22}{7}\times21\times21 = 231\ cm^2$$

iii) Area of segment

$$\frac{\theta}{360}\pi r^2 - \frac{1}{2}r^2\sin\theta$$

$$r^2\left(\frac{\theta}{360}\pi - \frac{1}{2}\sin 60\right) = 21\times21\left(\frac{\overset{1}{\cancel{60}}}{\cancel{360}_{\cancel{6}\,3}}\times\frac{\overset{11}{\cancel{22}}}{7} - \frac{1}{2}\times\frac{\sqrt{3}}{2}\right)$$

$$441\left(\frac{11}{21} - \frac{\sqrt{3}}{4}\right)$$

$$= 441\left(\frac{44 - 21\sqrt{3}}{84}\right) = 21\left(\frac{44 - 21\sqrt{3}}{4}\right)$$

$$\frac{21\times44}{4} - \frac{21\times21\sqrt{3}}{4}$$

$$231 - \frac{441\sqrt{3}}{4}$$

6. $r = 15\ cm$

$\theta = 60^\circ$

$\pi = 3.14$

$\sqrt{3} = 1.73$

Area of segment $= \frac{\theta}{360}\pi r^2 - \frac{1}{2}\sin\theta$

$$r^2\left(\frac{\theta}{360}\pi - \frac{1}{2}\sin 60^\circ\right)$$

$$15\times15\left(\frac{\overset{1}{\cancel{60}}}{\cancel{360}_6}\times3.14 - \frac{1}{2}\frac{\sqrt{3}}{2}\right)$$

$$225\left(\frac{1.57}{3} - \frac{1.73}{4}\right)$$

$$= 225\left(\frac{6.28 - 5.19}{12}\right)$$

$$\frac{225\times1.06}{12}$$

$$= \frac{39.75}{2} = 19.875\ cm^2$$

Area of major segment $= \pi r^2 - 19.875$

$3.14 \times 15 \times 15 - 19.875$

$806.50 - 19.875$

$= 786.625\ cm^2$

7. $r = 12\ cm$

$\theta = 120^\circ$

$\pi = 3.14$

$\sqrt{3} = 1.73$

Ar. of segment $= r^2\left(\frac{\theta\pi}{360} - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)$

$= 12 \times 12\left(\frac{120}{360} \times 3.14 - \sin 60 \cos 60\right)$

$= 144\left(\frac{3.14}{3} - \frac{\sqrt{3}}{4}\right)$

$= \cancel{144}^{12}\left(\frac{12.56 - 5.19}{\cancel{12}\ 1}\right)$

$= 12(7.37)$

$= 88.44\ cm^2$

8. Side of square $= 15\ m$

(i) length of rope $= 5m\ (r)$

$\theta = 90^\circ, \pi = 3.14$

Area horse can graze $= \frac{\theta}{360}\pi r^2$

$\frac{90}{360} \times 3.14 \times 5 \times 5$

$= \frac{78.5}{4}$

$= 19.625\ m^2$

(ii) $r = 10\ m$

$\theta = 90^\circ$

$\pi = 3.14$

Ar. horse can graze $= \frac{\theta}{360}\pi r^2$

$\frac{90}{360} \times 3.14 \times 10 \times 10$

$\frac{314}{4} = 78.5\ m^2$

$\therefore$ Increase in grazing area $= 78.5 - 19.625$

$= 58.875\ m^2$

9. diameter $= 35\ mm$

$r = \frac{35}{2}\ mm$

(a) wire used $= 2\pi r + 5 \times$ dia.

$2 \times \frac{22}{7} \times \frac{35}{2} + 5 \times 35$

$= 22 \times 5 + 5 \times 35$

$= 110 + 175$

$= 285\ mm$

(b) Area of each sector $= \frac{\pi r^2}{10}$

$\frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} \times \frac{1}{10}$

$= \frac{385}{4}$

$= 96.25\ mm^2$

10. There are 8 ribs in the umbrella

$r = 45$ cm

Ar. b/w two consecutive ribs $= \frac{\pi r^2}{8}$

$$\frac{22}{7} \times 45 \times 45 \times \frac{1}{8}$$

$$= \frac{22275}{28}$$

$$= 795.535 \text{ cm}^2$$

11. $r = 25$ cm

$\theta = 115°$

since there are 2 wipers.

