Chapter - 1
Number Systems

Numbers
- Real Numbers
  - Rational
    - Whole No.s: 0, 1, 2 . . .
    - Natural No.s: 1, 2, 3, 4 . . .
    - Integers: -1, 0, 1, 2 . . .
  - Irrational
- Imaginary Numbers

Definations

1. Natural No.s :- 1, 2, 3, 4 . . . .

2. Whole No.s : 0, 1, 2, 3, 4 . . . .

3. Integers : -2, -1, 0, 1, 2, 3 . . . .

4. Rational No.s : The real numbers which can be written in the form of $\frac{p}{q}$ where p is an integer and q is $\neq$ to 0.

eg: $\frac{2}{3}$

2. The real no.s which have either terminating or non-terminating repeating decimal expansion.

eg: $1.5, 1.23, 1.\overline{23}, 1.\overline{5}$

★ There are infinitely many rational no.s between any two given rational no.s.

Exercise 1.1

1. Yes, 0 is a rational no. We can write 0 in $\frac{p}{q}$ form where p and q are integers and $q \neq 0$

eg→ $0 = \frac{0}{2}$ or $\frac{0}{5}$ or $\frac{0}{6}$

2. 3 and 4

$\frac{3 \times 10}{1 \times 10} = \frac{30}{10}$ $\frac{4 \times 10}{1 \times 10} = \frac{40}{10}$

Six rational no.s are = $\frac{31}{10}, \frac{32}{10}, \frac{33}{10}, \frac{34}{10}, \frac{35}{10}, \frac{36}{10}$

3. $\frac{3}{5}$ and $\frac{4}{5}$

$\frac{3 \times 10}{5 \times 10} = \frac{30}{50}$ $\frac{4 \times 10}{5 \times 10} = \frac{40}{50}$

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Five rational no.s are $\frac{31}{50}, \frac{32}{50}, \frac{33}{50}, \frac{34}{50}, \frac{35}{50}$

Q: Find six rational numbers between $\frac{3}{2}$ and $\frac{7}{3}$

$\frac{3}{2}$ and $\frac{7}{3}$

LCM of 2 and 3 is 6

$\frac{3 \times 3}{2 \times 3}$ and $\frac{7 \times 2}{3 \times 2}$

$\frac{9}{6}$ and $\frac{14}{6}$

$\frac{9 \times 10}{6 \times 10}$ and $\frac{14 \times 10}{6 \times 10}$

$\frac{90}{60}$ and $\frac{140}{60}$

∴ Six rational no.s between $\frac{3}{2}$ and $\frac{7}{3}$ are :-

$\frac{91}{60}, \frac{92}{60}, \frac{93}{60}, \frac{94}{60}, \frac{95}{60}, \frac{96}{60}$

Exercise 1.2

1 (i) True, because real no. contain all rational no.
(ii) False, because we can't write 0 in this form.
(iii) False, because real no. contains rational numbers also.

2. False, because $\sqrt{9}$ is a rational number.

Exercise 1.3

Q1:

(i) $\frac{36}{100} = 0.36$ Terminating

(ii) $\frac{1}{11} = 0.\overline{09}$

Non Terminating Repeating

$$\begin{array}{r|l} 11 & 1 \;\; (0.0909 \\ & -0 \\ & 10 \\ & 00 \\ & 100 \\ & 99 \\ & 1 \end{array}$$

(iii) $4\frac{1}{8} = \frac{33}{8} = 4.125$ Terminating

(iv) $\frac{3}{13} = 0.\overline{230769}$ Non Terminating Repeating

(v) $\frac{2}{11} = 0.\overline{18}$ Non Terminating

(vi) $\frac{329}{400} = 0.8225$ Terminating

Q2: $\frac{1}{7} = 0.\overline{142857}$

$\frac{2}{7} = 2 \times \frac{1}{7} = 2 \times 0.\overline{142857}$

$\frac{3}{7} = 3 \times \frac{1}{7} = 3 \times 0.\overline{142857}$

$\frac{4}{7} = 4 \times \frac{1}{7} = 4 \times 0.\overline{142857}$

$\frac{5}{7} = 5 \times \frac{1}{7} = 5 \times 0.\overline{142857}$

$\frac{6}{7} = 6 \times \frac{1}{7} = 6 \times 0.\overline{142857}$

Q3:

(i) $0.\overline{6}$

Let $x = 0.6$ — (1)

Multiply by 10

$10x = 6.6$ — (2)

subtract ① from ②

$10x - x = 6.\bar{6} - 0.\bar{6}$

$\Rightarrow 9x = 6$

$\Rightarrow x = \frac{\cancel{6}\,2}{\cancel{9}\,3}$

So req. form is $\frac{2}{3}$

(ii) 0.47

Let $x = 0.47$

Multiply by 10

$10x = 4.\bar{7}$ – ①

Multiply by 10

$100x = 47.\bar{7}$ – ②

Subtract ① from ②

$100x - 10x = 47.\bar{7} - 4.\bar{7}$

$\Rightarrow 90x = 43$

$\Rightarrow x = \frac{43}{90}$

So req. form is $\frac{43}{90}$

(iii) $0.\overline{001}$

Let $x = 0.\overline{001}$ – ①

Multiply by 1000

$1000x = 1.\overline{001}$ – ②

Subtract ① from ②

$1000x - x = 1.\overline{001} - 0.\overline{001}$

$999x = 1$

$x = \frac{1}{999}$

So req. form is $\frac{1}{999}$

Q4: $0.\bar{9}$

Let $x = 0.\bar{9}$ – ①

Multiply by 10

$10x = 9.\bar{9}$ – ②

Subtract ① from ②

$10x - x = 9.\bar{9} - 0.\bar{9}$

$9x = 9$

$x = 1$

So req. form is 1

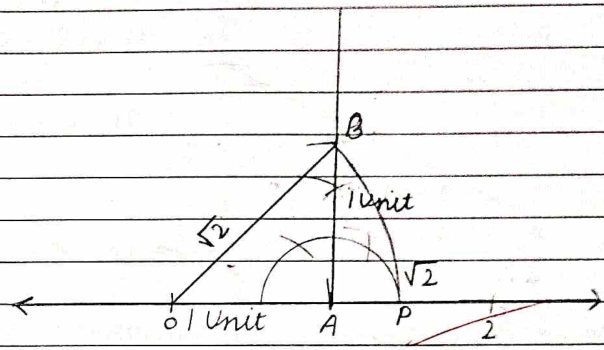
Q5: $\frac{1}{17}$

$17 \overline{)\,1\,}$ 0.0588235294117647

-0
10
-0
100
-85
150
-136
140
-136
40
-34
60
-51
90
-85
50
-34
160
153
70
68
20
-17
30
17
130
119
110
102
80
68
120
119
1

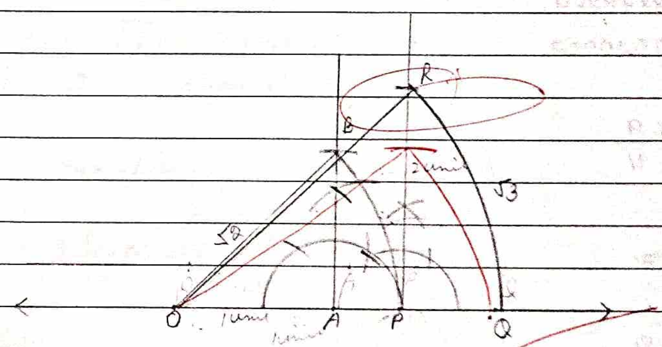
Exercise 1.2

$\sqrt{2}$



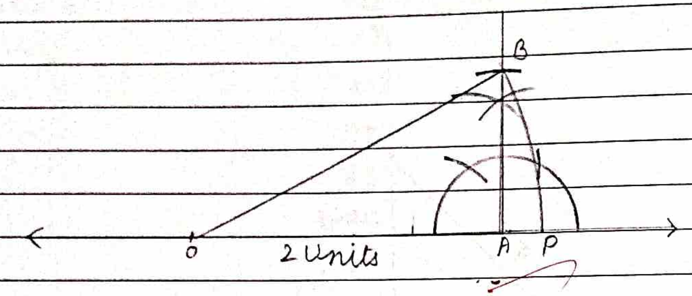
Result - Point P represents $\sqrt{2}$ on number line
So OP $\sqrt{2}$ Unit

$\sqrt{3}$



Result - Point P represents $\sqrt{3}$ on number line
So OQ = $\sqrt{3}$ unit

$\sqrt{5}$



Result - Point P represents $\sqrt{5}$ on number line

6. The prime factorisation of q has only powers of 2 or 5 or both.

7. (i) 1.70700700007.....
(ii) 6.9898898889.....
(iii) 4.20200200002.....

8. $\frac{5}{7}$ and $\frac{9}{11}$

$\frac{5}{7} = 0.71$

$\frac{9}{11} = 0.81$

2 irrational no.s between $\frac{5}{7}$ and $\frac{9}{11}$ are -

(i) 0.720720072000.....
(ii) 0.741741174111.....
(iii) 0.782782278222...

★ Note:-
$R + IR = IR$
$IR + IR = R$ or IR
$R - IR = IR$
$IR \times IR = R$ or IR

9.
(i) $\sqrt{23}$ Irrational
(ii) $\sqrt{225} = 15$ Rational
(iii) 0.3796 Rational
(iv) 7.478478.... Irrational
(v) 1.101001000100001.... Irrational

Exercise 1.4

Q1:
(i) $2 - \sqrt{5}$ Irrational
(ii) $(3 + \sqrt{23}) - \sqrt{23} =$ Rational
(iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$ $\frac{2}{7}$ = Rational
(iv) $\frac{1}{\sqrt{2}}$ = Irrational
(v) $2(\pi)$ = Irrational

Q2: Simplify

(i) $(3+\sqrt{3})(2+\sqrt{2})$

$3(2+\sqrt{2}) + \sqrt{3}(2+\sqrt{2})$
$6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{6}$

(ii) $(3+\sqrt{3})(3-\sqrt{3})$

$= 3^2 - (\sqrt{3})^2$
$= 9-3$
$= 6$

(iii) $(\sqrt{5}+\sqrt{2})^2$

$= (\sqrt{5})^2 + (\sqrt{2})^2 + 2\sqrt{5}.\sqrt{2}$
$= 5+2+2\sqrt{10}$
$= 7+2\sqrt{10}$

(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

$= (\sqrt{5})^2 - (\sqrt{2})^2$
$= 5-2$
$= 3$

Q5: Rationalise

(i) $\frac{1}{\sqrt{7}}$

Multiply Num. and deno. by $\sqrt{7}$

$= \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$
$= \frac{\sqrt{7}}{7}$

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

Multiply num. and deno. by $\sqrt{7}+\sqrt{6}$

$\frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}$

$= \frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7})^2-(\sqrt{6})^2}$

$= \frac{\sqrt{7}+\sqrt{6}}{7-6}$

$= \sqrt{7}+\sqrt{6}$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$

Multiply num. and deno. by $\sqrt{5}-\sqrt{2}$

$= \frac{1}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

$= \frac{\sqrt{5}-\sqrt{2}}{(\sqrt{5})^2-(\sqrt{2})^2}$

$= \frac{\sqrt{5}-\sqrt{2}}{5-2}$

$= \frac{\sqrt{5}-\sqrt{2}}{3}$

Page 16

(iv) $\frac{1}{\sqrt{7}-2}$

Multiply Num and deno. by $\sqrt{7}+2$

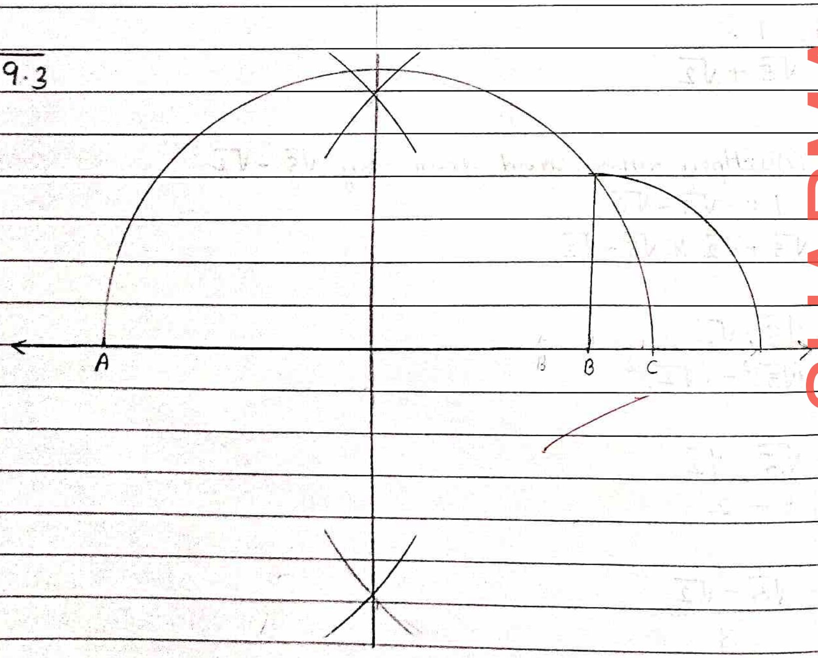
$= \frac{1}{\sqrt{7}-2} \times \frac{\sqrt{7}+2}{\sqrt{7}+2}$

$\frac{\sqrt{7}+2}{(\sqrt{7})^2-(2)^2}$

$\frac{\sqrt{7}+2}{7-4}$

$\frac{\sqrt{7}+2}{3}$

Q4: $\sqrt{9.3}$



Page 17

Laws of exponents

1. $a^m \times a^n = a^{m+n}$
2. $(a^m)^n = a^{mn}$
3. $\frac{a^m}{a^n} = a^{m-n}$, $m > n$
4. $a^m b^m = (ab)^m$

Exercise 1.5

Q1: Find

1. $64^{\frac{1}{2}} = 8^{2 \times \frac{1}{2}}$
$= 8$

2. $32^{\frac{1}{5}}$
$= 2^{5 \times \frac{1}{5}}$
$= 2$

3. $125^{\frac{1}{3}}$
$= 5^{3 \times \frac{1}{3}}$
$= 5^1$

Q2:

1. $9^{\frac{3}{2}}$
$= 3^{2 \times \frac{3}{2}}$
$= 3^3$
$= 27$

2. $32^{\frac{2}{5}}$

$= 2^{5\times\frac{2}{5}}$

$= 2^2 = 4$

3. $16^{\frac{3}{4}}$

$= 2^{4\times\frac{3}{4}}$

$= 2^3$

$= 8$

4. $125^{\frac{-1}{3}}$

$= 5^{3\times-\frac{1}{3}}$

$= 5^{-1}$

$= \frac{1}{5}$

Q3:

1. $2^{\frac{2}{3}} \times 2^{\frac{1}{5}}$

$= 2^{\frac{2}{3}+\frac{1}{5}}$

$= 2^{\frac{10+3}{15}}$

$= 2^{\frac{13}{15}}$

2. $\left(\frac{1}{3^3}\right)^7$

$= \frac{1}{(3^3)^7}$

$= \frac{1}{3^{4}}$

3. $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$

$= 11^{\frac{1}{2}-\frac{1}{4}}$

$= 11^{\frac{2-1}{4}}$

$= 11^{\frac{1}{4}}$

4. $7^{\frac{1}{2}} \times 8^{\frac{1}{2}}$

$= (7\times 8)^{\frac{1}{2}}$

$= 56^{\frac{1}{2}}$

Page 20

Assignment

1. $\frac{0}{4}$

2. $x^{-2} = 64$

$\left(\frac{1}{x}\right)^2 = 8^2$

$\frac{1}{x} = 8$

$x = \frac{1}{8}$

Now, $x^{\frac{1}{3}} + x^0$

$= \left(\frac{1}{8}\right)^{\frac{1}{3}} + 1$

$= \left(\frac{1}{2^3}\right)^{\frac{1}{3}} + 1$

$= \frac{1}{2} + 1$

$= \frac{1+2}{2}$

$= \frac{3}{2}$

3. $\sqrt{7 + \sqrt{48}}$

$= \sqrt{7 + 2\sqrt{4 \times 3}}$

$= \sqrt{4 + 3 + 2\sqrt{12}}$

$= \sqrt{(\sqrt{4})^2 + (\sqrt{3})^2 + (\sqrt{4 \times 3}}$

$= \sqrt{(\sqrt{4} + \sqrt{3})^2}$

$= 2 + \sqrt{3}$

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Page 21

4. $(125)^{\frac{-1}{3}}$

$= (5^3)^{\frac{-1}{3}}$

$= 5^{-1}$

$= \frac{1}{5}$

5. $(625)^{0.16} \times (625)^{0.09}$

$= (625)^{0.25}$

$= (625)^{\frac{25}{100}}$

$= (5^4)^{\frac{1}{4}}$

$= 5$

6. $\sqrt{3 + 2\sqrt{2}}$

$= \sqrt{2 + 1 + 2\sqrt{2}}$

$= \sqrt{(\sqrt{2})^2 + 1^2 + 2\sqrt{2}}$

$= \sqrt{(\sqrt{2} + 1)^2}$

$= \sqrt{2} + 1$

7. $\sqrt[3]{500}$

$\sqrt[3]{2 \times 2 \times 5 \times 5 \times 5}$

$5\sqrt[3]{4}$

8. $\sqrt{5} + 2$

9. $\frac{1}{5 + \sqrt{2}} \times \frac{5 - \sqrt{2}}{5 - \sqrt{2}}$

$= \frac{5 - \sqrt{2}}{5^2 - (\sqrt{2})^2}$

$= \frac{5 - \sqrt{2}}{25 - 2}$

$= \frac{5-\sqrt{23}}{23}$

10. $\frac{1}{2}$ and $\frac{7}{8}$

= LCM of 2 & 8 is 8

$\frac{1}{2} = \frac{1\times4}{2\times4} = \frac{4}{8}$

$\frac{7}{8}$

$\frac{4}{8}$ and $\frac{7}{8}$

So, required rational no.s are $\frac{5}{8}$ & $\frac{6}{8}$

11. 7 and 8

$\frac{7\times10}{1\times10} = \frac{70}{10}$ $\quad$ $\frac{8\times10}{1\times10} = \frac{80}{10}$

So, required rational no.s are $\frac{71}{10}, \frac{72}{10}, \frac{73}{10}$

12. 0.12 and 0.13

So, required rational no.s are :-

0.123123312333.....

0.124124412444....

13. a) $5.\bar{2}$

$x = 5.\bar{2}$

Multiply by 10

$10x = 52.\bar{2}$

$10x - x = 52.2 - 5.2$

$9x = 47$

$x = \frac{47}{9}$

b) $0.2\overline{35}$

Multiply by 10

$10x = 2.\overline{35}$

Multiply by 100

$1000x = 235.\overline{35}$

Subtract

$1000x - 10x = 235.\overline{35} - 2.35$

$990x = 233$

$x = \frac{233}{990}$

c) $x = 0.00\overline{352}$

Multiply by 100

$100x = 0.\overline{352}$

Multiply by 1000

$100000x = 352.352$

Subtract

$100000x - 100x = 352.352$

$99900x = 352$

$x = \frac{352}{99900}$

14. $\frac{4\sqrt{3}+5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}} \times \frac{4\sqrt{3}-3\sqrt{2}}{4\sqrt{3}-3\sqrt{2}}$

$= \frac{48-12\sqrt{6}+20\sqrt{6}-30}{48-18}$

$$\frac{18+8\sqrt{5}}{30} = \frac{\cancel{2}(9+4\sqrt{5})}{\cancel{30}\,15}$$

$$= \frac{9+4\sqrt{5}}{15}$$

15. $x+1 = \sqrt{2}$

$$\frac{1}{x} = \frac{1}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}}$$

$$= \frac{1}{x} = \frac{1-\sqrt{2}}{1^2-(\sqrt{2})^2} = \frac{1}{x} = \frac{1-\sqrt{2}}{1-2}$$

$$= \frac{1}{x} = \frac{-(1-\sqrt{2})}{-1} \text{ (sign change)}$$

$$= \frac{1}{x} = -1+\sqrt{2}$$

$$= \left(x-\frac{1}{x}\right)^3 = \left[1+\sqrt{2}-(-1+\sqrt{2})\right]^3$$

$$= (1+\sqrt{2}+1-\sqrt{2})^3$$

$$= 2^3$$

$$= 8$$

16. $x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, $y = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$$x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$= \frac{(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2} = \frac{3+2+2\sqrt{6}}{3-2}$$

$$= 5+2\sqrt{6}$$

$$y = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{(\sqrt{3}-\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2} = \frac{3+2-2\sqrt{6}}{3-2}$$

$$5-2\sqrt{6}$$

$$= (x+y)^2$$

$$= (5+2\sqrt{6}+5-2\sqrt{6})^2$$

$$= (10)^2$$

$$= 100$$

17. $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a+b\sqrt{3}$

$$= \frac{5+2\sqrt{3}}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} = a+b\sqrt{3}$$

$$= \frac{5+2(\sqrt{3})(7-4\sqrt{3})}{7^2-(4\sqrt{3})^2} = a+b\sqrt{3}$$

$$= \frac{35-20\sqrt{3}+14\sqrt{3}-24}{49-48} = a+b\sqrt{3}$$

$$11-6\sqrt{3} = a+b\sqrt{3}$$

Comparing both sides

$a = 11$, $b = -6$

18. $4^x - 4^{x-1} = 24$

$= 4^x - 4^x . 4^{-1} = 24$

$= 4^x - \frac{4^x}{4} = 24$

$= 4^x \left[\frac{1-1}{4} \right] = 24$

$= 4^x \left[\frac{3}{4} \right] = 24$

$= 4^x = \frac{24 \times 4}{3}$

$= 4^x = 32$

$= 2^{2x} = 2^5$

since bases are same so powers will be equal

$2x = 5$

$x = \frac{5}{2}$

R. Sharma 19/4/23

$(2x)^x$

$(2x)^{\frac{5}{2}}$

$\left(\cancel{2} \times \frac{5}{\cancel{2}}\right)^{\frac{5}{2}}$

$5^{\frac{5}{2}}$

Chapter-2
Polynomials

☆ Polynomial is an algebraic expression in which power of variable is a whole no.

☆ Highest power of variable is called degree of polynomial.

☆ Degree of non zero polynomial is 0

☆ Degree of zero polynomial is not defined.

☆ 1. A polynomial having 1 term is called monomial
2. A polynomial having 2 termis is called Binomial
3. " 3 terms is called
4. " 4 terms is called Quadrinomial

☆ A polynomial of degree 1 is called Linear Polynomial
" 2 is called Qudratic Polynomial
" 3 is called Cubic Polynomial
" 4 is called Biquadratic Polynomial

☆ Zero of zero polynomial is a real no.

Exercise 2.2

1. Let $P(x) = 5x - 4x^2 + 3$

(i) $x = 0$

$P(0) = 5(0) - 4(0)^2 + 3$
$= 0 - 0 + 3$
$= 3$

(ii) $x = -1$

$P(-1) = 5(-1) - 4(-1)^2 + 3$
$= -5 - 4 + 3$
$= -9 + 3$
$= -6$

(iii) $x = 2$

$P(2) = 5(2) - 4(2)^2 + 3$
$P(2) = 10 - 16 + 3$
$= 13 - 16$
$= -3$

2. (i) $P(y) = y^2 - y + 1)$

$P(0) = 0^2 - 0 + 1 = 1$
$P(1) = 1^2 - 1 + 1 = 1$
$P(2) = 2^4 - 2 + 1 = 3$

(ii) $P(t) = 2 + t + 2t^2 - t^3$

$P(0) = 2 + 0 + 2.0^2 - 0^3 = 2$
$P(1) = 2 + 1 + 2(1)^2 - (1)^3 = 4$
$P(2) = 2 + 2 + 2(2)^2 - (2)^3 = 4$

(iii) $P(x) = x^3$

$P(0) = 0^3 = 0$

$P(1) = 1^3 = 1$
$P(2) = 2^3 = 8$

(iv) $P(x) = (x-1)(x+1)$
$P(0) = (0-1)(0+1) = 0^2 - 1^2 = -1$
$P(1) = (1-1)(1+1) = 0$
$P(2) = (2-1)(2+1) = 2^2 - 1^2 = 3$

3. $P(x) = 3x+1$
Put $x = \frac{-1}{3}$

$P\left(\frac{-1}{3}\right) = \cancel{3}\left(\frac{-1}{\cancel{3}}\right) + 1$

$-1+1$
$= 0$

since $P\left(\frac{-1}{3}\right) = 0$

so $x = \frac{-1}{3}$ is 0 of $P(x)$

(ii) $P(x) = 5x - \pi$, $x = \frac{4}{5}$
put $x = \frac{4}{5}$

$P\left(\frac{4}{5}\right) = \cancel{5}\left(\frac{4}{\cancel{5}}\right) - \pi$

$= 4 - \pi$

Since $P\left(\frac{4}{5}\right) \neq 0$

So $\frac{4}{5}$ is not 0 of $P(x)$

19/4/23

(iii) $P(x) = x^2 - 1$, $x = 1, -1$

Put $x = 1$ in $P(x)$
$(P)1 = 1^2 - 1 = 1 - 1 = 0$
$P(-1) = (-1)^2 - 1 = 1 - 1 = 0$
So 1 and -1 are zeroes of $P(x)$

(iv) $P(x) = (x+1)(x-2)$
$x = -1, 2$
Put $x = -1$
$P(-1) = (-1+1)(-1-2)$
$= 0(-3)$
$= 0$
Put $x = 2$
$P(2) = (2+1)(2-2)$
$= 3 \times 0$
$= 0$
$\therefore -1, 2$ are zeroes of $P(x)$

(v) $P(x) = x^2$
Put $x = 0$ in $P(x)$
$P(0) = 0^2 = 0$

$\therefore x = 0$ is zero of $p(x)$

(vi) $P(x) = lx + m$
put $x = \frac{-m}{l}$

$P\left(\frac{-m}{l}\right) = \cancel{l}\left(\frac{-m}{\cancel{l}}\right) + m$

$-m+m=0$

$P\left(\frac{-m}{l}\right)=0$

$\therefore -\frac{m}{l}$ is 0 of $P(x)$

(vii) $P(x) = 3x^2-1$

$x = \frac{-1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

Put $x = \frac{-1}{\sqrt{3}}$

$= P\left(\frac{-1}{\sqrt{3}}\right) = 3\times\left(\frac{-1}{3}\right)^2-1$

$= P\left(\frac{-1}{\sqrt{3}}\right) = \cancel{3}\times\left(\frac{1}{\cancel{3}}\right)-1$

$= P\left(\frac{-1}{\sqrt{3}}\right) = 1-1$

$= P\left(\frac{-1}{\sqrt{3}}\right) = 0$

Put $x = \frac{2}{\sqrt{3}}$

$= P\left(\frac{2}{\sqrt{3}}\right) = 3\times\left(\frac{4}{3}\right)-1$

$= P\left(\frac{2}{\sqrt{3}}\right) = \cancel{3}\times\left(\frac{4}{\cancel{3}}\right)-1$

$= P\left(\frac{2}{\sqrt{3}}\right) = 4-1$

$= P\left(\frac{2}{\sqrt{3}}\right) = 3$

$\therefore$ No $\frac{2}{\sqrt{3}}$ is not 0 of $3x^2-1$

(viii) $P(x) = 2x+1$, $x = \frac{1}{2}$

Put $x = \frac{1}{2}$

$P\left(\frac{1}{2}\right) = \cancel{2}\left(\frac{1}{\cancel{2}}\right)+1$

$P\left(\frac{1}{2}\right) = 1+1$

$P\left(\frac{1}{2}\right) = 2$

No, $\frac{1}{2}$ is not 0 of $2x+1$

Q4. (i) $P(x) = x+5$

$x+5=0$

$x=-5$

So (-5) is zero of $P(x)$

(ii) $P(x) = x-5$

$x-5=0$

$x=5$

So (5) is zero of $P(x)$

(iii) $P(x) = 2x+5$

$2x+5=0$

$2x=-5$

$x=\frac{-5}{2}$

So $-\frac{5}{2}$ is zero of $P(x)$

(iv) $P(x)=3x-2$

$3x-2=0$

$3x=2$

$x=\frac{3}{2}$

So $\frac{3}{2}$ is zero of $P(x)$

(v) $P(x) = 3x$

$3x=0$

$x=0$

So zero is zero of $P(x)$

(vi) $P(x) = ax,\ a \neq 0$

$ax = 0$

$x = 0$

So 0 is zero of $P(x)$

(vii) $P(x)\ cx + d,\ c \neq 0$

$P(x) = cx + d = 0$

$cx = -d$

$x = \frac{-d}{c}$

★ Factor Theorem:- $x - a$ is a factor of $P(x)$, if $P(a) = 0$. Also if $x - a$ is a factor of $p(x)$ then $P(a) = 0$.

Exercise 2.3

1. $x + 1$

Let $P(x) = x^3 + x^2 + x + 1$

$x + 1 = 0$

$x = -1$

$P(-1) = (-1)^3 + (-1)^2 + (-1) + 1$

$P(-1) = -1 + 1 - 1 + 1$

$= 0$

So by factor ~~poto~~ theorem $(x+1)$ is a factor of given polynomial.

(ii) $x^4 + x^3 + x^2 + x + 1$

$x + 1 = 0$

$x = -1$

$P(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$

$P(-1) = 1 + (-1) + 1 + (-1) + 1$

$1 - 1 + 1 - 1 + 1$

$= 1$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$

$x + 1 = 0$

$x = -1$

$P(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$

$P(-1) = 1 + (-3) + 3 - 1 + 1$

$1 - \cancel{3} + \cancel{3} - \cancel{1} + \cancel{1}$

$= 1$

So by factor theorem $(x+1)$ is not a factor of $x^4 + 3x^3 + 3x^2 + x + 1$

(iv) $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

$x + 1 = 0$

$x = (-1)$

$(-1)^3 - (-1)^2 - (2 + \sqrt{2})(-1) + \sqrt{2}$

$-1 - 1 - 2 - \cancel{\sqrt{2}}(-1) + \cancel{\sqrt{2}}$

$-2 - 2 - 1 = -4 - 1$

$= -5$

So by factor theorem $(x+1)$ is not the factor of $x^3 - x^2 - (2\sqrt{2})x + \sqrt{2}$

Q2 (i) $P(x) = 2x^3+x^2-2x-1$

$g(x) = x+1$

$x+1=0$

$x=-1$

$P(-1) = 2(-1)^3+(-1)^2-2(-1)-1$

$P(-1) = 2(-1)+1+2-1$

$P(-1) = -2+1+2-1$

$P(-1) = 0$

So $g(x)=x+1$ is a factor of $P(x)$ by factor theorem.

(ii) $P(x) = x^3+3x^2+3x+1$

$G(x) = x+2$

$x+2=0$

$x=-2$

$P(-2) = (-2)^3+3(-2)^2+3(-2)+1$

$= -8+3(4)+3(-2)+1$

$= -8+12-6+1$

$= -1$

So by factor theorem $g(x)$ is a factor of $P(x)$

(iii) $P(x) = x^3-4x^2+x+6$, $g(x) = x-3$

$x-3=0$

$x=3$

$P(3) = (3)^3-4(3)^2+3+6$

$P(3) = 27-36+3+6$

$= 0$

So by factor Theorem $g(x)$ is a factor of $P(x)$

Q3: $x-1$, is a factor of $P(x)$

(i) $P(x) = x^2+x+k$

$P(x-1=0$

$x=1$

Since $x-1$ is a factor of $P(x)$

$P(1)=0$

$1^2+1+k=0$

$2+k=0$

$k=-2$

(ii) $P(x) = 2x^2+kx+\sqrt{2}$

$x-1=0$

$x=1$

$P(1) = 2(1)^2+k(1)+\sqrt{2}$

$2+k+\sqrt{2}$

$k+\sqrt{2} = -2$

$k = -2-\sqrt{2}$

$k = -(2+\sqrt{2})$

(iii) $P(x) = kx^2-\sqrt{2}x+1$

$x-1=0$

$x=1$

$P(1) = k(1)^2-\sqrt{2}(1)+1$

$k-\sqrt{2}+1=0$

$k=\sqrt{2}-1$

(iv) $P(x) = kx^2-3x+k$

$x-1=0$

$x=1$

$P(1) = k(1)^2-3(1)+k$

$k(1)-3+k$

$k-3+k$

Page 38

$2k-3=0$

$2k=3$

$k=\frac{3}{2}$

Q4: Factorise

1. $12x^2-7x+1$

$12x^2-3x-4x+1$

$3x(4x-1)-1(4x-1)$

$(3x-1)(4x-1)$

2. $2x^2+7x+3$

$2x^2+6x+x+3$

$2x(x+3)+1(x+3)$

$(2x+1)(x+3$

3. $6x^2+5x-6$

$=6x^2+9x-4x-6$

$=3x(2x+3)-2(2x+3)$

$=3x(2x+3)-2(2x+3)$

$=(3x-2)(2x+3)$

4. $3x^2-x-4$

$3x^2-4x+3x-4$

$x(3x-4)+1(3x-4)$

$(x+1)(3x-4)$

classmate

Page 39

★ Factors:-

Dividend = Divisor x Quotient + Remainder

Q5: Factorise

(i) Let $P(x)=x^3-2x^2-x+2$

Possible factors of 2 are $\pm 1, \pm 2$

$\boxed{P(-1)}=(-1)^3-2(-1)^2-(-1)+2$

$=\cancel{1}-\cancel{2}+\cancel{1}+\cancel{2}$

$=0$

So -1 is zero of $P(x)$ $\quad x+1=0 \quad x=-1$

$x+1$ is factor of $P(x)$

$$
\begin{array}{r|l}
x+1 & \cancel{x^3}-2x^2-x+2 \;\; \big[\, x^2-3x+2 \\
 & -\cancel{x^3}\pm x^2 \\
 & -3\cancel{x^2}-x+2 \\
 & \mp 3\cancel{x^2}\mp 3x \\
 & 2\cancel{x}+\cancel{2} \\
 & 2\cancel{x}+\cancel{2} \\
 & 0
\end{array}
$$

Dividend $= D\times Q+R$

$=(x+1)(x^2-3x+2)$

$(x+1)\left[x(x-2)-1(x-2)\right]$

$(x+1)(x-2)(x-1)$

(ii) Let $P(x) = x^3 - 3x^2 - 9x - 5$

Possible factors of -5 are $\pm 1, \mp 5$

$P(-1) = (-1)^3 - 3(-1)^2 - 9(-1) - 5$

$= -1 - 3 + 9 - 5$

$= -9 + 9$

$= 0$

Therefore (-1) is zero of $P(x)$

$x+1$ is factor of $P(x)$

$x+1 \overline{)\ x^3 - 3x^2 - 9x - 5\ } (x^2 - 4x - 5$

$-\cancel{x^3} \pm x^2$

$-4x^2 - 9x - 5$

$\mp 4x^2 \mp 4x$

$-5x - 5$

$\mp 5x \mp 5$

0

Dividend $= D \times Q + R$

$x^3 - 3x^2 - 9x - 5 = (x+1)(x^2 - 4x - 5)$

$= (x+1)(x^2 - 5x + x - 5)$

$= (x+1)[x(x-5) + 1(x-5)]$

$= (x+1)(x-5)(x+1)$

(iii) $x^3 + 13x^2 + 32x + 20$

Possible factors of $+20$ are $\pm 1, \pm 2, \pm 4, \pm 5, \pm 20$

$P(-1) = (-1)^3 + 13(-1)^2 + 32(-1) + 20$

$= (-1)^3 + 13 - 32 + 20$

$= 0$

Therefore (-1) is 0 of $P(x)$

$x+1$ is the factor of $P(x)$

$x+1 \overline{)\ x^3 + 13x^2 + 32x + 20\ } (x^2 + 12x + 20$

$-\cancel{x^3} \pm x^2$

$12x^2 + 32x + 20$

$-\cancel{12x^2} \pm 12x$

$20x + 20$

$\cancel{20x} \pm \cancel{20}$

0

Dividend $= D \times Q + R$

$(x+1)(x^2 + 12x + 20)$

$(x+1)(x^2 + 10x + 2x + 20)$

$(x+1)[x(x+10) + 2(x+10)]$

$(x+1)(x+10)(x+2)$

(iv) $2y^3 + y^2 - 2y - 1$

$(P)(+1) = 2(+1)^3 + (+1)^2 - 2(+1) + 1$

$= -\cancel{2} + \cancel{1} + \cancel{2} - \cancel{1}$

$= 0$

$2(+1)^3 + (1)^2 - 2(1) - 1$

$2 + 1 - 2 - 1$

$2(-1)^3 + (-1)^2 - 2(-1) - 1$

$-2 + 1 + 2 - 1$

$\therefore y+1$ is factor of $P(x)$

$y+1 \overline{)\ 2y^3 + y^2 - 2y - 1\ } (2y^2 + 3y + 1$

$-\cancel{2y^3} \pm 2y^2$

$3y^2 - 2y - 1$

$\cancel{3y^2} \mp 3y$

$y - 1$

$\cancel{y} \mp 1$

0

$(y-1)[2y^2 + 3y + 1]$

$(y-1)(2y^2 + 2y + y + 1)$

$(y-1)(2y(y+1) + (y+1))$

$(y-1)(2y+1)(y+1)$

Algebraic Identities

1. $(a+b)^2 = a^2+b^2+2ab$
2. $(a-b)^2 = a^2+b^2-2ab$
3. $(a+b)(a-b) = a^2-b^2$
4. $(x+a)(x+b) = x^2+(a+b)x+ab$
5. $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$
6. $(a+b)^3 = a^3+b^3+3ab(a+b) = a^3+b^3+3a^2b+3ab^2$
7. $(a-b)^3 = a^3-b^3-3ab(a-b) = a^3-b^3-3a^2b-3ab^2$
8. $a^3+b^3 = (a+b)(a^2+b^2-ab)$
9. $a^3-b^3 = (a-b)(a^2+b^2+ab)$
10. $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

Exercise 2.4

Q1:

(i) $(x+4)(x+10)$

Using $(x+a)(x+b) = x^2+(a+b)x+ab$

$x^2+(4+10)x+4\times 10$

$= x^2+14x+40$

(ii) $(x+8)(x-10)$

Using $(x+a)(x+b) = x^2+(a+b)x+ab$

$x^2+(8-10)x+8\times(-10)$

$x^2-2x-80$

(iii) $(3x+4)(3x-5)$

Using $(x+a)(x+b) = x^2+(a+b)x+ab$

$(3x)^2+(4-5)3x+4\times(-5)$

$= 9x^2-3x-20$

(iv) $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

Using $(x+a)(x+b) = x^2+(a+b)x+ab$

$= (y^2)^2+\left(\frac{3}{2}-\frac{3}{2}\right)y^2+\frac{3}{2}\times\frac{(-3)}{2}$

$= y^2-\frac{9}{4}$

(v) $(3-2x)(3+2x)$

Using $(x+a)(x+b) = x^2+(a+b)x+ab$

$= 3^2+(-2x+2x)3+2x\times(-2x)$

$= 9-4x^2$

Q2:

(i) 103×107

$= (100+3)(100+7)$

Using $(x+a)(x+b) = x^2+(a+b)x+ab$

$= (100)^2+(3+7)100+3\times 7$

$= 10000+1000+21$

$= 11021$

(ii) 95×96

$(90+5)(90+6)$

Using $(x+a)(x+b) = x^2+(a+b)x+ab$

$(90)^2+(5+6)90+(5\times 6)$

$= 8100+990+30$

$= 9120$

(iii) (104×96)

$(100+4)(100-4)$

$(100)^2+(4-4)100+(4\times(-4))$

$10000+0-16$

$= 9984$

Q3:

(i) $9x^2 + 6xy + y^2$

$(3x)^2 + 2.3x.y + (y)^2$

Using $a^2 + b^2 + 2ab = (a+b)^2$

$= (3x+y)^2$

$= (3x+y)(3x+y)$

(ii) $4y^2 - 4y + 1$

$(2y)^2 - 2.2y.1 + (1)^2$

Using $a^2 + b^2 + 2ab$

$= (2y-1)^2$

$= (2y-1)(2y-1)$

(iii) $x^2 - \frac{y^2}{100}$

$(x)^2 - \left(\frac{y}{10}\right)^2$

Using $a^2 - b^2 = (a+b)(a-b)$

$= \left(x + \frac{y}{10}\right)\left(x - \frac{y}{10}\right)$

Q4: (i) $(x + 2y + 4z)^2$

Using $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$x^2 + (2y)^2 + (4z)^2 + 2.x.2y + 2.2y.4z + 2.4z.x$