so, ar. cleaned at each sweep = 2 × Ar. of sector

$$= \frac{2\theta}{360} \pi r^2$$

$$2 \times \frac{115}{360} \times \frac{22}{7} \times 25 \times 25$$

$$= \frac{158125}{126}$$

$$= 1254.960 \text{ cm}^2$$

12. $\pi = 3.14$

$\theta = 80°$

$r = 16.5$ km

Ar. of sea in which ships can be warned =

$$\frac{\theta}{360} \pi r^2$$

$$= \frac{80}{360} \times \frac{22}{7} \times 16.5 \times 16.5$$

$$= \frac{80}{360} \times 3.14 \times 16.5 \times 16.5 \quad = 189.97 \text{ km}^2$$

13. since there are 6 symmitrical designs

so $\theta = \frac{360}{6} = 60°$

$\sqrt{3} = 1.7$

Ar. of 1 design = Ar. of segment

$$r^2\left(\frac{\theta}{360}\pi - \frac{1}{2}\sin\theta\right)$$

$$= 28 \times 28 \left(\frac{60}{360} \times \frac{22}{7} - \frac{1}{2}\frac{\sqrt{3}}{2}\right)$$

$$= 28 \times 28 \left(\frac{11}{21} - \frac{\sqrt{3}}{4}\right)$$

$$= \cancel{28} \times 28 \left(\frac{44 - 21 \times 1.7}{\cancel{84}}\right)$$

$$= 28\left(\frac{44 - 35.7}{3}\right)$$

$$= \frac{28 \times 8.3}{3}$$

Ar. of 6 designs $= \frac{6 \times 28 \times 8.3}{3} = 56 \times 8.3$

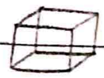
$= 464.8 \text{ cm}^2$

Cost of 1 cm² design = ₹0.35

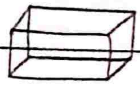
Cost of 464.8 cm² design = 0.35 × 464.8

= ₹162.68

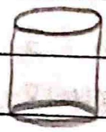
Chapter-12
Surface Areas and Volumes

1. Cube 

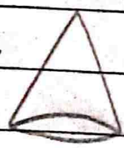
TSA = $6a^2$
LSA = $4a^2$
Vol. = a^3
D. = $\sqrt{3}\,a$

2. Cuboid 

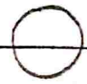
TSA = $2(lb + bh + hl)$
LSA = $2h(l+b)$
Vol. = lbh
D. = $\sqrt{l^2+b^2+h^2}$

3. Cylinder 

CSA = $2\pi rh$
TSA = $2\pi rh + 2\pi r^2$
$= 2\pi r(r+h)$
$V = \pi r^2 h$

4. Right circular cone 

$l^2 = h^2 + r^2$
CSA = πrl
TSA = $\pi rl + \pi r^2$
$\pi r(r+l)$
$V = \frac{1}{3}\pi r^2 h$

5. Sphere 

SA = $4\pi r^2$
$V = \frac{4}{3}\pi r^3$

6. Hemisphere

TSA = $3\pi r^2$
CSA = $2\pi r^2$
$V = \frac{2}{3}\pi r^3$

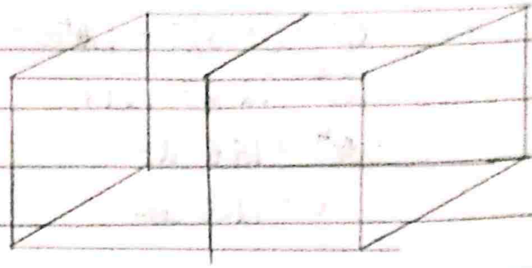
Exercise 12.1

1. Volume of each cube = $64\ cm^3$

$\therefore side^3 = 64$

$side = \sqrt[3]{64}$

$side = 4cm$



When 2 cubes are joined end to end, we get a cuboid of $l = 8\ cm$, $b = 4\ cm$, $h = 4\ cm$

TSA of cuboid $= 2(lb + bh + hl)$

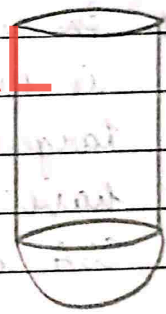
$= 2(8\times4 + 4\times4 + 8\times4)$

$= 2(32 + 16 + 32)$

$= 2\times 80$

$= 160\ cm^2$

2. In fig a hollow hemisphere is mounted by a hollow cylinder.



$r = 7cm$

Total height of vessel $= 13\ cm$

so height of cylinderical part $= 13 - 7$

$= 6cm$

Inner surface area of vessel $= 2\pi r^2 + 2\pi rh$

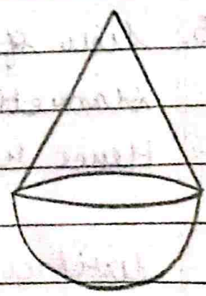
$= 2\pi r(r + h)$

$= 2\times\frac{22}{\cancel{7}}\times\cancel{7}(7+6)$

$= 2\times 22\times 13$

$= 572\ cm^2$

3. Radius (r) of hemisphere and cone $= 3.5\ cm$



height of toy $= 15.5\ cm$

height of cone $= 13.5 - 3.5 = 12\ cm$

CSA of toy $= \pi rl + 2\pi r^2$

$\pi r(l + 2r)$

$l^2 = r^2 + h^2$

$l^2 = (3.5)^2 + 12^2$

$l^2 = 12.25 + 144$

$l^2 = 156.25$

$l = 12.5$ cm

$\pi r(l + 2r)$

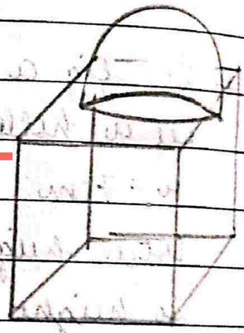
$\frac{22}{7} \times 3.5(12.5 + 7)$

$22 \times 0.5\,(19.5)$

$22 \times \frac{5}{10} \times \frac{195}{10} = \frac{21450}{100}$

$= 214.50 \text{ cm}^2$

4. In a fig a cubical box of side 7cm is surmounted by a hemisphere largest diameter a hemisphere can have is 7cm.



CSA of solid $= 6a^2 + 2\pi r^2 - \pi r^2$

$6a^2 + \pi r^2$

$6 \times 7 \times 7 + \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$6 \times 49 + \frac{77}{2} = 294 + 38.5$

$= 332.5 \text{ cm}^2$

5. Side of cubical block = l unit

Diameter of hemispherical depression = l unit

Hence radius = $\frac{l}{2}$ unit.

Surface area of remaining object = $6a^2 + 2\pi r^2 - \pi r^2$

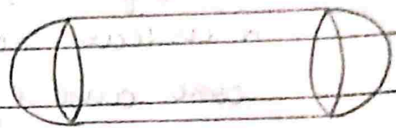
$6a^2 + \pi r^2$

$$= 6(l^2) + \pi\left(\frac{l}{2}\right)^2$$

$$6l^2 + \pi\frac{l^2}{4} = \frac{24l^2 + \pi l^2}{4}$$

$$= l^2\left(\frac{24+\pi}{4}\right) \text{ unit}^2$$

6. A capsule is in the shape of cylinder with 2 hemispheres at its each end.



$d = 5mm$

$r = \frac{5}{2} mm$

Total length = 14 mm

Length of cylindrical part $(h) = 14 - 5 = 9mm$

Surface area of capsule $= 2\pi r^2 + 2\pi r^2 + 2\pi rh$

$$= 2\pi r(r + r + h)$$

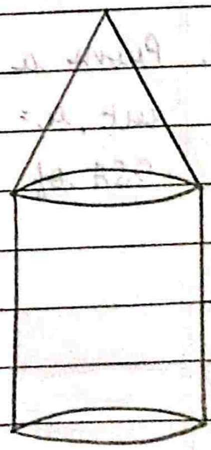
$$= 2\pi r(2r + h)$$

$$= \cancel{2} \times \frac{22}{7} \times \frac{5}{\cancel{2}}\left(\cancel{2} \times \frac{5}{\cancel{2}} + 9\right)$$

$$= \frac{110}{7}(14)$$

$$= 220 mm^2$$

7. A tent is made up of a cylinder with a conical top. Radius of cylinder and cone = 2m, slant height (l) of cone = 2.8m



Area of canvas req. $= \pi rl + 2\pi rh$

$$\pi r(l + 2h)$$

$$\frac{22}{7} \times 2(2.8 + 2(2.1))$$

$$\frac{22}{7} \times 2(2.8 + 4.2)$$

$= \frac{22}{7} \times 2 \times 7$

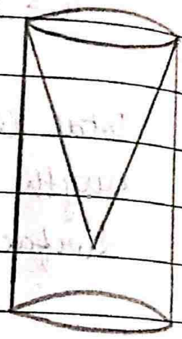
$= 44 m^2$

Rate of canvas $1 m^2 = ₹500$

44×500

$= ₹22000$

8. In fig a cylinderical solid of $h = 2.4$ cm
A conical cavity is hollowed out. Radius of cone and cylinder $(r) = 0.7$ cm
TSA of remaining solid = CSA of cone + ar. of base + CSA of cylinder