$x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8zx$

(ii) $(2x - y + z)^2$

Using $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$(2x)^2 + (-y)^2 + (z)^2 + 2.2x.(-y) + 2.(-y).(z) + 2(z)(2x)$

$4x^2 + y^2 + z^2 - 4xy - 2yz + 4zx$

(iii) $(-2x + 3y + 2z)^2$

Using $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$(-2x)^2 + (3y)^2 + (2z)^2 + 2(-2x)(3y) + 2(3y)(2z) + 2(2z)(-2x)$

$4x^2 + 3y^2 + 4z^2 - 12xy + 12yz - 8zx$

(iv) $(3a - 7b - c)^2$

Using $(a+b+c)^2 = a^2 + b^c + c^2 + 2ab + 2bc + 2ca$

$(3a)^2 + (-7b)^2 + (-c)^2 + 2(3a)(-7b) + 2(-7b)(-c) + 2(-c)(3a)$

$9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ca$

(v) $(-2x + 5y - 3z)^2$

Using $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$(-2x)^2 + (5y)^2 + (-3z)^2 + 2(-2x)(+5y) + 2(5y)(-3z) + 2(-3z)(-2x)$

$4x^2 + 25y^2 + 9z^2 + 20xy - 30yz + 12xz$

(vi) $\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2$

Using $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$= \left(\frac{1}{4}a\right)^2 + \left(\frac{-1}{2}b\right)^2 + (1)^2 + 2\left(\frac{1}{4}a\right)\left(\frac{-1}{2}b\right) + 2\left(\frac{-1}{2}b\right)(1) + 2(1)\left(\frac{1}{4}a\right)$

$= \frac{a^2}{16} + \frac{b^2}{4} + 1 - \frac{ab}{4} - b + \frac{a}{2}$

Q5: Factorise

(i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

$=(2x)^2+(3y)^2+(-4z)^2+2(2x)(3y)+2(3y)(-4z)+2(2x)(-4z)$

= Using $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

$(2x+3y-4z)^2$

$(2x+3y-4z)(2x+3y-4z)$

(ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

$(-\sqrt{2}x)^2+y^2+(2\sqrt{2}z)^2+2(-\sqrt{2}x).y+2.y(2\sqrt{2}z)+2(2\sqrt{2}z)($

Using $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

$(-\sqrt{2}x+y+2\sqrt{2}z)^2$

$(-\sqrt{2}x+y+2\sqrt{2}z)(-\sqrt{2}x+y+2\sqrt{2}z)$

Q6: $(2x+1)^3$

Using $(a+b)^3 = a^3+b^3+3a^2b+3ab^2$

$(2x)^3+(1)^3+3(2x)^2+1+3(2x)+1^2$

$8x^3+1+12x^2+6x$

$8x^2+12x^2+6x+1$

(ii) $(2a-3b)^3$

Using $a^3-b^3-3a^2b+3ab^2$

$(2a)^3-(3b)^3-3(2a)^2(3b)+3(2a)(3b)^2$

$8a^3-27b^3-36a^2b+54ab^2$

(iii) $\left[\frac{3}{2}x+1\right]^3$

Using $(a+b)^3 = a^3+b^3+3a^2b+3ab^2$

$= \left(\frac{3}{2}x\right)^3+(1)^3+3\left(\frac{3}{2}x\right)^2(1)+3\left(\frac{3}{2}x\right)+1^2$

$= \frac{27}{8}x^3+1+\frac{27}{4}x^2+\frac{9}{2}x$

$= \frac{27}{8}x^3+\frac{27}{4}x^2+\frac{9}{2}x+1$

(iv) $\left[x-\frac{2}{3}y\right]^3$

Using $a^3-b^3-3ab(a-b)$

$= (x)^3-\left(\frac{2}{3}y\right)^3-3(x)\left(\frac{2}{3}y\right)\left(x-\frac{2}{3}y\right)$

$= x^3-\frac{8}{27}y^3-2x^2y+\frac{4xy^2}{3}$

Q7: Evaluate

(i) $(99)^3$

$=(100-1)^3$

using $(a-b)^3 = a^3+b^3+3a^2b+3ab^2$ or $a^3-b^3-3ab(a-b)$

$100^3-1^3-3\times100\times1(100-1)$

$=1000000-1\times99-300$

$=1000000-1-300\times99$

$=1000000-1-29700$

$=1000000-29701$

$=970299$

(ii) $(998)^3$

$=(1000-2)^3$

Using $a^3-b^3-3ab(a-b)$

$(1000)^3-2^3-3\times1000\times2(1000-2)$

$= 1000000000-8-6000\times998$

$= 1000000000-8-5988000$

$= 1000000000-5988009$

$= 994011992$

classmate Date Page 48

(iii) $(102)^3 = (100+2)^3$

Using $a^3+b^3+3a^2b+3ab^2 = (a^3+b^3+3ab(a+b)$

$= 100^3+2^3+3(100)(2)(100+2)$

$= 1000000+8+600\times102$

$= 1061208$

Q8: Factorise

(i) $8a^3+b^3+12a^2b+6ab^2$

$= (2a)^3+b^3+3(2a)^2b+3(2a)b^2$

Using $a^3+b^3+3a^2b+3ab^2 = (a+b)^3$

$= (2a+b)^3$

$= (2a+b)(2a+b)(2a+b)$

(ii) $8a^3-b^3-12a^2b+6ab^2$

$= (2a)^3+b^3+3(2a)^2b+3(a)b^2$

Using $a^3+b^3+3a^2b+3ab^2 = (a+b)^3$

$= (2a-b)^3$

$= (2a-b)(2a-b)(2a-b)$

(iii) $27-125a^3-135a+225a^2$

$= 3^3-(5a)^3-3(3)^2(5a)+3(3)(5a)^2$

Using $a^3-b^3-3a^2b+3ab^2 = (a-b)^3$

$= (3-5a)^3$

$= (3-5a)(3-5a)(3-5a)$

(iv) $64a^3-27b^3-144a^2b+108ab^2$

$(4a)^3-(3b)^3-3(4a)^2(3b)+3(4a)(3b)^2$

Using $a^3-b^3-3(a^2b)+3ab^2$

$(4a-3b)^3$

$(4a-3b)(4a-3b)(4a-3b)$

classmate Date Page 49

(v) $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$

$3p^3-\left(\frac{1}{8}\right)^3-3(3p)^2\left(\frac{1}{8}\right)+3(3p)\left(\frac{1}{6}\right)^2$

Using $a^3-b^3-3(a^2b)+3b^2 = (a-b)^3$

$\left(3p-\frac{1}{8}\right)^3 = \left(3p-\frac{1}{8}\right)\left(3p-\frac{1}{8}\right)\left(3p-\frac{1}{8}\right)$

Q9: $x^3+y^3 = (x+y)(x^2-xy+y^2)$

RHS $= (x+y)(x^2-xy+y^2)$

$= x(x^2-xy+y^2)+y(x^2-xy+y^2)$

$= x^3-\cancel{x^2y}+\cancel{xy^2}+\cancel{x^2y}-\cancel{xy^2}+y^3$

$= x^3+y^3$

$=$ LHS

Since LHS = RHS

Hence, verified

(ii) $x^3-y^3 = (x-y)(x^2+xy+y^2)$

RHS $= (x-y)(x^2+xy+y^2)$

$= x(x^2+xy+y^2)-y(x^2+xy+y^2)$

$= x^3+\cancel{x^2y}+\cancel{xy^2}-\cancel{x^2y}-\cancel{xy^2}-y^3$

$= x^3-y^3$

$=$ LHS

Since LHS = RHS

Hence, verified

Q10: Factorise

1. $27y^3 + 125z^3$

$= (3y)^3 + (5z)^3$

using $a^3+b^3 = (a+b)(a^2-ab+b)^2$

$(3y+5z)[(3y)^2 - 3y(5z) + (5z)^2]$

$(3y+5z)[9y^2 - 15yz + 25z^2]$

2. $64m^3 - 343n^3$

$= (4m)^3 - (7n)^3$

Using $a^3-b^3 = (a-b)(a^2+ab+b^2)$

$= (4m-7n)[(4m^2) + 4m(7n) + (7n)^2]$

$= (4m-7n)[16m^2 + 28mn + 49n^2]$

Q11: $27x^3 + y^3 + z^3 - 9xyz$

$= (3x)^3 + y^3 + z^2 - 3(3x)(y)(z)$

Using $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2+b^2+c^2-ca-bc-ab)$

$= (3x+y+z)[(3x)^2 + (y)^2 + z^2 - (3x)(y) - (y)(z) - (z)(3x)$

$= (3x+y+z)(9x^2+y^2+z^2-3xy-yz-3xz)$

Q13: If $x+y+z=0$

show $x^3+y^3+z^3 = 3xyz$

$x+y+z=0$

$x+y=-z$ —(1)

cubing both sides

$(x+y)^3 = (-z)^3$

$x^3+y^3+3xy(x+y) = -z^3$

$x^3+y^3+3xy(-z) = -z^3$ (using eq. (1))

$x^3+y^3-3xyz = -z^3$

$x^3+y^3-3xyz = \cancel{-z^3}$

$x^3+y^3+z^3 = 3xyz$

Hence Proved

Q14:

(i) $(-12)^3+(7)^3+(5)^3$

Here $x=-12, y=7, z=5$

$x+y+z = -12+7+5$

$= 0$

we know if $x+y+z=0$ then $x^3+y^3+z^3 = 3xy$

$\therefore (-12)^3+7^3+5^3 = 3(-12)(7)(5)$

$= -1260$

(ii) $(28)^3+(-15)^3+(-13)^3$

Here $x=28, y=-15, z=-13$

$x+y+z = 28-15-13$

~~If x+v~~ $= 0$

If $x+y+z=0$ then $x^3+y^3+z^3 = 3xyz$

$\therefore (28)^3+(-15)^3+(-13)^3 = 3(28)(-15)(-13)$

$= 16380$

Q15:

(i) Area of Rectangle $= 25a^2 - 35a + 12$

$= 25a^2 - 20a - 15a + 12$

$= 5a(5a-4) - 3(5a-4)$

$(5a-3)(5a-4)$

Length $= (5a-3)$ unit

Breadth $= (5a-4)$ unit

2	300
2	210
3	105
5	35
7	7
	1

(ii) Area of rectangle

$= 35y^2 + 13y - 12$

$= 35y^2 + 28y - 15y - 12$

$= 7y(5y+4) - 3(5y+4)$

$= (7y-3)(5y+4)$

Length $= (5y+4)$ Unit

Breadth $= (7y-3)$ Units.

Q16: (i) $V = 3x^2 - 12x$

$= 3x(x-4)$

So possible dimensions are :-

$l = 3$ Units

$b = x$ unit

$h = (x-4)$ unit

(ii) $V = 12k^2y + 8ky - 20k$

$V = 4k(3y^2 + 2y - 5)$

$= 4k(3y^2 + 5y - 3y - 5)$

$= 4k[y(3y+5) - 1(3y+5)$

$= 4k(y-1)(3y+5)$

So possible dimensions are

$l = (4k)$ units

$b\ (y-1)$

$h = (3y+5)$

Assignment
Chapter-2
Polynomials

6. $a-b=8$, $ab=12$, $a^3-b^3=?$

$(a-b)^3 = a^3-b^3-3ab(a-b)$

$=(-8)^3 = a^3-b^3-3\times(-12)(-8)$

$-512 = a^3-b^3-288$

$a^3-b^3=-224$

7. $\dfrac{(2.3)^3-(0.027)}{(2.3)^2+0.69+0.09}$

$\dfrac{(2.3)^3-(0.3)^3}{(2.3)^2+2.3\times0.3+0.3^2}$

$a^3-b^3 = (a-b)(a^2+b^2+ab)$

$= \dfrac{(2.3-0.3)[(2.3)^2+(0.3)^2+(2.3)(0.3)]}{(2.3)^2+2.3\times0.3+(0.3)^2}$

$= 2.3-0.3$

$=2$

Subjective type questions

1. $P(x) = 69+11x-x^2+x^3$

$x+3=0$

$x=-3$

$P(-3) = 69+11(-3)-(-3)^2+(-3)^3$

$=69-33-9-27$

$=69-69$

$=0$

Since $P(-3)=0$, So by factor theorem $(x+3)$ is a factor of $P(x)$

2. $(x-2)^2-(x+2)^2=0$

Using $a^2-b^2 = (a+b)(a-b)$

$(x-2+x+2)(x+2-x-2)=0$

$2x(-4)=0$

$-8x=0$

$x=\dfrac{0}{8}$ or $x=0$

3. $64m^3-343n^3$

$(4m)^3-(7n)^3$

$a^3-b^3=(a-b)(a^2+b^2+2ab)$

$=(4-7)[(4)^2+(7)^2+3(4)(7)]$

$=3[16+49+84]$

4. $l+m+n=0$

LHS $\dfrac{l^2}{mn}+\dfrac{m^2}{nl}+\dfrac{n^2}{ml}$

$=\dfrac{l^3+m^3+n^3}{lmn}$

We know if $x+y+z=0$ then $x^3+y^3+z^3=3xyz$

Page 56

~~3.4mn~~
~~4mn~~

-3

$= RHS$

5. Let $P(x) = 3x^2 + kx + 6$

$x + 3 = 0$

$x = -3$

$P(-3) = 3(-3)^2 + k(-3) + 6$

$P(-3) = 27 + (-3k + 6)$

6. $x+2$ and $x+1$ are factors of $x^3 + 3x^2 - 2mx + n$

Let $P(x) = x^3 + 3x^2 - 2mx + n$

$x + 2 = 0$	$x + 1 = 0$
$x = -2$	$x = -1$
ATQ	$P(-1) = 0$
$P(-2) = 0$	$(-1)^3 + 3(-1)^2 - 2m(-1) + n$
$(-2)^3 + 3(-2)^2 - 2m(-2) + n$	$2 + 2m - 4 - 4m$
$-8 + 12 + 4m + n = 0$	$-2m = -2$
$4 + 4m + n = 0$	$m = \frac{-2}{-2} = -1$
$n = -4 \neq 4m$	
	$P(m) = -1 \quad = -4 - 4(-1)$
	$n = -4 + 4$
	$n = -4 + 4 = 0$

~~S~~o(m) $= -1$, $n = 0$

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Page 57

8. If $x - 2$ and $x - \frac{1}{2}$ are factors of $px^2 + 5x + r$
prove that $p = r$
So let $f(x) = px^2 + 5x + r$

$x - 2 = 0$

$x = 2$

$F(2) = 0$

$P(2)^2 + 5(2) + r = 0$

$4p + 10 + r = 0$

$4p + r = -10 \quad -(1)$

$x - \frac{1}{2} = 0$

$x = \frac{1}{2}$

$F\left(\frac{1}{2}\right) = 0$

$P\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) + r = 0$

$\frac{p}{4} + \frac{5}{2} + r = 0$

Multiplying by 4

$P + 10 + 4r = 0$

$P + 4R = -10 \quad -(2)$

From (1) & (2)

$4R + r = P + 4R$

$4P - R = 4R - r$

$3p = 3r$

$p = r$

9. $2\sqrt{2}a^3+8b^3-27c^3+18\sqrt{2}abc$

$= (\sqrt{2}a)^3+(2b)^3+(-3c)^3-3(\sqrt{2}a)(2b)(-3c)$

$a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$= (\sqrt{2}a+2b-3c)[(\sqrt{2}a)^2+(2b)^2+(-3c)^2-\sqrt{2}a(2b)-(2b)(-3c)-(-3c)(\sqrt{2}a)]$

$= (\sqrt{2}a+2b-3c)(2a^2+4b^2+9c^2-2\sqrt{2}ab+6bc+3\sqrt{2}a)$

10. If $x-\frac{1}{x}=2$ find $x^4+\frac{1}{x^4}$

$x-\frac{1}{x}=2$

Squaring both sides

$\left(x-\frac{1}{x}\right)^2 = 2^2$

$x^2+\frac{1}{x^2}-2=4$

$x^2+\frac{1}{x^2}=6$

Squaring both sides

$\left(x^2+\frac{1}{x^2}\right)^2 = 6^2$

$x^4+\frac{1}{x^4}+2=36$

$x^4+\frac{1}{x^4}=34$

11. $x^2+\frac{1}{x^2}=14$

$=\left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2$

$=\left(x+\frac{1}{x}\right)^2 = 14+2$

$=\left(x+\frac{1}{x}\right)^2 = 16$

$= x+\frac{1}{x} = 4$

Cubing both sides

$\left(x+\frac{1}{x}\right)^3 = 4^3$

$x^3+\frac{1}{x^3}+3=64$

$x^3+\frac{1}{x^3}=64-3$

$x^3+\frac{1}{x^3}=61$

Chapter-4

Linear Equations in two variables

Exercise 4.1

Q1: Let cost of notebook be ₹x

Let cost of pen be ₹y

ATQ

cost of notebook = 2 × Cost of pen

$x = 2y$

$x - 2y = 0$ is req. equation.

Q2: (i) $2x + 3y = 9.3\overline{5}$

$2x + 3y = 9.3\overline{5}$

$2x + 3y - 9.3\overline{5} = 0$

Here $a = 2x$, $b = 3y$, $c = -9.3\overline{5}$

(ii) $x - \frac{y}{5} - 10 = 0$

$a = x$, $b = \frac{-y}{5}$, $c = -10$

(iii) $-2x + 3y = 6$

$-2x + 3y - 6 = 0$

Here $a = -2x$, $b = 3y$, $c = -6$

(iv) $x = 3y$

$x - 3y = 0$

$x - 3y + 0 = 0$

Here $a = x$, $b = -3y$, $c = 0$

(v) $2x = -5y$

$2x + 5y + 0 = 0$

$a = 2x$, $b = 5y$, $c = 0$

(vi) $3x + 2 = 0$

$3x + 2 + 0 = 0$

$a = 3x$, $b = 2$, $c = 0$

(vii) $y - 2 = 0$

$y - 2 + 0 = 0$

$a = y$, $b = -2$, $c = 0$

(viii) $5 = 2x$

$5 - 2x + 0 = 0$

$a = 5$, $b = -2x$, $c = 0$

V. Good

6/5/23

Exercise 4.2

Q1: (iii) Infinitely many solutions

→ Because of every value of x, there is a corresponding value of y and vice a versa.

Q4: $x = 2$, $y = 1$

$2x + 3y = k$

$2 \times 2 + 3 \times 1 = k$

$4 + 3$ k

$k = = -7$

Q2:

(i) $2x + y = 7$

$y = 7 - 2x$ — (1)

Put $x = 0$ so $y = 7$

Put $x = 1$ so $y = 7 - 2 = 5$

Put $x = 3$ so $y = 7 - 6 = 1$

Put $x = 4$ so $y = 7 - 8 = -1$

so $(0,7)$; $(1,5)$; $(3,1)$; $(4,-1)$ are 4 solutions

(ii) $\pi x + y = 9$

$y = 9 - \pi x$ — (1)

Put $x = 0$, so $y = 9$

Put $x = 1$, So $y = 9 - \pi$

Put $x = 2$, So $y = 9 - 2\pi$

Put $x = 3$, So $y = 9 - 3\pi$

so $(0,9)$; $(1, 9-\pi)$; $(2, 9-2\pi)$; $(3, 9-3\pi)$

(iii) $x = 4y$

Put $y = 0$, $x = 0$

Put $y = 1$, $x = 4$

Put $y = 2$, $x = 8$

Put $y = -1$, $x = -4$

so 4 Solutions are $(0,0)$; $(1,4)$; $(2,8)$; $(-1,-4)$

Q3: (i) $(0,2)$

Put $x = 0, y = 2$ in given eq.

$0 - 2(2) = 4$

$-4 = 4$

which is false

so $(0,2)$ is not sol. of given eq.

(ii) $(2,0)$

Put $x = 2, y = 0$ in given eq.

$2 - 2(0) = 4$

$2 - 2 = 4$

$0 \neq 4$

which is false

so $(2,0)$ is not sol. of given eq.

(iii) $(4,0)$

Put $x = 4, y = 0$ in given eq.

$4 - 2(0) = 4$

$4 = 4$

which is true

so $(4,0)$ is sol. of given eq.

(iv) $(\sqrt{2}, 4\sqrt{2})$

Put $x = \sqrt{2}, y = 4\sqrt{2}$ in given eq.

$\sqrt{2} - 2(4\sqrt{2}) = 4$

$\sqrt{2} - 8\sqrt{2} \neq 4$

which is false

so, $(\sqrt{2}), 4\sqrt{2})$ is not sol. of given eq.

(v) (1,1)

Put $x=1, y=1$ in given eq.

$1-2(1)=4$

$1-2=4$

$-1 \neq 4$

which is false

So (1,1) is not sol. of given eq.

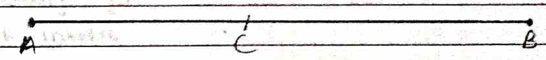
Chapter - 5

Introduction to Euclid's Geometry

Q4: In fig point C lie between point A and B

$AC = BC$

To prove $AC = \frac{1}{2} AB$



Proof:- $AC = BC$ | Given

Add AC on both sides

$AC + AC = BC + AC$

$2AC = AB$

$AC = \frac{1}{2} AB$

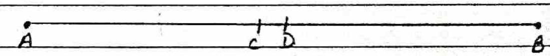
hence proved

Q5: In fig C is mid point of AB

Let AB has one more mid point which is D

If C is mid pt. of AB

$AC = BC$ — ①



If D is mid pt. of AB

$AD = BD$ — ②

Sub. eq ② from eq. ①

$AC - AD = BC - BD$

$DC = -DC$

$DC + DC = 0$

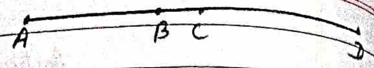
$2DC = 0$

$DC = 0$

It means that it has 2 mid points

Page 66

Q6: Given = $AC = BD$



To prove :- $AB = CD$

Proof $AC = BD$ — (1) | Given

Sub. BC from both sides

$AC - BC = BD - BC$ | If equals are subtracted from equal remainders are also equal

$AB = CD$

Hence Proved

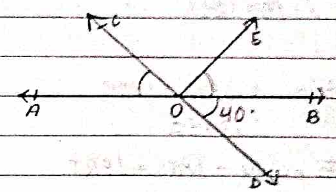
15/5/23

Page 67

Chapter - 6

Lines and Angles

Exercise 6.1



1. $\angle AOC + \angle BOE = 70°$

$\angle BOD = 40°$

To find $\angle BOE$ and reflex $\angle COE$

$\angle BOD = \angle AOC = 40°$ [V.O.A]

$\angle AOC + \angle BOE = 70°$ — (1)

So $40° + \angle BOE = 70°$

$\angle BOE = 30°$

$\angle AOC + \angle COE + \angle BOE = 180°$ [Sum of ∠s on a straight line]

Using (1)

$70 + \angle COE = 180°$

$\angle COE = 110°$

reflex $\angle COE = 360° - 110°$

$= 250°$

hence $\angle BOE = 30°$ and reflex $\angle COE = 250°$

2. $\angle POY = 90°$

$a : b = 2 : 3$

To find $\angle c$

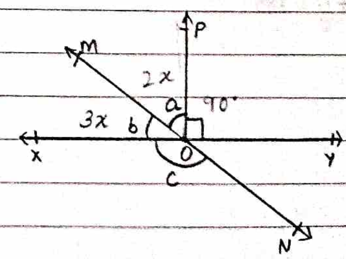
$a + b + \angle POY = 180°$ [Sum of ∠s on a straight line]

$a + b + 90 = 180°$

$a + b = 90°$ — (1)

Let $a = 2x°$ and $b = 3x°$

From (1)



$2x + 3x = 90°$

$5x = 90°$

$x = 18°$

Page 68

So $b = 3x = 3 \times 18 = 54$

$\angle b + \angle c = 180^\circ$ [Linear Pair]

$54 + \angle c = 180^\circ$

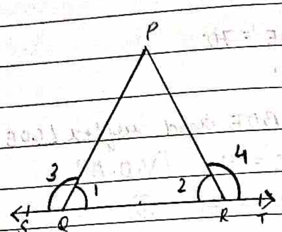
$\angle c = 126^\circ$

3. Given → $\angle PQR = \angle PRQ$

$\angle 1 = \angle 2$

To prove - $\angle PQS = \angle PRT$

Or $\angle 3 = \angle 4$



Proof → $\angle 1 + \angle 3 = 180^\circ$ - ① [Linear Pair]

$\angle 2 + \angle 4 = 180^\circ$ - ② [Linear Pair]

From ① and ②

$\cancel{\angle 1} + \angle 3 = \cancel{\angle 2} + \angle 4$

$\angle 3 = \angle 4$

hence $\angle PQS = \angle PRT$

4. Given - $x + y = w + z$ - ①

To prove AOB is a st line

Proof -

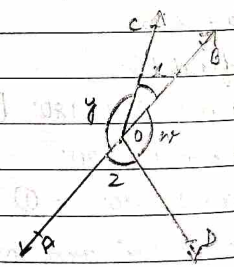
$x + y + w + z = 360^\circ$ [sum of $\angle$s around a point]

$x + y + x + y = 360^\circ$

$2x + 2y = 360^\circ$

$2(x + y) = 360$

$x + y = 180^\circ$

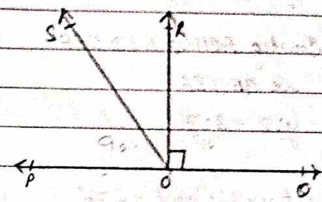


Page 69

5. Given :- POQ is a line

OR ⊥ PQ

To prove - $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$



Proof :- OR ⊥ PQ

$\angle ROP = \angle ROQ$ [90° each]

$\angle ROS + \angle POS = \angle QOS - \angle ROS$

$\angle ROS + \angle ROS = \angle QOS - \angle POS$

$2\angle ROS = \angle QOS - \angle POS$

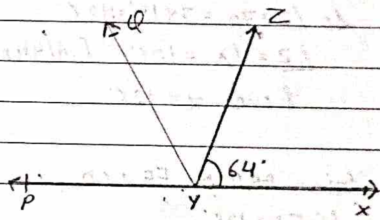
$\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$

hence proved.

6. Given :- $\angle XYZ = 64^\circ$

YQ Bisects $\angle ZYP$

To find :- $\angle XYQ$ & reflex $\angle PYQ$



$\angle PYZ + \angle XYZ = 180^\circ$ [Linear Pair]

$\angle PYZ + 64 = 180^\circ$

$\angle PYZ = 116^\circ$

Since YQ bisects $\angle PYZ$ so

$\angle PYQ = \frac{116}{2}$

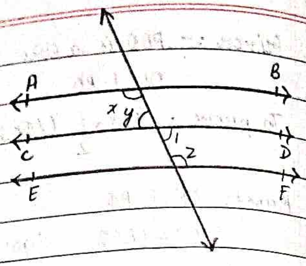
$\angle PYQ = 58^\circ$

reflex $\angle PYQ = 360^\circ - 58$

$= 302^\circ$

Date
Page 70

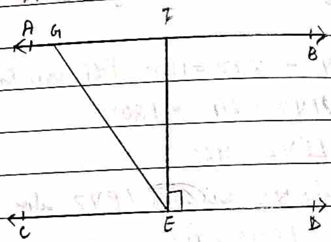
Exercise 6.2



1. In fig $AB \parallel CD$ & $CD \parallel EF$
So $AB \parallel EF$
$y:z = 3:7$

Let $y = 3p$ and $z = 7p$
$\angle 1 = \angle y = 3p$ [VOA]
$\angle 1 + z = 180°$ [Co-interior angles]
$3p + 7p = 180°$
$10p = 180°$
$p = 18°$

So $\angle z = 7p = 7 \times 18° = 126°$
$\angle z = \angle x = 126°$ [Alternate interior angles]
hence $x = 126°$



2. Given $AB \parallel CD$, $EF \perp CD$
$\angle GED = 126°$
To find :- $\angle AGE$, $\angle GEF$, $\angle FGE$

$AB \parallel CD$, GE is transversal
$\angle AGE = \angle GED = 126°$ [Alternate interior angles]
$\angle AGE = 126°$
$\angle GEF = \angle GED - \angle FED$
$= 126 - 90$
$= 36°$

$\angle FGE + \angle AGE = 180°$ [Linear Pair]
$\angle FGE + 126 = 180$
$\angle FGE + 126 = 180$
$\angle FGE = 54°$

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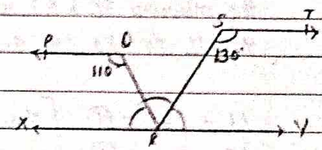
Date
Page 71

So angles are :- $\angle AGE = 126°$, $\angle GEF = 36°$, $\angle FGE = 54°$

3. $PQ \parallel ST$, $\angle PQR = 110°$
$\angle RST = 130°$
To find $\angle QRS$
Construction :- draw $XY \parallel PQ \parallel ST$ through R

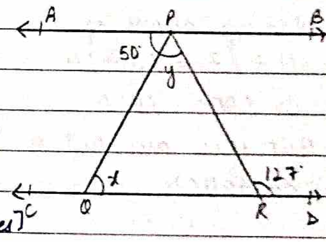


$\angle S + \angle SRY = 180°$ [Co-interior angles]
$130° + \angle SRY = 180°$
$\angle SRY = 50°$

$\angle Q + \angle QRX = 180°$
$110° + \angle QRX = 180°$
$\angle QRX = 70°$

$\angle QRX + \angle SRY + \angle SRQ = 180°$
$50 + 70 + \angle SRQ = 180°$
$\angle SRQ = 60°$

4. $AB \parallel CD$, $\angle APQ = 50°$ & $\angle PRD = 127°$



$\angle x = 50°$ [Alternate interior angle]
$AB \parallel CD$, PR is transversal
$\angle APR = 127°$ [Alternate interior Angles]
$50° + y = 127°$
$y = 127° - 50°$
$y = 77°$
So $x = 50°$
$y = 77°$

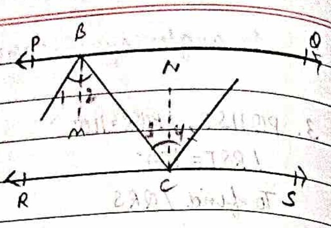
Page 72

5. Given:- PQ || RS

To prove:- AB || CD

Proof:- Draw BM and CN normal to the mirrors PQ & RS respectively

Let ∠1, ∠2, ∠3, ∠4 in fig.



∠1 = ∠2 - ① [Laws of reflection]

∠3 = ∠4 - ② [Laws of reflection]

BM || CN, BC is transversal

So ∠2 = ∠3 - ③ [Alternate interior angles]

From ①, ②, ③

∠1 = ∠2 = ∠3 = ∠4 - ④

From ④

∠1 = ∠3

∠2 = ∠4

Adding above eq.

∠1 + ∠2 = ∠3 + ∠4

So ∠ABC = ∠BCD

But these are A.I.A

So AB || CD

Date 11/7/23 Page 73

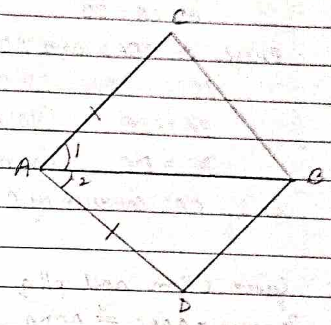
Chapter-7
Triangles

Exercise 7.1

Q1: Given- In quad ABCD

AC = AD, AB Bisect ∠A

To prove - ΔABC ≅ ΔABD



Proof :- In ΔABC and ΔABD

AC = AD [Given]

∠1 = ∠2 [AB bisects ∠A]

AB = AB [Common]

Hence ΔABC ≅ ΔABD by SAS

BC = BD [By CPCT]

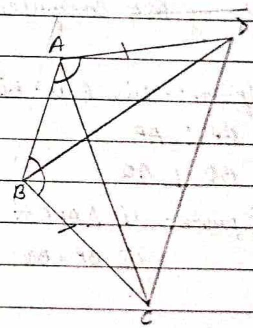
Q2: Given:- In quad ABCD

AD = BC, ∠DAB = ∠CBA

To prove:- (i) ΔABD ≅ ΔBAC

(ii) BD = AC

(iii) ∠ABD = ∠BAC



Proof:- (i) In ΔABD and ΔBAC

AD = BC [Given]

∠DAB = ∠CBA [Given]

AB = BA [Common]

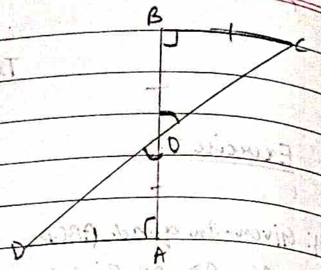
By SAS Congruency ΔABD ≅ ΔBAC

Q3: Given :- $\angle A = \angle B = 90°$

$AD = BC$

To prove :- CD bisects AB

or $OA = OB$



Proof - In $\triangle OAD$ and $\triangle OBC$

$\angle AOD = \angle BOC$ [V.O.A.]

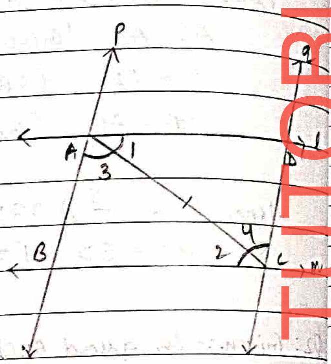
$\angle A = \angle B$ [90° each]

$DC = DC$ [Common]

So by AAS conruency $\triangle OAD \cong \triangle OBC$

Q4: Given $l \parallel m$ and $p \parallel q$

To prove - $\triangle ABC \cong \triangle CDA$



Proof In $\triangle ABC$ and $\triangle CDA$

$\angle 1 = \angle 2$ [Alternate interior $\angle$s]

$AC = AC$ [Common]

$\angle 4 = \angle 3$ [Alternate interior $\angle$s]

So by ASA congruency $\triangle ABC \cong \triangle CDA$

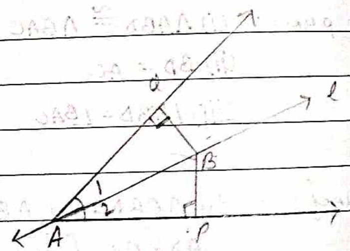
Q5: Given :- line l is bisector of $\angle A$

$BP \perp AP$

$BQ \perp AQ$

To prove :- (i) $\triangle APB \cong \triangle AQB$

(ii) $BP = BQ$



Proof - In $\triangle APB$ and $\triangle AQB$

$\angle P = \angle Q$ [90° each]

$\angle 2 = \angle 1$ [line bisector of $\angle A$]

$AB = AB$ [Common]

$\therefore$ By AAS congruency

$\triangle APB \cong \triangle AQB$

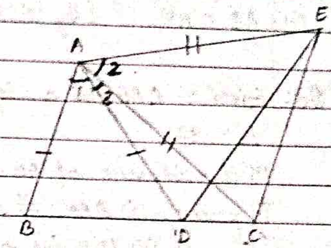
(ii) $BP = BQ$ [By CPCT]

or B is equidistant from arms of $\angle A$

Q6: Given :- $AC = AE$, $AB = AD$

$\angle BAD = \angle EAC$

To prove :- $BC = DE$



Proof :- $\angle 1 = \angle 2$ [Given]

Add $\angle 3$ on both sides

$\angle 1 + \angle 3 = \angle 2 + \angle 3$

$\angle BAC = \angle DAE$ — ①

In $\triangle ABC$ and $\triangle ADE$

$AB = AD$ [Given]

$\angle BAC = \angle DAE$ [From ①]

$AC = AE$ [Given]

$\therefore$ By SAS congruency $\triangle ABC \cong \triangle ADE$

Hence, $BC = DE$ [CPCT]

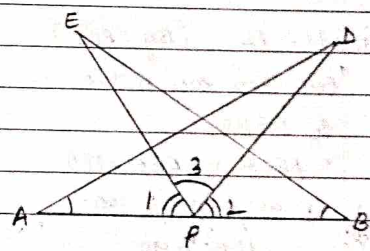
Q7: Given :- P is midpoint AB

$\angle BAD = \angle ABE$

$\angle EPA = \angle DPB$

To prove :- (i) $\triangle DAP \cong \triangle EBP$

(ii) $AD = BE$



Proof :- $\angle 1 = \angle 2$ [Given]

Add $\angle 3$ on both sides

$\angle 1 + \angle 3 = \angle 2 + \angle 3$

$\angle APE = \angle BPD$ — ①

In $\triangle DAP$ and $\triangle EBP$

$\angle A = \angle B$ [Given]

$AP = PB$ [P is midpoint of AB]

$\angle APD = \angle BPE$ [From ①]

$\therefore \triangle DAP \cong \triangle EBP$ by ASA congruency

Page 76

(ii) $AD = BE$ [By CPCT]

Q8: Given :- ΔABC is a right angled triangle

$\angle C = 90°$

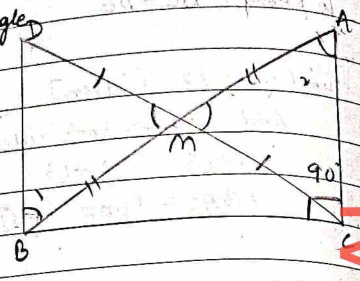
M is midpoint of AB. $DM = CM$

To prove :- (i) $\Delta AMC \cong \Delta BMD$

(ii) $\angle DBC$ is a right angle

(iii) $\Delta DBC \cong \Delta ACB$

(iv) $CM = \frac{1}{2} AB$



Proof :- (i) In ΔAMC and ΔBMD

$AM = MB$ [M is the midpoint of AB]

$\angle AMC = \angle BMD$ [V.O.A]

$MC = MD$ [Given]

$\therefore$ By SAS congruency $\Delta AMC \cong \Delta BMD$

$\angle 1 = \angle 2$ [CPCT]

$BD = AC$

(ii) $\angle 1 = \angle 2$ [By CPCT]

But these are A.I.A

So $DB \parallel AC$

$\therefore \angle BCA + \angle DBC = 180°$

$90° + \angle DBC = 180°$

$\angle DBC = 90°$

(iii) In ΔDBC and ΔACB

$DB = AC$ [By CPCT]

$\angle DBC = \angle ACB$ [90° each]

$BC = BC$ [Common]

By SAS congruency

(iv) $\Delta DBC \cong \Delta ACB$

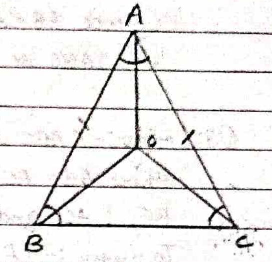
Page 77

(iv) $CM = \frac{1}{2} CD$

But $CD = AB$ [CPCT]

So $CM = \frac{1}{2} AB$

Exercise 7.2



Q1: Given :- Let in fig ΔABC in which $AB = AC$

BO and CO are bisectors of $\angle B$ and $\angle C$

To prove :- (i) $OB = OC$

(ii) AO bisects $\angle A$

Proof :- (i) In ΔABC

$AB = AC$ [Given]

So $\angle ABC = \angle ACB$ [Angle opposite to equal sides]

$\frac{1}{2} \angle ABC = \frac{1}{2} \angle ACB$

$\angle OBC = \angle OCB$ — (1)

Hence $OB = OC$ [sides opposite equal angles]

(ii) In ΔAOB & ΔAOC

$AO = AO$ [Given]

$AB = AC$ [Given]

$OB = OC$ [Proved in (i)]

$\therefore$ So by SSS $\Delta AOB \cong \Delta AOC$

Hence $\angle BAO = \angle CAO$

$\therefore$ AO bisects $\angle A$

Q2: Given - In ΔABC, $AD \perp BC$

and AD bisects BC

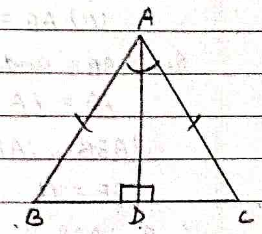
So $BD = CD$

To prove :- ΔABC is isosceles where $AB = AC$

Proof :- In ΔADB and ΔADC

$AD = AD$ [Common]

$\angle ADB = \angle ADC$ [90° each]



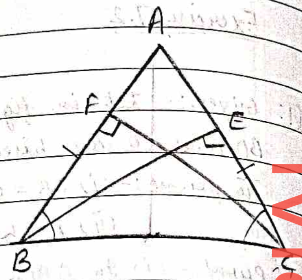
$BD = CD$ [AD Bisects BC]

$\therefore$ By SAS congruency

$\Delta ADB \cong \Delta ADC$

hence $AB = AC$ [CPCT]