$l^2 = r^2 + h^2$

$l^2 = 2.4^2 + 0.7^2$

$l^2 = 0.49 + 5.76$

$l^2 = 6.25$

$l = 2.5$ cm

$\pi rl + \pi r^2 + 2\pi rh$

$\pi r(l + r + 2h)$

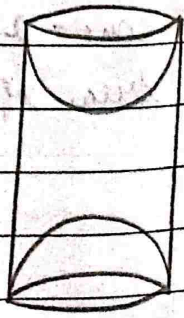
$\frac{22}{7} \times \frac{7}{10} (2.5 + 0.7 + 4.8)$

$= 2.2(8)$

$= 17.6 \text{ cm}^2$

$= 18 \text{ cm}^2$ (approx).

9. From a solid cylinder two hemispheres are scooped out, $r = 3.5$ cm, $h = 10$ cm



CSA of solid $= 2\pi rh + 2\pi r^2 + 2\pi r^2$

$= 2\pi r(h + 2r)$

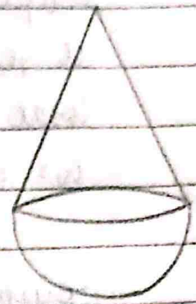
$= 2 \times \frac{22}{7} \times 3.5(10 + 7)$

$= 22 \times 17$

$= 374 \text{ cm}^2$

Exercise 12.2

1. In fig a solid is in the shape of cone standing on a hemisphere. $r = 1$ cm, $h = 1$ cm



volume of solid $= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

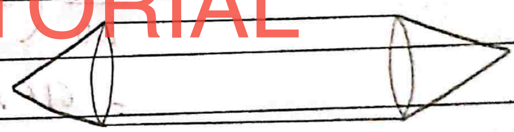
$$\pi r^2\left(\frac{1}{3}h + \frac{2}{3}r\right)$$

$$\pi(1)^2\left(\frac{1}{3}(1) + \frac{2}{3}(1)\right)$$

$$\pi\left(\frac{3}{3}\right) = \pi \times 1$$

$$= \pi \text{ cm}^3$$

2. In fig a model is in the shape of cylinder with two cones at its two ends. $r = \frac{3}{2}$ cm, $d = 3$ cm



$h = 12 - 4$ $\qquad h' = 2$ cm

$= 8$ cm

volume of model = vol. of 2 cones + vol. of cylinder.

$$\frac{1}{3}\pi r^2 h' + \frac{1}{3}\pi r^2 h' + \pi r^2 h$$

$$2\left(\frac{1}{3}\pi r^2 h'\right) + \pi r^2 h$$

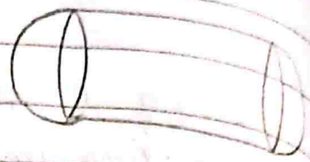
$$\cancel{2}\left(\frac{1}{\cancel{3}} \times \frac{\overset{11}{\cancel{22}}}{7} \times \frac{\cancel{3}}{\cancel{2}} \times \frac{3}{\cancel{2}} \times \cancel{2}\right) + \frac{22}{7} \times \frac{3}{\cancel{2}} \times \frac{3}{\cancel{2}} \times \cancel{8}\,\overset{}{2}$$

$$2\left(\frac{11}{7} \times 3\right) + \frac{22 \times 9 \times 2}{7}$$

$$2\left(\frac{33}{7}\right) + \frac{396}{7}$$

$$\frac{66}{7} + \frac{396}{7} = \frac{462}{7} = 66 \text{ cm}^3$$

3. In fig a gulab jamun is in the shape of cylinder with two hemisphere at the two ends.



Total (h) = 5 cm, d = 2.8 cm, r = 1.4 cm

$h = 5 - 1.4 - 1.4$

$= 2.2$ cm

volume of one gulab jamun $= \pi r^2 h + \left(\frac{2}{3}\pi r^3\right) 2$

$$= \pi r^2 \left(h + \frac{4}{3} r\right)$$

$$= \frac{22}{7} \times 1.4 \times 1.4 \left(2.2 + \frac{4}{3} \times 1.4\right)$$

$$= \frac{22}{7} \times \frac{14}{10} \times \frac{14}{10} \left(2.2 + \frac{4}{3} \times 1.4\right)$$

$$= \frac{616}{100} \left(\frac{6.6 + 5.6}{3}\right)$$

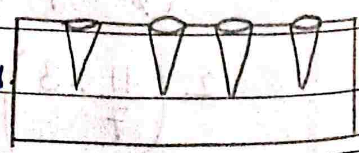
$$= \frac{616 \times 12.2}{100 \times 3} \text{ cm}^3$$