So ΔABC is isosceles triangle



Q3: Given:- ΔABC is isosceles triangle

where $AB = AC$

$BE \perp AC$ and $CF \perp AB$

To prove :- $CF = BE$

Proof:- Since $AB = AC$

So $\angle ABC = \angle ACB$ -① [Angles opp. to equal sides]

In ΔBFC and ΔCEB

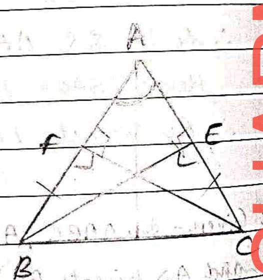
$\angle BFC = \angle CEB$ [90° each]

$\angle ABC = \angle ACB$ [From ①]

$BC = CB$ [Common]

So by AAS congruency $\Delta BFC \cong \Delta CEB$

So $CF = BE$ by CPCT



Q4: Given :- $BE \perp AC$

$CF \perp AB$

To prove :- (i) $\Delta ABE \cong \Delta ACF$

(ii) $AB = AC$

i.e ABC is an isosceles triangle

Proof:- (i) In $\Delta ABE \cong \Delta ACF$

(ii) $AB = AC$

In ΔABE and ΔACF

$\angle A = \angle A$ [Common]

$\angle AEB = \angle AFC$ [90° each]

$BE = CF$ [Given]

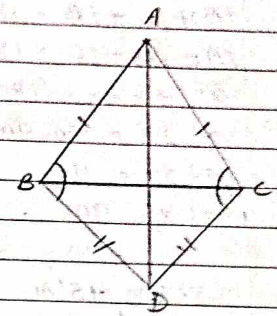
$\therefore$ By AAS congruency

$\Delta ABE \cong \Delta ACF$

(ii) $AB = AC$ [CPCT]

hence ΔABC is isosceles Triangle

Q5:



2 isosceles triangls on same base BC

Construction - Join AD

To prove:- $\angle ABD = \angle ACD$

Proof:- In ΔABD and ΔACD

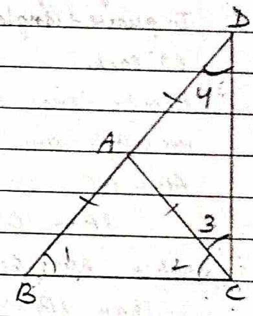
$AB = AC$ [Given]

$BD = CD$

$AD = AD$ [common]

$\therefore$ By SSS $\Delta ABD \cong \Delta ACD$

So $\angle ABD = \angle ACD$ By CPCT



Q6: ΔABC is an isosceles triangle

$AB = AC$, $AD = AB$

To prove :- $\angle C = 90°$

Proof:- we know angeles opposite to equal sides are equal

So $AB = AC$

hence $\angle 1 = \angle 2$ -①

$AD = AC$ So $\angle 3 = \angle 4$ - ②

Using A.S.P in ΔBCD

$\angle B + \angle BCD + \angle D = 180°$

$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180°$

$\angle 2 + \angle 2 + \angle 3 + \angle 3 = 180°$

$2\angle 2 + 2\angle 3 = 180°$

$2(\angle 2 + \angle 3) = 180°$

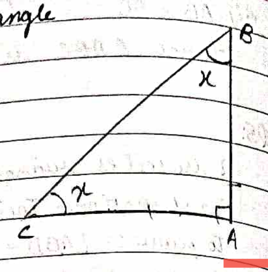
$\angle 2 + \angle 3 = 90°$

hence

$\angle BCD = 90°$

classmate Date Page 80

Q7. Given:- ABC is a right angled triangle
$\angle A = 90^\circ$, $AB = AC$



$\angle A + \angle C + \angle B = 180^\circ$
$\angle A + \angle B + \angle C = 180^\circ$
$90^\circ + x + x = 180^\circ$
$\Rightarrow 2x = 180 - 90$
$\Rightarrow 2x = 90^\circ$
$\Rightarrow x = \frac{90^\circ}{2}$
$\Rightarrow x = 45^\circ$
So $\angle B \pm \angle C = 45^\circ$

Q8: Let ing fig ABC be an equilateral triangle
To prove - Angles of equilateral triangle are 60° each
Proof- Since all sides of equilateral triangle are equal
$AB = BC$
$\therefore \angle A = \angle C$ — (1) [Angles opposite to equal sides]
when $AB = AC$
than $\angle B = \angle C$ - (2)
From (1) and (2)

$\angle A = \angle B = \angle C = x^\circ$ (say)
Using ASP in $\triangle ABC$
$\angle A + \angle B + \angle C = 180^\circ$
$x^\circ + x^\circ + x^\circ = 180^\circ$
$3x = 180^\circ$
$x = 60^\circ$
So $\angle A = \angle B = \angle C = 60^\circ$
hence each any angle of an equilateral triangle is 60°.

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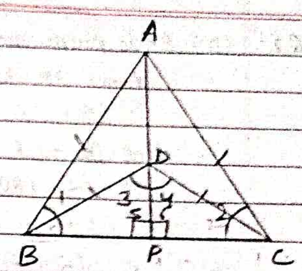
classmate Date Page 81

Exercise 7.3



Q1: Given:- $\triangle ABC$ and $\triangle DBC$ are 2 isosceles triangles on same base BC
$\therefore AB = AC$
and $BD = CD$
To prove:- (i) $\triangle ABD \cong \triangle ACD$
(ii) $\triangle ABP \cong \triangle ACP$
(iii) AP bisects $\angle A$ and $\angle D$
(IV) AP is perpendicular bisector of BC
Proof:- (i) In $\triangle ABD$ and $\triangle ACD$
$AB = AC$ [Given]
$BD = DC$ [Given]
$AD = AD$ [Common]
So by SSS congruency $\triangle ABD \cong \triangle ACD$

(ii) In $\triangle ABP$ and $\triangle ACP$
$AB = AC$ [Given]

$\angle 1 = \angle 2$ [CPCT]
$AP = AP$ [Common]
So by SAS congruency $\triangle ABP \cong \triangle ACP$

(iii) Since $\angle 1 = \angle 2$ by CPCT
So AP bisects $\angle A$
In $\triangle BDP$ and $\triangle CDP$
$BD = CD$ [Given]
$DP = DP$ [Common]
$BP = PC$ [CPCT]
$\therefore$ By SSS $\triangle BDP \cong \triangle CDP$
hence $\angle 3 = \angle 4$ by CPCT
AP bisects $\angle D$

Date
Page 82

Q1. (iv) AP is perpendicular bisector of BC

hence AP bisect BC

$\angle 5 = \angle 6$ [CPCT]

But $\angle 5 + \angle 6 = 180°$

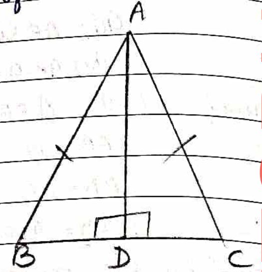
$\angle 5 + \angle 5 = 180°$ [$\angle 5 = \angle 6$]

$2\angle 5 = 180$

$\angle 5 = 90°$

hence AP is perpendicular bisector of BC

Q2: Given:- Let in fig $\triangle ABC$ is isosceles triangle $AB = AC$, $AD \perp BC$



To prove:- (i) AD bisects BC

(ii) AD bisects $\angle A$

Proof:- In $\triangle ADB$ and $\triangle ADC$

$\angle ADB = \angle ADC$ [90° each]

$AB = AC$ [Given]

$AD = AD$ [Common]

So by RHS $\triangle ADB \cong \triangle ADC$

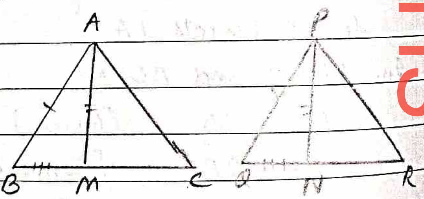
hence $BD = CD$

So AD bisects BC [CPCT]

(ii) $\angle BAD = \angle CAD$ [CPCT]

so AD bisects $\angle A$

Q3: Given:- In $\triangle ABC$ and $\triangle PQR$, AM and PN are medians $AB = PQ$, $BC = QR$, $AM = PN$



To prove:- (i) $\triangle ABM \cong \triangle PQN$

(ii) $\triangle ABC \cong \triangle PQR$

Proof:- $BC = QR$ [Given]

$\frac{1}{2} BC = \frac{1}{2} QR$ [AM and PN are median]

Date
Page 83

$BM = QN$

(i) In $\triangle ABM$ and $\triangle PQN$

$AB = PQ$ [Given]

$AM = PN$ [Proved above]

So by SSS $\triangle ABM \cong \triangle PQN$

(ii) In $\triangle ABC$ and $\triangle PQR$

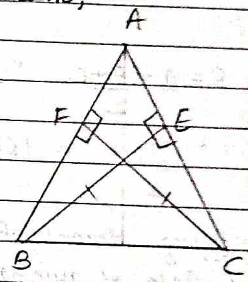
$AB = PQ$ [Given]

$\angle B = \angle Q$ [CPCT]

$BC = QR$ [Given]

So by SAS $\triangle ABC \cong \triangle PQR$

Q4: Given:- Let in fig a $\triangle ABC$ in which $BE \perp AC$, $CF \perp AB$, $BE = CF$



To prove:- $\triangle ABC$ is isosceles triangle

Proof:- In $\triangle BFC$ and $\triangle CEB$

$\angle CFB = \angle BEB$ [90° each]

$BC = BC$ [Common]

$BE = CF$ [Given]

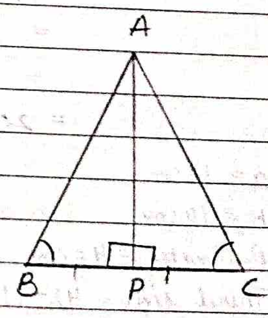
$\therefore$ By RHS congruency $\triangle BFC \cong \triangle CEB$

$\therefore$ Hence $\angle ABC = \angle ACB$ by CPCT

So $AB = AC$ [Sides opp. to equal angles]

hence $\triangle ABC$ is isoceles triangle.

Q5: Given:- Let in fig $\triangle ABC$ in which $AB = AC$ and $AP \perp BC$



To prove:- $\angle B = \angle C$

Proof:- In $\triangle APB$ and $\triangle APC$

$\angle APB = \angle APC$ [90° each]

$AB = AC$ [Given]

$AP = AP$ [Common]

~~$\angle APB = \angle APC$ [Given]~~

$\therefore$ By RHS

Chapter-10
Heron's Formula

$S = \frac{a+b+c}{2}$

Heron's Formula

Area of triangle $= \sqrt{S(S-a)(S-b)(S-C)}$

Exercise-10.1

3. Here, $a = 15m$

$b = 11m$

$c = 6m$

$S = \frac{a+b+c}{2} = \frac{15+11+6}{2}$

$\frac{32}{2} = 16m$

Using Heron's Formula

Area of triangle $= \sqrt{S(S-a)(S-b)(S-C)}$

$= \sqrt{16(16-15)(16-11)(16-6)}$

$= \sqrt{16 \times 1 \times 5 \times 10}$

$= \sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5}$

$= 2 \times 2 \times 5\sqrt{2}$

$= 20\sqrt{2}$

$= 20\sqrt{2}\ cm^2$

4. $a = 18\ cm$

$b = 10\ cm$

Perimeter $= 42\ cm$

Third side $= 42 - (18+10)$

$c = 14\ cm$

Using heron's formula :-

Area of triangle $= \sqrt{S(S-a)(S-b)(S-C)}$

where $S = \frac{42}{2} = 21\ cm$

Area of triangle $= \sqrt{21(21-18)(21-10)(21-14)}$

$= \sqrt{21 \times 3 \times 11 \times 7}$

$= \sqrt{7 \times 3 \times 3 \times 11 \times 7}$

$= 7 \times 3\sqrt{11}$

$= 21\sqrt{11}\ cm^2$

5. Let the sides of given triangle be $12x$, $17x$ & $25x$

Perimeter of triangle $= 540\ cm$

$12x + 17x + 25x = 540$

$54x = 540$

$x = 10$

So sides are :-

$12 \times 10 = 120\ cm$

$17 \times 10 = 170\ cm$

$25 \times 10 = 250\ cm$

$S = \frac{a+b+c}{2}$

$= \frac{540}{2} = 270\ cm$

Using Heron's Formula

Area of $\Delta = \sqrt{S(S-a)(S-b)(S-C)}$

$= \sqrt{270(270-120)(270-170)(270-150)}$

$= \sqrt{270 \times 150 \times 100 \times 20}$

$= \sqrt{3 \times 3 \times 3 \times 3 \times 5 \times 100 \times 100 \times 2 \times 5 \times 5}$

$= 3 \times 3 \times 5 \times 2 \times 100$

$= 9000\ cm^2$

6. 2 equal sides of isosceles triangle are 12 cm each.

$so, a = 12 cm$

$b = 12 cm$

$Perimeter = 30 cm$

$= 30 - 24$

$c = 6 cm$

$s = \frac{a+b+c}{2}$

$s = \frac{12+12+6}{2}$

$s = 15 cm$

Using Heron's Formula:-

where $s = 15 cm$

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{15(15-12)(15-12)(15-6)}$

$= \sqrt{15 \times 3 \times 3 \times 9}$

$= \sqrt{3 \times 3 \times 3 \times 3 \times 3 \times 5}$

$= 9\sqrt{15}\ cm^2$

1. Let in fig the equilateral triangle with side a

$s = \frac{a+a+a}{2} = \frac{3a}{2}$

Using Heron's Formula

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{\frac{3a}{2}\left(\frac{3a-2a}{2}\right)\left(\frac{3a-2a}{2}\right)\left(\frac{3a-2a}{2}\right)}$

$= \sqrt{\frac{3a}{2}\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)}$

$= \frac{a}{2} \times \frac{a}{2} \sqrt{2} = \frac{a^2}{4} = \frac{\sqrt{3}a^2}{4}$

$Perimeter = 180 cm$

$Side = \frac{180}{3} = 60 cm$

$= \frac{\sqrt{3}}{4} \times 60 \times 60$

$= 900\sqrt{3}\ cm^2$

2. Here, $a = 122 m$, $b = 22 m$, $c = 120 m$

$s = \frac{a+b+c}{2}$

$s = \frac{122+22+120}{2}$

$s = \frac{264}{2} = 132 m$

Using Heron's Formula:-

Area of wall $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{132(132-122)(132-22)(132-120)}$

$= \sqrt{132 \times 10 \times 12 \times 110}$

$= \sqrt{11 \times 12 \times 10 \times 12 \times 11 \times 10}$

$= \sqrt{11 \times 11 \times 12 \times 12 \times 10 \times 10}$

$= 11 \times 12 \times 10$

$= 1320\ m^2$

Rent for $1 m^2$ for 12 months $= ₹5000$

Rent for $1320 m^2$ for 12 months $= 5000 \times 1320$

" " $1320 m^2$ for 12 months $= \frac{5000 \times 1320}{12}$

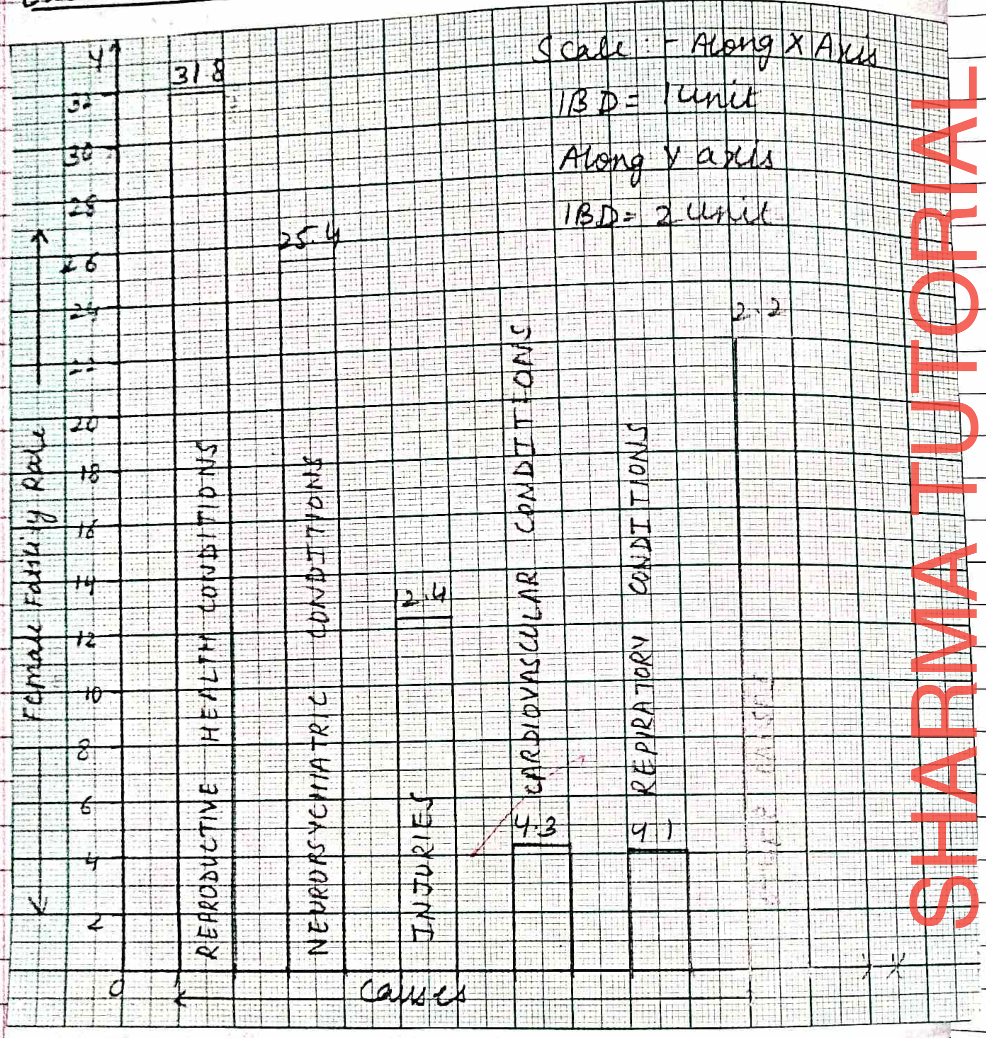
3 months $= \frac{5000 \times 1320 \times 3}{12}$

$= ₹1650000$

Chapter -12
Statistics

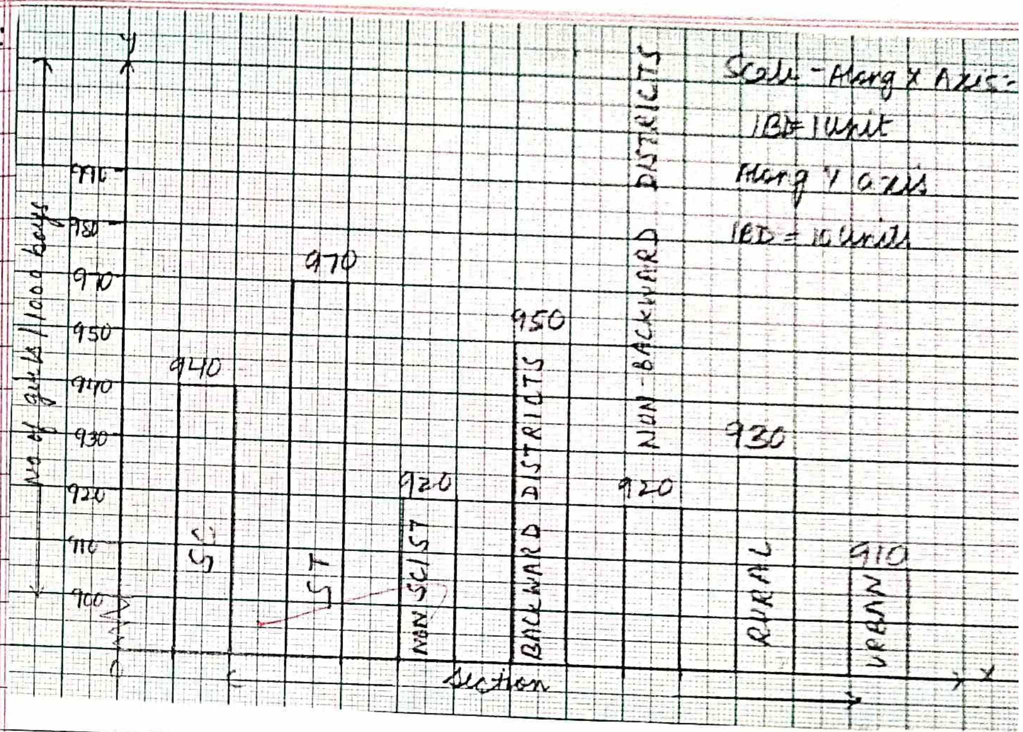
Exercise 12.1

Q1:



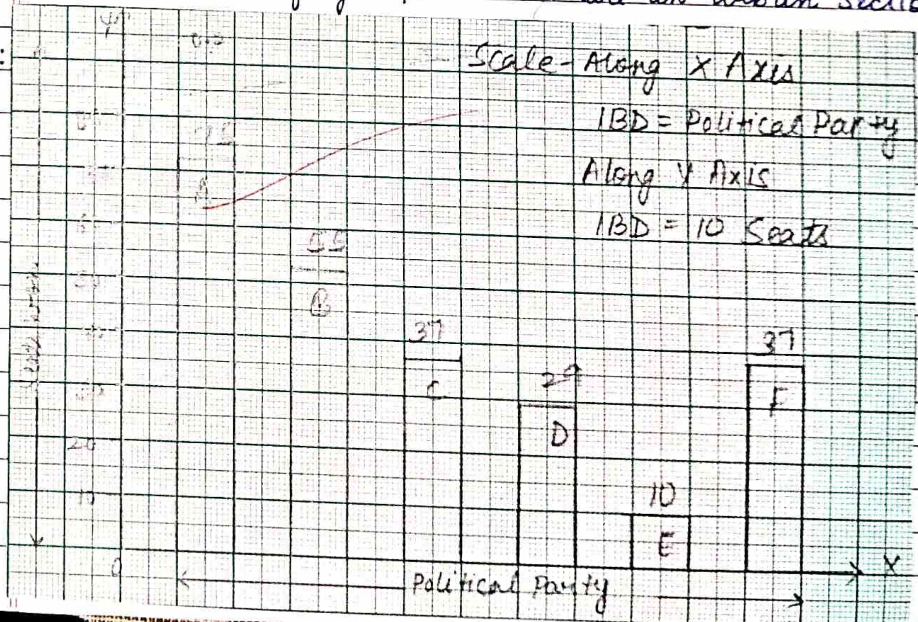
a) Reproductive health conditions

Q2:



maximum no. of girls/1000 boys are in ST Section and Minimum no. of girls/1000 boys are in Urban Section.

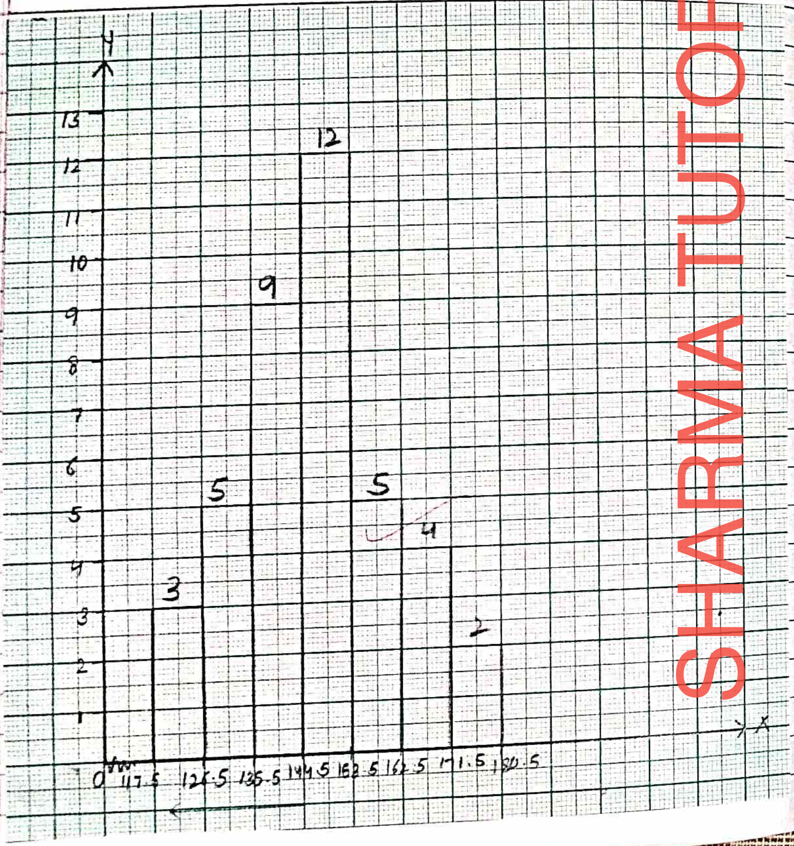
Q3:



(ii) Seats won by A = 75

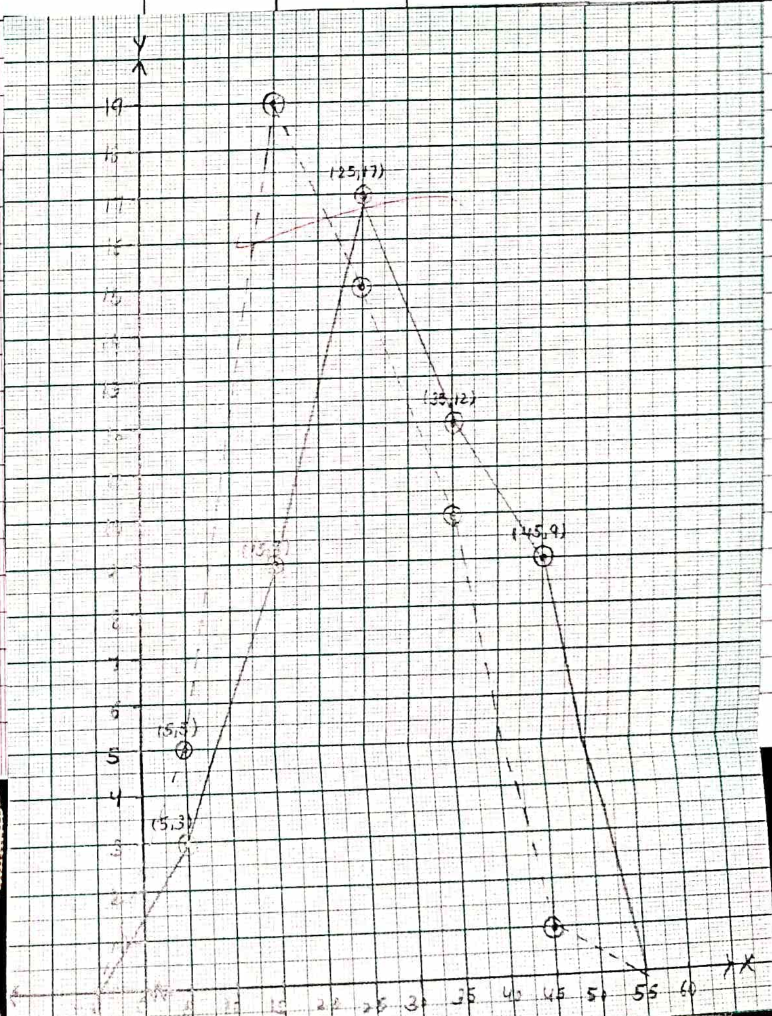
Q4

LENGTH	NO. OF LEAVES
117.5-126.5	3
126.5-135.5	5
135.5-144.5	9
144.5-153.5	12
153.5-162.5	5
162.5-171.5	4
171.5-180.5	2

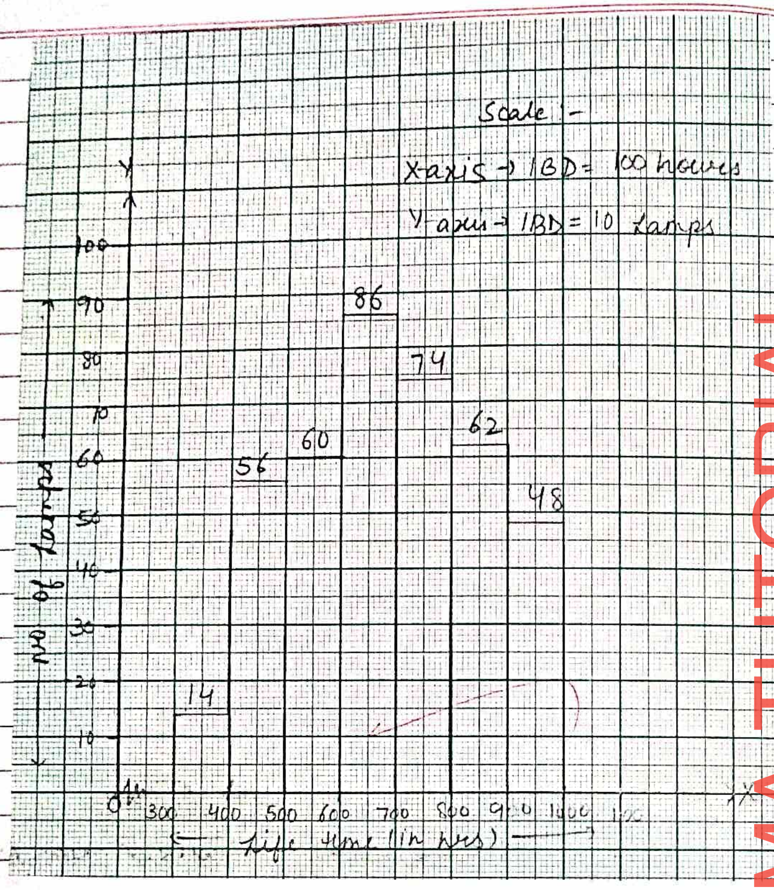


Q6:

Marks	Mid Value	sec-A	Points	sec-B	Points
0-10	5	3	(5,3)	5	(5,5)
10-20	15	9	(15,9)	19	(15,19)
20-30	25	17	(25,17)	15	(25,15)
30-40	35	12	(35,12)	10	(35,10)
40-50	45	9	(45,9)	1	(45,1)

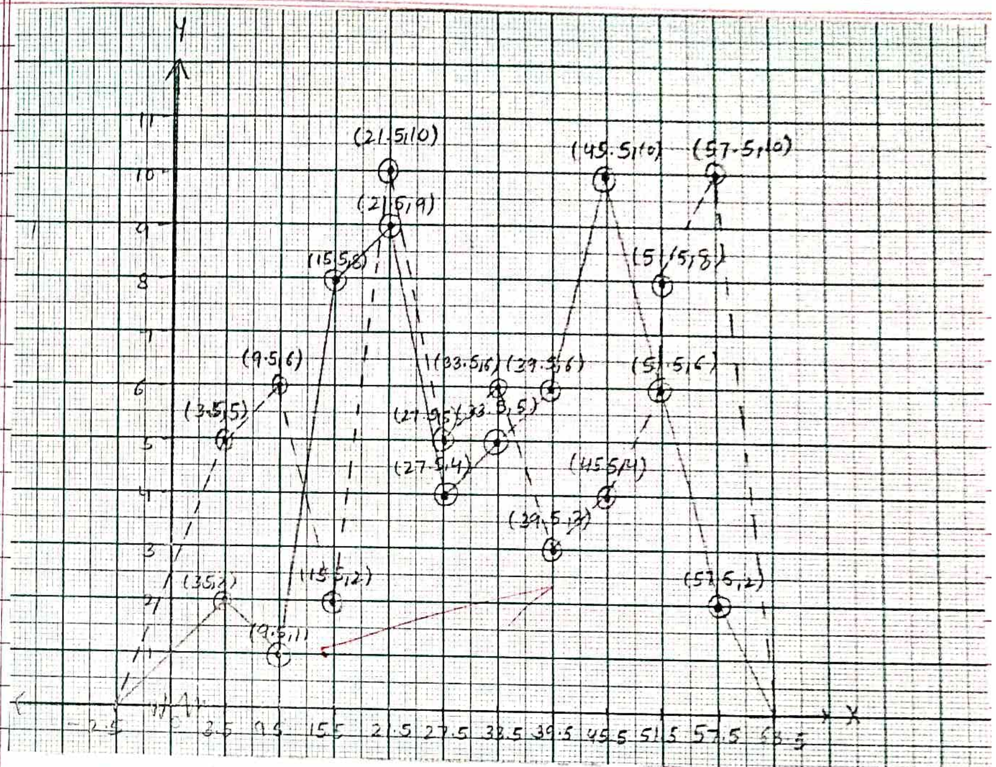


Q5:



Q7:

No. of Balls	Mid Value	Team A	Points	Team B	Points
0.5 - 6.5	3.5	2	(3.5,2)	5	(3.5,5)
6.5 - 12.5	9.5	1	(9.5,1)	6	(9.5,6)
12.5 - 18.5	15.5	8	(15.5,8)	2	(15.5,2)
18.5 - 24.5	21.5	9	(21.5,9)	10	(21.5,10)
24.5 - 30.5	27.5	4	(27.5,4)	5	(27.5,5)
30.5 - 36.5	33.5	5	(33.5,5)	6	(33.5,6)
36.5 - 42.5	39.5	6	(39.5,6)	3	(39.5,3)
42.5 - 48.5	45.5	10	(45.5,10)	4	(45.5,4)
48.5 - 54.5	51.5	6	(51.5,6)	8	(51.5,8)

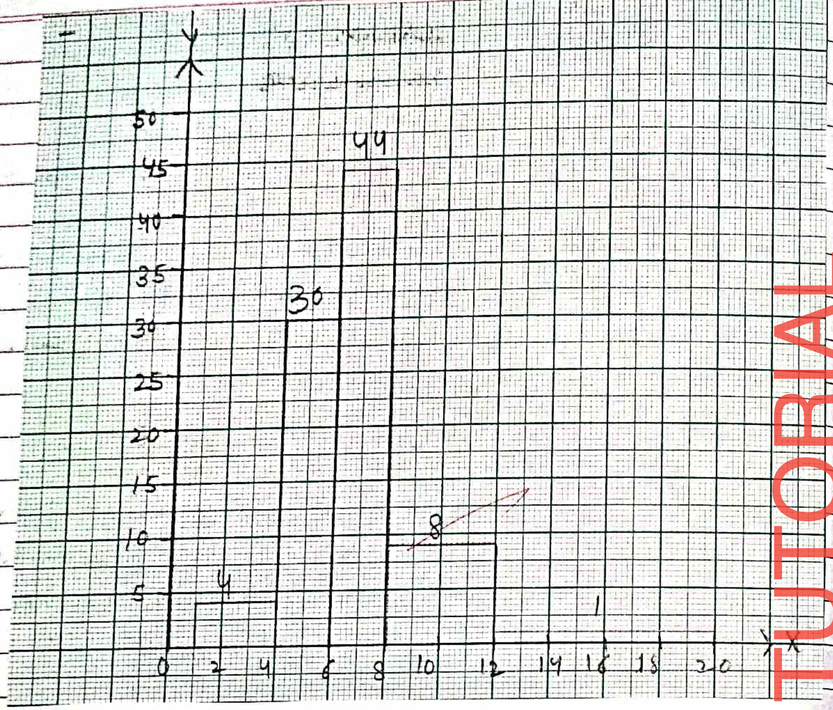


Q9:

No. of Letters	No. of Surnames	Adjusted Frequency $\left[\frac{\text{Min. C.S} \times F}{C.S}\right]$
1 - 4	6	$\frac{2}{3} \times 6 = 4$
4 - 6	30	$\frac{2}{2} \times 30 = 30$
6 - 8	44	$\frac{2}{2} \times 44 = 44$
8 - 12	16	$\frac{2}{4} \times 16 = 8$
12 - 20	4	$\frac{2}{8} \times 4 = 1$

Excellent!

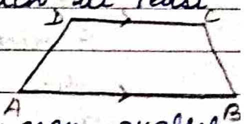
23/8/23



Chapter - 8
Quadrilaterals

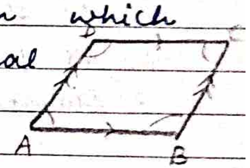
★ Quadrilateral - It is a closed figure having 4 sides

★ Trapezium - It is a quadrilateral in which at least one pair of opposite sides are parallel



★ Isoceles Trapezium - A trapezium in which non parallel sides are equal.

★ Parallelogram - It is a quadrilateral in which both pairs of opposite sides are equal



Properties of ||gm :-

1. Opposite sides are equal
2. Opposite angles are equal
3. Diagonals bisect each other.
4. Adjecent ∠s are supplementry.

Paralellogram

Rectangle | Rhombus | Square

1. Rectangle
A rectangle is a 11gm in which ~~all~~ one of the angles are 90°.

Properties of Rectangle -
1. All the properties of 11gm.
2. All the angles are 90° each.
3. Diagonals are equal.

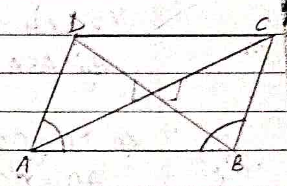
2. Square
Square is a 11gm in which one angle is 90° and one pair of adjectnt sides are equal.

Properties of Square -
1. All properties of 11gm
2. angle bisect at 90°
3. Diagonals are equal.

3. Rhombus
Rhombus is a 11gm in which one pair of adject sides is equal.

Properties of Rhombus -
1. All the properties of 11gm
2. Diagonals bisect at 90°
3. One pair of adject sides is equal.

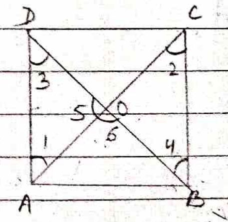
Exercise 8.1



Q1: Given :- Let in fig ABCD is given 11gm in which AC = BD
To prove :- ABCD is a rectangle.

Proof :- In ΔDAB and ΔCBA
$AD = BC$ | opp. sides of a 11gm
$AB = AB$ | Common
$AC = BD$ | Given
So by SSS $\Delta DAB \cong \Delta CBA$
hence $\angle DAB = \angle CBA$ | By CPCT — (1)
But $\angle DAB + \angle CBA = 180°$ | Co-interior angles
$\angle DAB + \angle DAB = 180°$ | Using (1)
$2\angle DAB = 180°$
$\angle DAB = 90°$
Since in this 11gm $\angle DAB = 90°$ so this is a rectangle.

Q2: Given :- Let ABCD is a given square
To prove :- Diagonals are equal and bisect at 90°.



Proof :- In ΔDAB and ΔCBA
$DA = CB$ | Sides of square
$\angle DAB = \angle CBA$ | 90° each
$AB = AB$ | Common
So by SAS $\Delta DAB \cong \Delta CBA$

BD = AC | CPCT

Hence diagonals are equal.

In ΔDOA and ΔBOC

$\angle 1 = \angle 2$ | AIA

$AD = BC$ | Sides of Square

$\angle 3 = \angle 4$ | AIA

$\therefore$ By ASA $\Delta DOA \cong \Delta BOC$

So $DO = OB$ [CPCT]

$AO = OC$

Hence diagonals bisect each other

In ΔAOD & ΔAOB

$AO = AO$ | Common

$AD = AB$ | Sides of square

$OD = OB$ | Proved above

So by SSS $\Delta AOD \cong \Delta AOB$

$\angle 5 = \angle 6$ | CPCT — (1)

But $\angle 5 + \angle 6 = 180°$

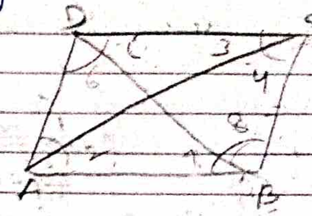
$\angle 5 + \angle 5 = 180°$ | using (1)

$2\angle 5 = 180°$

$\angle 5 = 90°$

Hence diagonals of square are equal and bisect at 90°.