V. of 45 gulab Jamuns $= \frac{45 \times 616 \times 12.2}{100 \times 3}$

$$30\% = \frac{30 \times 45 \times 616 \times 12.2}{100 \times 100 \times 3}$$

$$= 338.184 \text{ cm}^3$$

4. In fig a pen stand is in the shape of cuboid with 4 conical depressions



Dimensions of cuboid are,

$l = 15$ cm, $b = 10$ cm, $h = 3.5$ cm

$r = 0.5$ cm, height of conical part (H) = 1.4 cm

Volume of wood in the entire stand $= lbh - 4\left(\frac{1}{3}\pi r^2 h\right)$

$$15 \times 10 \times 3.5 - \frac{4}{3} \times \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times \frac{14}{10}$$

$$15 \times 35 - \frac{4}{3} \times \frac{11}{10} \quad = 15 \times 35 - \frac{44}{30}$$

$$= 525 - \frac{44}{30}$$

$$= 525 - 1.47$$

$$= 523.53 \text{ cm}^3$$

6. In fig there is a big cylinder mounted by small cylinder.

$$\pi R^2 h + \pi r^2 h$$

$$3.14 \times 144 \times 220 + 3.14 \times 64 \times 60$$

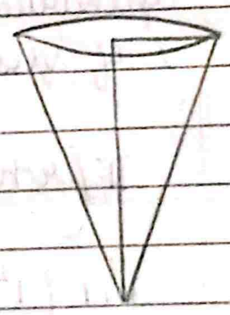
$$99475.2 + 12057.6$$

$$= 11532.8 \text{ cm}^3$$

Mass $= 11532.8 \times 8$

$= 892262.4$

$= 892.262$ kg



5. An inverted cone of height $(h) = 8$ cm

$R = 5$ cm, radius of lead shots $(r) = 0.5 = \frac{1}{2}$ cm

Let req. no. of lead shots be x

ATQ

$$x \times \text{V. of sphere} = \frac{1}{4} \text{ V. of cone}$$

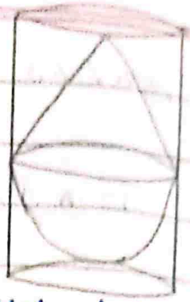
$$x \times \frac{4}{3}\pi r^3 = \frac{1}{4} \times \frac{1}{3}\pi r^2 h$$

$$x \times \frac{4}{3} \times \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \times \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 8$$

$$x = 50 \times 2$$

$$x = 100$$

7. Let in fig a solid of cone, $h = 120$ cm, radius $= 60$ cm, hemisphere ($r = 60$ cm) and cylinder ($r = 60$ cm, $h = 180$ cm)



ATQ

V. of water left in cylinder = V. of cylinder − V. of hemi. + V. of cone

$$\pi r^2 h - \left(\frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h\right)$$

$$\pi r^2 h - \frac{1}{3}\pi r^2 (2r + h)$$

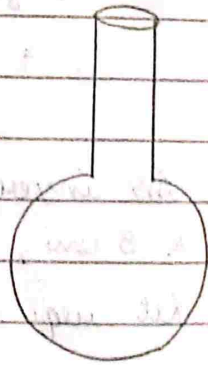
$$\frac{22}{7} \times 60 \times 60 \times 180 - \frac{1}{\cancel{3}} \times \frac{22}{7} \times \overset{20}{\cancel{60}} \times 60\,(120+120)$$

$$\frac{14256000}{7} - \frac{6336000}{7}$$

$$= \frac{7920000}{7}$$

$$= 1131428\frac{4}{7}\ cm^3$$

8. Let in fig a spherical vessel has cylindrical neck. $d = 2$ cm, $h = 8$ cm, $d' = 8.5$ cm



V. of vessel $= \pi r^2 h + \frac{4}{3}\pi r^3$

$$\pi\left(r^2 h + \frac{4}{3}R^3\right)$$

$$3.14\left(1^2 \times 8 + \frac{4}{3}\left(\frac{17}{4}\right)^3\right)$$

$$3.14\left(8 + \frac{\overset{1}{\cancel{4}}}{3} \times \frac{4913}{\underset{16}{\cancel{64}}}\right)$$

$$3.14\left(\frac{8 + 4913}{48}\right) = 3.14\left(\frac{384 + 4913}{48}\right) = 346.51\ cm^3$$

She is not correct.