5. Given - Let in fig ABCD is a given rhombus



To Prove :- AC bisects $\angle A$ & $\angle C$

BD bisects $\angle B$ & $\angle D$

Proof :- In ΔABC & ΔADC

$AB = AD$
$BC = CD$ [Adj. sides of rhombus]

$AC = AC$ | Common

By SSS $\Delta ABC \cong \Delta ADC$

$\angle 1 = \angle 2$
$\angle 3 = \angle 4$ [CPCT]

So AC bisects $\angle A$ & $\angle C$

In ΔDAB & ΔDCB

$DA = DC$
$BA = BC$ [Adj. Sides of rhombus]

$DB = DB$ | Common

By SSS $\Delta DAB \cong \Delta DCB$

$\angle 5 = \angle 6$
$\angle 7 = \angle 8$ [CPCT]

So BD bisects $\angle B$ & $\angle D$

5.5. Given :-

ABCD is a ||gm DP = BQ

To prove :- (i) $\Delta APD \cong \Delta CQB$

(ii) AP = CQ

(iii) $\Delta AQB \cong \Delta CPD$

(IV) AQ = CP

(V) AQCP is a ||gm

Proof :- (i) In ΔAPD and ΔCQB

$PD = BQ$ | Given

$\angle 1 = \angle 2$ | Alt. interior $\angle s$)

$BC = AD$ [opp. sides are equal]

$\Delta APD \cong \Delta CQB$ [SAS Congruency]

$AP = CQ$ [CPCT]

(ii) Proved Above

$AP = CQ$

(iii) In ΔABQ & ΔCPD

$AB = CD$ — $AQ = CP$ | Opp. sides of ||gm

$\angle 4 = \angle 3$ — $BQ = CD$ | Given

$BQ = DP$ — $\angle 3 = \angle 4$ | Alt. int. $\angle s$

$\Delta ABQ \cong \Delta CPD$ by SAS

$AQ = CP$ | CPCT

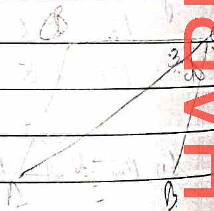
(iv) Proved Above $AQ = CD$

(v) In Quad. AQCP

$AQ = CP$

$AP = QC$

Since if both sides of opp. sides are equal then it is a ||gm.



3. Given - ABCD is a ||gm

AC bisects $\angle A$

So $\angle 1 = \angle 2$

To prove - (i) AC bisects $\angle C$

(ii) ABCD is a rhombus

Proof - $\angle 1 = \angle 2$ | Given

$\angle 2 = \angle 3$ | Alt. int. $\angle s$

So $\angle 1 = \angle 4$ | Alt. int. $\angle s$

So $\angle 1 = \angle 2 = \angle 3 = \angle 4$

$\angle 3 = \angle 4$

So AC bisects $\angle C$ also.

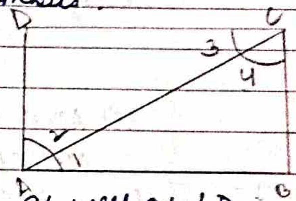
4. (ii) $\angle 2 = \angle 4$

So $AB = BC$ | Sides opp. to equal $\angle s$ are equal.

$\angle 1 = \angle 2$ & $\angle 2 = \angle 3$

So $\angle 1 = \angle 3$ so $DA = DC$ | sides opp. to equal $\angle s$ are equal

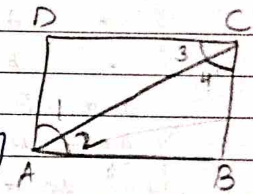
Since in ||gm ABCD one pair of adjacent sides is equal to its a rhombus.



4. In fig ABCD is a rectangle in which $\angle A$ & $\angle C$

To prove - (i) ABCD is a square

(ii) diagonals BD bisects $\angle B$ as well as $\angle D$



Proof - (i) $\angle 1 = \angle 2$ [Given]

$\angle 3 = \angle 4$

But $\angle 2 = \angle 3$ [Alt. interior $\angle s$]

$\angle 4 = \angle 1$

So $\angle 1 = \angle 3$

$AD = CD$ [sides opp. to equal sides are equal]

So $\angle 2 = \angle 4$ [sides opp. to equal sides are equal.]

Page 102

7. In fig ABCD is a Trapezium

Given - $AD = BC$, $AB \parallel CD$

To prove :- (i) $\angle A = \angle B$

(ii) $\angle C = \angle D$

(iii) $\triangle ABC \cong \triangle BAD$

(iv) diagonal AC = diagonal BC

Proof - (i) Construct → Extent AB and draw a line through C $\parallel$ to DA intersecting AB produced at E

(i) $AD = BC$ - (1)

$AB \parallel CD$

Now ABCD is a trapezium

$AB \parallel DC$

$AE \parallel DC$ | Parts of $\parallel$ lines are $\parallel$)

and $CE \parallel AD$ by construction

If both pairs of opp. sides are $\parallel$ its a $\parallel$gm.

$AD = CE$ - (2)

| opp. sides of a $\parallel$gm

From (1) & (2)

$BC = CE$

In $\triangle BCE$

$BC = CE$

$\angle 2 = \angle 1$ - (3)

| $\angle$s opp. to equal sides are equal.

Now $\angle 4 + \angle 1 = 180^\circ$ | Linear Pair - (4)

and $\angle 3 + \angle 2 = 180^\circ$ | Cointerior $\angle$s - (5)

From (1) & (5)

$\angle 4 + \cancel{\angle 1} = \angle 3 + \cancel{\angle 2}$

$\angle 4 = \angle 3$ $(\because \angle 1 = \angle 2)$

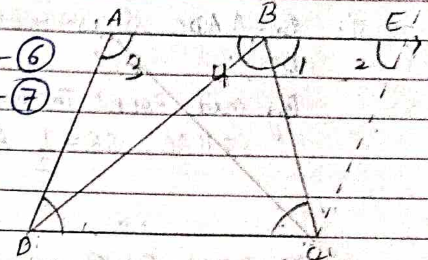
$\angle B = \angle A$

Page 103

(ii) Now

$\angle A + \angle D = 180^\circ$ | Co-int. $\angle$s - (6)

$\angle B + \angle C = 180^\circ$ - (7)



From (6) & (7)

$\cancel{\angle A} + \angle D = \cancel{\angle B} + \angle C$

$\angle D = \angle C$

(iii) In $\triangle ABC$ & $\triangle BAD$

$AB = AB$ | Common

$\angle B = \angle A$

$BC = AD$ | Given

$\triangle ABC \cong \triangle BAD$ [SAS]

(iv) By CPCT

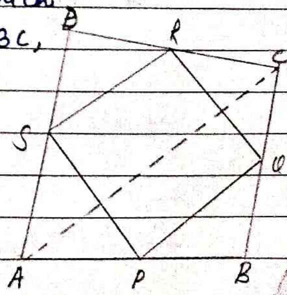
Mid Point Theorem

* The line segment Joining the mid-points of two sides of triangle is parallel to the third side.

* Converse Mid Point - The line drawn through the mid point of one side of a triangle is parallel to third side bisects the third side.

Exercise 8.2

Q1: Given ABCD is a quadrilateral in which P, Q, R, S are midpoints of sides AB, BC, CD, DA respectively



To Prove - (i) $SR \parallel AC$, ~~AC =~~ $SR = \frac{1}{2} AC$

(ii) $PQ = SR$

(iii) PQRS is a $\parallel$gm.

Page 104

(i) In ΔADC, S is midpoint of AD, R is midpoint of DC.
By mid Point Theorem.
$SR \parallel AC$, $SR = \frac{1}{2} AC$ — (1)

(ii) In ΔABC, P & Q are midpoints of sides AB and BC respectively.
∴ By mid Point Theorem
Since $SR = \frac{1}{2} AC$ [Proved Above]

$PQ \parallel SR$, $PQ = \frac{1}{2} AC$ — (2)

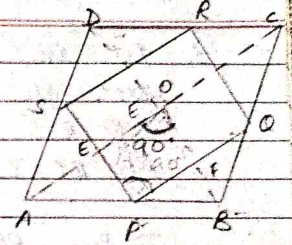
Since $SR = \frac{1}{2} AC$
$PQ = \frac{1}{2} AC$
[Proved Above]

So $SR = PQ$

(iii) Since
$SR \parallel AC$
$PQ \parallel AC$
[Proved Above]
∴ $SR \parallel PQ$
But $SR = PQ$
Since is quad. PQRS one pair of opposite sides is equal and parallel, so it is a $\parallel gm$.

Page 105

Q2 Given- Let in fig ABCD is a given rhombus.
P,Q,R,S are mid points of sides AB, BC, CD, DA respectively
To prove- PQRS is a rectangle



Proof:- In ΔADC, S and R are midpoints of AC & DC respectively
So by mid Point Theorem
$SR \parallel AC$, $SR = \frac{1}{2} AC$ — (1)

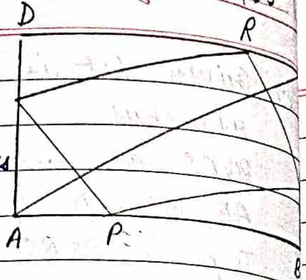
In ΔABC, P & Q are midpoints of sids AB & BC
So by mid Point Theorem
$PQ \parallel AC$, $PQ = \frac{1}{2} AC$ — (2)

From (1) & (2)
$PQ \parallel SR$, $PQ = SR$
Since is quad. PQRS one pair of opposite sides is equal and parallel so it's a $\parallel gm$.

$\angle AOB = 90°$ | Diagonals of rhombus bisect at 90°
In ΔABC, $OE \parallel BP$
~~In ΔSDC, $SR \parallel AC$~~,
In ΔABD, ~~$SP \parallel DB$~~
$PE \parallel OF$
hence of OFPE is a $\parallel gm$
$\angle EPB = 90°$ | Opp. angles of a $\parallel gm$

In $\parallel gm$, PQRS one angle is 90° so it is a rectangle.

3. Let in fig ABCD is a rectangle.
P, Q, R, S are mid points of sides AB, BC, CD, DA



To Prove - PQRS is a Rhombus

Proof - In ΔADC, S & R are mid points of sides DC & DA respectively
So by mid Point Theorem
$SR \parallel AC, \frac{1}{2} AC$ - (1)

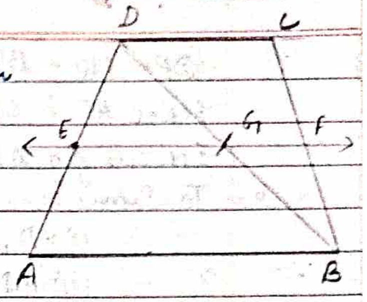
In ΔABC, P & Q are mid Points of sides AB & CB respectively
So by mid Point Theorem
$PQ \parallel AC$, $PQ = \frac{1}{2} AC$ - (2)

From (1) & (2)
$PQ \parallel SR$, $PQ = SR$
since in quad. PQRS, one pair of opposite sides is equal & parallel so it is a ||gm.

In ΔASP & ΔBQP
AS = BQ | Half of opp. sides of a rect.
AP = PB | P is mid Point
$\angle SAB = \angle QBP$ | 90° each

So by SAS
$\Delta ASP \cong \Delta BQP$
PS = PQ | CPCT
Since in ||gm PQRS one pair of adjecent side is equal so it is a rhombus.

4. Given - ABCD is a trapezium in which AB || CD
E is mid point of BD

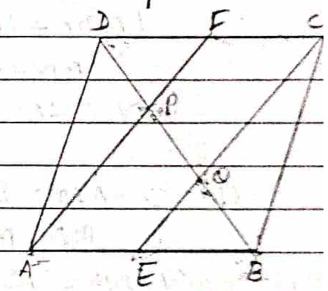


Proof :- AB || CD | Given
But EF || AB
so AB || EF || CD

In ΔABD, E is mid point of AD & EG || AB therefore by converse of mid point theorem G is mid point of DB

In ΔBGE, G is midpoint of DB, therefore by converse of mid point theorem F is mid point of CB.

5. Given - ABCD is a ||gm, E & F are the mid points of sides AB & CD respectively
To prove :- AF & EC trisects BD



Proof :-

Therefore AECF is a ||gm

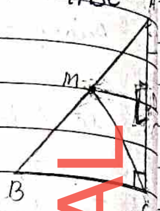
In ΔABP, E is midpoint of AB, EQ || AP [opp. sides of ||gm]
Therefore by converse of midpoint theorem, BQ = PQ - (1)

In ΔCDQ, F is midpoint of CD & FP || CQ, Therefore, by converse of midpoint theorem, P is midpoint of DQ
DP = PQ - (2)
Therefore from (1) & (2)

$DP = PQ = BQ$

Hence AF & CE trisect BD

Let ine fig ΔABC, $\angle C = 90°$, M is mid point of AB, $MD \parallel BC$



6. To Prove :-

(i) D is mid point of AC

(ii) $MD \perp AC$

(iii) $CM = MA \frac{1}{2} AB$

Proof :- In ΔABC, M is midpoint of AB, $MD \parallel BC$, therefore by converse of midpoint theorem, D is mid point of AC

(ii) $MD \parallel BC$, AD is transversal, therefore angle $\angle MDC + \angle DCA$

$\angle MDC + \angle DCA = 180°$ | Cointerior

$\angle MDC = 90°$

$MD \perp AC$

(iii) In ΔAMD & ΔCMD

$MD = MD$ | Common

$\angle ADM = \angle CDM = 90°$ each

$AD = CD$ | D is mid point

$\therefore$ By SAS

ΔAMD & ΔCMD

$\Delta AMD \cong \Delta CMD$

So $CM = AM$ | CPCT

But $AM = \frac{1}{2} AB$

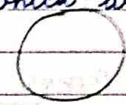
So $CM = \frac{1}{2} AB$

So $CM = MA \frac{1}{8} AB$

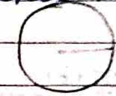
Chapter-9

Circles

☆ Circle - Circle is the collection of points which are at equal distance from the centre



☆ Radius is the line segment joining the centre to any point on the circle.



☆ Chord is a line segment joining any two points on the circle.

☆ Diameter is a chord that passes throught the centre and it is the longest chord.

☆ Arc is a part of a circle

☆ Semicircle is the half of a circle.

☆ Minor Arc is an arc whose length is smaller than the semicircle.

☆ Major Arc is an arc whose length is more than the semicircle.

☆ Sector is the area bounded by two radii and an arc.

☆ Minor Sector is the area bounded by two raddii and minor arc.

☆ Major Sector is the area bounded by two radii and major arc.

☆ Segment is the area bounded by a chord and an arc

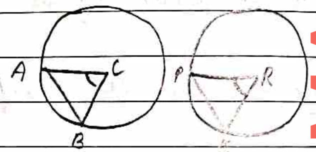
☆ Minor segment is the area bounded by a chord & a minor arc.

☆ Major segment is the area bounded by a chord and a major arc.

Exercise 9.1

Q1: Let in fig two congruents circle with centre C & R
$AB = PQ$

To prove – $\angle C = \angle R$

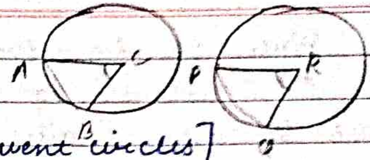


Proof – In ΔACB & ΔPRQ
$AC = PR$ [Radii of congruent circles]
$BC = QR$
$AB = PQ$ | Given

By SSS $\Delta ACB \cong \Delta PRQ$
Hence $\angle C = \angle R$ by CPCT
Hence Proved.

Q2: Given :- Let in fig two congruent circles with centre C & R
$\angle C = \angle R$

To prove – $AB = PQ$

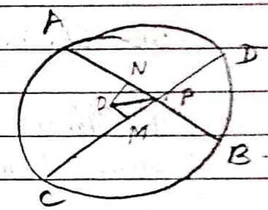


Proof – In ΔACB & ΔPRQ
$AC = PR$ [Radii of congruent circles]
$BC = QR$
$\angle C = \angle R$ | Given

By SAS $\Delta ACB \cong \Delta PRQ$
$AB = PQ$ | CPCT
Hence Proved.

Exercise 9.2

3. Let in fig. a circle with centre O
AB & CB are two equal chords which intersect at point P.



To prove ~~Given~~ $\angle OPA = \angle OPC$

Construction :- Draw a perpendicular from O ~~that~~
Draw $OM \perp DC$ & $ON \perp AB$

Proof :- In ΔOPN and ΔOPM
$\angle N = \angle M$ | 90° each
$OP = OP$ | Common
$ON = OM$ | equal chords are at equal distance.
By RHS
$\therefore \angle OPA = \angle OPC$ | CPCT

V. Good

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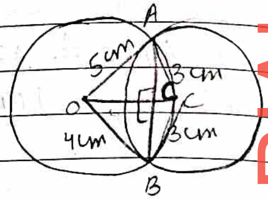
1. Given- In fig. there are two circles with centre O & C intersecting at point A & B

OB = OA = 5cm [Radii of the same (bigger) circle]

AC = BC = 3cm [Radii of the same (smaller) circle]

OB = 4cm [distance between the centres in



In ΔAOC

$(AO)^2 = (OC)^2 + (AC)^2$

$5^2 = 4^2 + 3^2$

$25 = 25$

$\therefore$ 3, 4, 5 are pythagorian triplet

Hence ΔAOC is a right angled Δ

$\angle ACO = 90^\circ$

Similarly we can prove $\angle BCO = 90^\circ$

Hence $\angle ACO + \angle BCO = 90^\circ + 90^\circ$

$= 180^\circ$

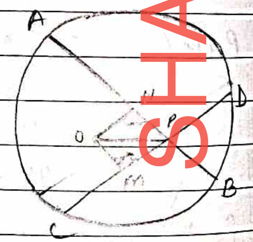
So AB is a straight line

Hence $AB = 3 + 3$

$= 6cm.$

2. Let in fig a circle with Centre O.

AB and CD are two equal chord intersect ant point. P



To Prove :- AP = CP

PD = PB

Construction- Draw $ON \perp AB$ & $OM \perp CD$ and join OP

Proof :- In ΔPON & ΔPOM

$\angle N = \angle M$ | 90° each

OP = OP | Common

ON = OM | equal chords are at equal distance from the circle.

$\therefore$ By RHS $\Delta PON \cong \Delta POM$

Therefore NP = MP —(1) [CPCT]

Since Perpendicular from the centre bisect the chord.

$\therefore AN = NB = \frac{1}{2} AB$

$CM = MD = \frac{1}{2} CD$

AB = CD | Given

$\frac{1}{2} AB = \frac{1}{2} CD$

AN = CM —(2)

Adding (1) & (2)

AN + NP = CM + MP

AP = CP

Again AB = CD

$\frac{1}{2} AB = \frac{1}{2} CD$

NB = MD —(3)

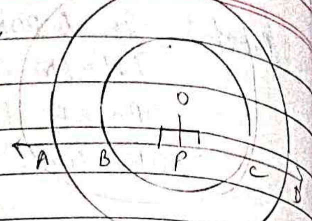
Subtract (1) from (3)

NB − NP = MD − MP

PB = PD

Hence Proved

4. In fig there are two consentric ~~...~~ circle with ~~circle~~ centre O
A, B, C, D are the point where a given line intersect the circles.



To prove :- AB = CD
Construction :- Draw $OP \perp AD$

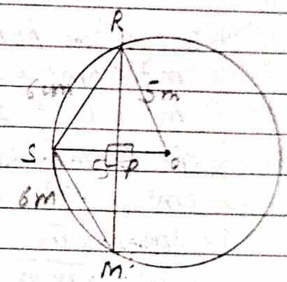
Proof - We know perpendicular from the centre bisect from the chord.
$\therefore AP = PD$ —(1)
& $BP = PC$ —(2)
Sub. (2) from (1)
$AP - BP = PD - PC$
$AB = CD$

5. Let in fig a circular park with a radius of 5m. R, S, M are the position of 3 girls.
$RS = SM = 6m$
$RO = 5m$
Join RM
We know perpendicular from the centre bisect the chord.
Draw $OP \perp$ to RM
So P is mid point of RM
$RP = PM$ —(1)

Since ΔRSM is an isosceles triangle & P is mid point of RM so $SP \perp RM$.
$\angle RPO + \angle RPS = 90° + 90° = 180°$
So SO is a straight line.
$SO = 5m$ radius.

In ΔRSO
$a = 5m$, $b = 5m$ & $c = 6m$
$S = \frac{5+5+6}{2} = 8m$

Area of a $\Delta RSO = \sqrt{s(s-a)(s-b)(s-c)}$
$= \sqrt{8(8-5)(8-5)(8-6)}$
$= \sqrt{8 \times 5 \times 3 \times 2}$
$= \sqrt{144}$
$= 12m^2$ —(2)



Again $\Delta RSO = \frac{1}{2} \times SO \times RP$

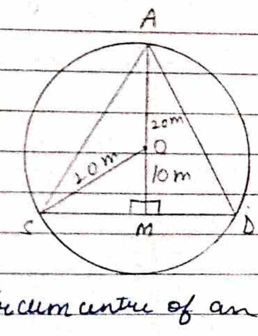
$\frac{1}{2} \times 5 \times RP = 12$

$RP = \frac{12 \times 2}{5} = 4.8m$

$RP = PM = 4.8m$ | using (1)

$\therefore RM = 4.8 + 4.8$
$= 9.6m$

6. Let in fig. a circular path of radius 20m with centre O. A, S & D are the position of 3 boys Ankur, Sayyed & David. which are at equal distance from each other. Each having a toy telephone in their hand.



Now ΔASD is an equilateral triangle.

We know orthocentre, centroid & circumcentre of an equilateral Δ are coincidened which is O.
Therefore AM is altitude as well as median and we know

Centroid divide the median 2:1.

$\therefore \frac{AO}{OM} = \frac{2}{1}$

$\frac{20}{OM} = \frac{2}{1}$

$OM = 10m$

In rt. $\angle$ed ΔSOM = By phytagoras Theorem

$SM^2 + OM^2 = SO^2$

$SM^2 + 10^2 = 20^2$

$SM^2 = 400 - 100$

$SM^2 = 300$

$SM = \sqrt{300}$

$\sqrt{3 \times 100}$

$= 10\sqrt{3}\ m$

$SM = MD = 10\sqrt{3}m$ /Am ~~joint~~ is a median

so $SD = SM + MD$

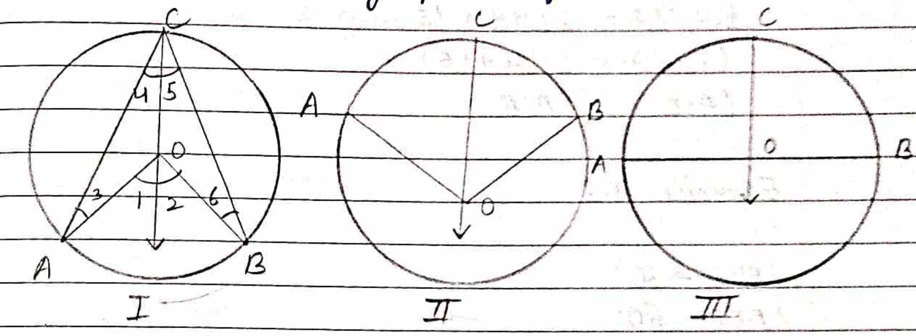
$= 10\sqrt{3} + 10\sqrt{3}$

$= 20\sqrt{3}$

So length of string of toy telephone is $20\sqrt{3}m$

SHARMA TUTORIAL

* Angle subtended by an arc at the centre of the circle is double the angle subtended by ~~the~~ it at any point on the remaining part of the circle.



Let in fig a circle with centre O. There are 3 cases. In case I there is a major arc, In case II there is a major arc & In case III there is a semicircle. Proof is same for all cases we will prove for the case I

To prove :-

$\angle AOB = 2\angle ACB$

Const. :- Join CO & extend

Proof :- In ΔAOC

$AO = OC$ /Radii of the same circle

$\angle 3 = \angle 4$ — (1) / $\angle$s opp. to equal sides

Similarly in ΔOBC

$\angle 5 = \angle 6$ — (2)

In ΔAOC by exterior angle property

$\angle 1 = \angle 3 + \angle 4$

$\angle 1 = \angle 4 + \angle 4$ / using (1)

$\angle 1 = 2\angle 4$ — (3)

Similarly in ΔBOC we can prove

$\angle 2 = 2\angle 5$ — (4)

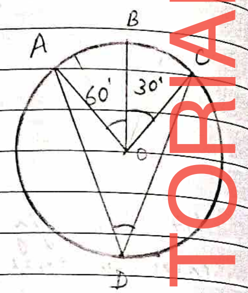
Adding (3) & (4)

$\angle 1 + \angle 2 = 2\angle 4 + 2\angle 5$

$\angle 1 + \angle 2 = 2(\angle 4 + \angle 5)$

$\angle AOB = 2\angle ACB$

Exercise 9.3



1. $\angle BOC = 30°$

$\angle AOB = 60°$

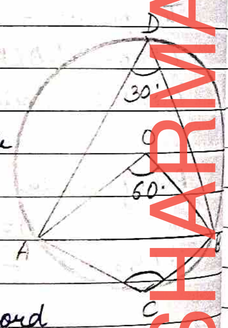
Minor arc AC subtend

$\angle AOC = 60° + 30° = 90°$ at the centre &

$\angle ADC$ at the remaining part of of the circle so, $\angle ADC = \frac{1}{2}\angle AOC$

$\angle ADC = \frac{1}{2} \times 90$

$= 45°$



2. Let in fig AB is a chord which is equal to the radius of the circle

$\therefore \Delta OAB$ is an equilateral triangle.

Hence $\angle AOB = 60°$

$\angle ADB$ & $\angle ACB$ are subtended by AB chord in the major segment & minor segment respectively

AB chord subtended $\angle AOB$ at the centre & $\angle ADB$ at the remaining part so.

$\angle AOB = 2\angle ADB$

So $\frac{1}{2}\angle ADB$

$\angle ADB = 30°$

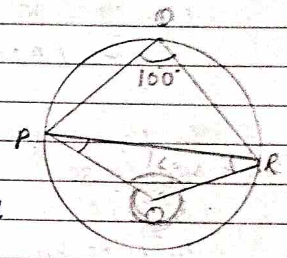
since quad. ACBD is cyclic

so $\angle C + \angle D = 180°$

$\angle C + 30° = 180°$

$\angle C = 150°$

So req. angles are $150°, 30°$



3. In fig $\angle PQR = 100°$

O is the centre of the circle

Major arc PR subtend $\angle PQR = 100°$ at the remaining part of the circle and Reflex $\angle POR$ at the centre

$\therefore$ Reflex $\angle POR = 2\angle PQR$

$\angle POR = 2 \times 100$

Reflex $\angle POR = 200$

$\angle POR = 360° - $ reflex $\angle POR$

$360° - 200°$

$\angle POR = 160°$

PO = RO | Radii of same circle

$\therefore \angle OPR = \angle ORP = x°$ (Let)

In ΔOPR by ASP [180 − 160]

$x + x = 20$

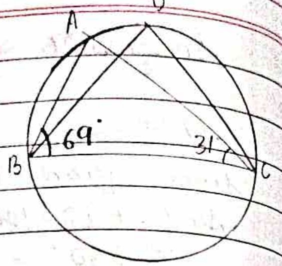
$2x = 20$

$2x = 20°$

$x = 10°$

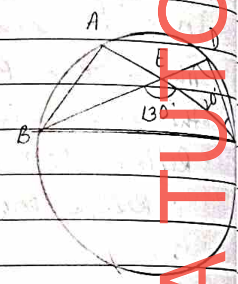
18/10/23

4. $\angle ABC = 69°$
$\angle ACB = 31°$
In ΔABC [By ASP]
$\angle A + \angle ABC + \angle ACB = 180°$
$\angle A + 69° + 31 = 180°$
$\angle A = 180 - 100$
$\angle A = 180°$

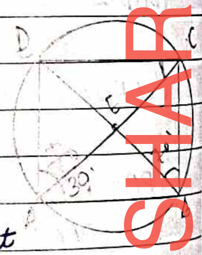


$\angle A = \angle D = 80°$ | $\angle s$ in the same segment
So $\angle BDC = 80°$

5. In fig
$\angle BEC = 130°$
$\angle ACD = 20°$
In ΔCDE by exterior angle property
$\angle BEC = \angle ECD + \angle D$
$130° = 20° + \angle D$
$\angle D = 110°$
$\angle A = \angle D = 110°$ | $\angle s$ in the same segment
So $\angle BDC = 110°$



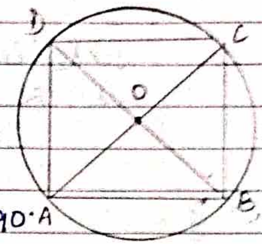
6. ABCD is a cyclic Quadrilateral
$\angle DBC = 70°$, $\angle BAC = 30°$
$\angle BCD$
$\angle CBD = \angle CAD = 70°$ | $\angle s$ in the same segment
$\angle DAB = 70° + 30° = 100°$



$\angle DAB + \angle BCD = 180°$ | opp. $\angle s$ of cyclic Quad.
$\angle DAB + \angle BCD = 180°$
$100 + \angle BCD = 180°$
$\angle BCD = 80°$

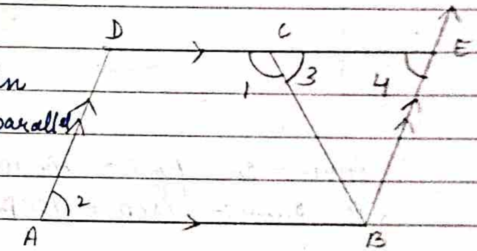
If $AB = BC$ so $\angle BAC = \angle BCA = 30°$ | Angles opp. to equal sides.
Now $\angle ACD = 80 - 30$
$= 50°$

7. Let in fig ABCD is a given cyclic quadrilateral.
In which Diagonal AC & BD are the diameter
$\therefore$ O is the centre of the circle.
We know angle in a semi circle is $90°$
So $\angle DAB = \angle ABC = \angle BCD = \angle CDA = 90°$



Since in quadrilateral all the angles are $90°$

Given:-
8. Let in Fig ABCD is a given Trapezium, in which non-parallel sides are equal.



$\therefore AB \parallel CD$ & $AB = CD$

To Prove:- ABCD is a cyclic Quadrilateral.
Const:- Draw a line $\parallel$ to AD through point B and extend DC that intersect at E.

Since $AB \parallel DE$ | Given
$AD \parallel BE$ | Construction
$\therefore$ ABDE is a $\parallel$ gm
Since $AD = BC$ | Given
But $AD = BE$ | opp. sides of a $\parallel$ gm
$\therefore BC = BE$

Hence, $\angle 3 = \angle 4$ — ① | $\angle$s opp. to equal sides

(L.P) $\angle 1 + \angle 3 = 180°$

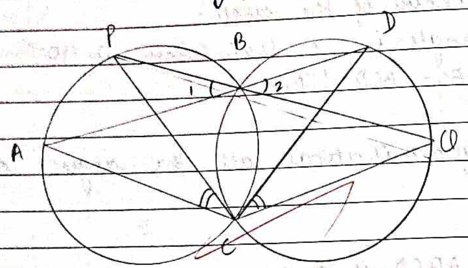
(using ①) $\angle 1 + \angle 4 = 180°$

$\angle 1 + \angle 2 = 180°$

Since in quad ABCD sum of 1 pair of opp. sides is 180° so it is a llgm.

It is a cyclic quadrilateral.

8.



Given – In fig 2 circles intersecting at point B & C.

To Prove :- $\angle ACP = \angle QCB$

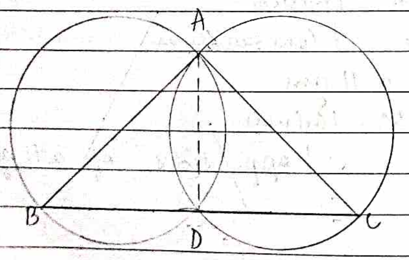
Prove :- $\angle 1 = \angle PCA$ [$\angle$s in the same segment]

$\angle 2 = \angle QCB$

But $\angle 1 = \angle 2$ | V.O.A

So $\angle ACP = \angle QCD$

10.



Let in fig ΔABC, taking AB & AC as diameters. Draw 2 circles intersecting at point A & D

To prove :- Point D lie on BC

Construction :- Join AD, BD, CD.

Prove :-

We know $\angle$s in a semicircle is 90° so

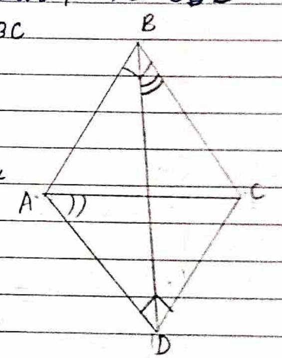
$\angle ADB = \angle ADC = 90°$

$\angle ADB + \angle ADC = 90° + 90°$

$= 180°$

∴ ABD & ADC are forming linear pair, Hence BDC is a straight line and D lies on BC

11. Given :- Let in fig 2 rt. led Δ ABC & ΔADC with common Hypotenuse



To prove :- $\angle CAD = \angle CBD$

Proof :- $\angle ABC + \angle ADC = 90° + 90°$

$= 180°$

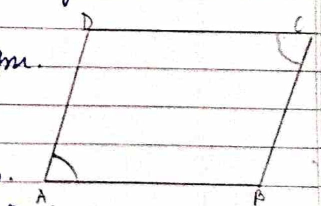
Since in quad. ABCD, one pair of opp. angles is 180° so,

It is a cyclic Quadrilateral &

Hence $\angle CAD = \angle CBA$ | $\angle$s in a same segment.

12. Let in fig ABCD is given cyclic llgm.

To prove :- ABCD is a rectangle



Proof :- $\angle A + \angle C = 180°$ | sum of opp. $\angle$s of cyclic quad.

But $\angle A = \angle C$ | opp. $\angle s$

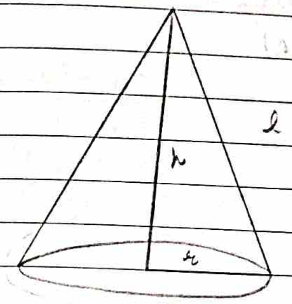
$\angle A + \angle A = 180°$

$2\angle A = 180°$

$\angle A = 90°$

Since in ||gm ABCD 1 angle is 90° so it is a rectangle.

Chapter - 11
Surface Areas and Volumes



Right Circular Cone

$C.S.A = \pi r l$

$T.S.A = \pi r l + \pi r^2$

$= \pi r (r + l)$

$l^2 = r^2 + h^2$ $\quad l = \sqrt{r^2 + h^2}$

$V = \frac{1}{3} \pi r^2 h$

Exercise 11.1

1 Diameter of cone $= 10.5$ cm

$r = \frac{10.5}{2}$ $\quad l = 10$ cm

$C.S.A = \pi r l$

$= \frac{22}{7} \times \frac{10.5}{2} \times 10$

$= 16.5 \times 10$

$= 165$ cm

2. $l = 21m$

$d = 24m$

$r = 12m$

T.S.A of cone $= \pi r(r+l)$

$= \frac{22}{7} \times 12(12+21)$

$= \frac{22 \times 12 \times 33}{7}$

$= \frac{8712}{7}$

$= 1244\frac{4}{7}\ cm^2$

3. $l = 14cm$

C.S.A $= 308\ cm^2$

$\pi rl = 308$

$\frac{22}{7} \times r \times 14 = 308$

$r = \frac{\cancel{308}^{1547} \times 7^{1}}{\cancel{22}_{1} \times \cancel{14}\ 21}$

$r = 7$

T.S.A $= \pi r(r+l)$

$= \frac{22 \times 7}{7}(7+14)$

22×21

$= 462\ cm^2$

4. For a conical tent: $h = 10m$

$r = 24m$

(i) Slant height of the tent

(ii) Cost of canvas required

$l = \sqrt{r^2+h^2}$

$l = \sqrt{24^2+10^2}$

$= \sqrt{576+100}$

$l = \sqrt{676}$

$l = 26m$

Area of canvas required = C.S.A of cone

πrl

$= \frac{22}{7} \times 24 \times 26$

Cost of $1m^2$ canvas $=$ ₹70

Cost of $1m^2$ canvas $= \frac{22}{7_1} \times 24 \times 26 \times \cancel{70}^{10}$

$=$ ₹137280

5. Breadth of Tarpaulin $= 3m$ $(\pi = 3.14)$

Let req. length be x m

For conical Tent $h = 8m$

$r = 6m$

So $l = \sqrt{r^2+h^2}$

$l = \sqrt{6^2+8^2}$

$l = \sqrt{36+64}$

$l = \sqrt{100}$

$l = 10\ m$

Since conical tent is made from rectangular Tarpaulin

So area of Tarpaulin = C.S.A of cone

$l \times b = \pi r l$

$x \times 3 = \frac{22}{7} \times 6 \times 10$

$x \times 3 = 3.14 \times 6 \times 10$

$x = \frac{3.14 \times 6 \times 10}{3}$

$= 62.8m$

Extra length required for stiching margin and wastage in cutting $= 20cm$

$= 0.2m$

Total length required $= 62.8 + 0.2$

$= 63m$

6. For conical Tomb

$l = 25m$

$d = 14m$

$r = 7m$

Rate of white washing = ₹210/100m²

CSA of Conical tomb $= \pi r l$

$\frac{22}{7} \times 7 \times 25$

$= 550m^2$

cost of 100m² white washing = ₹210

Cost of 1m² " " = ₹$\frac{210}{100}$

" " 550m² " " $= \frac{210}{100} \times 550$

= ₹1155

7. For Joker's Conical cap:-

Radius $(r) = 7cm$

Height $(h) = 24cm$

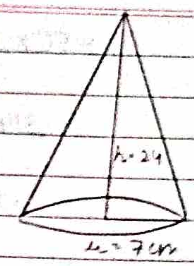
Slant height $(l) = \sqrt{r^2+h^2}$

$= \sqrt{7^2+24^2}$

$= \sqrt{49+576}$

$= \sqrt{625}$

$= 25cm$



Area of sheet req. for 1 cap = CSA of cone

$= \pi r l$

" " " " 10 caps $= 10\pi r l$

$= 10 \times \frac{22}{7} \times 7 \times 25$

$= 5500 cm$

8. For hollow cones

diameter $(d) = 40cm$

$(r) = 20cm$

$= 0.2m$ (1m = 100 cm)

Height $(h) = 1m$

$\pi = 3.14$

$\sqrt{1.04} = 1.02$

cost of painting = ₹12/m²

Slant Height $(l) = \sqrt{r^2+h^2}$

$= \sqrt{0.2^2+1^2}$

$= \sqrt{0.04+1}$

$= \sqrt{1.04}$

$l = 1.02m$

Area to be painted in 1 cone $= \pi r l$

" " 50 cone $= 50\pi r l$

$= 50 \times 3.14 \times 0.21 \times 1.02$

$= 50 \times \frac{314}{100} \times \frac{2}{10} \times \frac{102}{100}$

$= \frac{314 \times 102}{1000}\ m^2$

$= 32.028\ m^2$

Cost of painting $1m^2$ = ₹12

" " " $32.028\ m^2$ = ₹12×32.028

= ₹384.336

Exercise 11.2

1. Radius (r) of sphere = 10.5 cm

Surface area of sphere = $4\pi r^2$

$= 4 \times \frac{22}{7} \times \frac{105}{10} \times \frac{105}{10}$

$= 1386\ cm^2$

2. Diameter of sphere = 14 cm

Radius (r) of sphere = 7cm

S.A of sphere = $4\pi r^2$

$= 4 \times \frac{22}{7} \times 7 \times 7$

$= 616\ cm^2$

3. $r = 10$ cm

$\pi = 3.14$

T.S.A of hemisphere = $3\pi r^2$

$3 \times 3.14 \times (10)^2$

$\frac{3 \times 314 \times \cancel{10} \times \cancel{10}}{1\cancel{00}}$

=

4. Case I

Radius (r) = 7cm

S.A. of spherical Balloon = $4\pi r^2$

$= 4\pi 7^2$

Case II

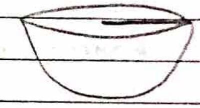
Radius (R) = 14cm

S.A of spherical Balloon = $4\pi 14^2$

$\frac{4\pi r^2}{4\pi R^2} = \frac{\cancel{4\pi} \times \cancel{7} \times \cancel{7}}{\cancel{4\pi} \times \cancel{14}_2 \times \cancel{14}_2} = \frac{1}{4}$

$= 1:4$

5. Radius of hemispherical bowl = $\frac{10.5}{2}$



$= \frac{105}{20} = \frac{21}{4}$ cm

Area to be tin plating = Inner C.S.A of bowl

$= 2\pi r^2$

$= \cancel{2}^1 \times \frac{22}{7_1} \times \frac{\cancel{21}^3}{\cancel{4}_2} \times \frac{21}{4}$

$= \frac{693}{4}\ cm^2$

Cost of tin plating $100\ cm^2$ = ₹16

Cost of tin plating $1\ cm^2 = \frac{16}{100}$

Cost of tin plating $\frac{693}{4}\ cm^2$ = ₹$\frac{16}{100} \times \frac{693}{4}$

$= \frac{2772}{100}$

= ₹27.72

6. Surface Area of sphere = $154 cm^2$

$S.A = 4\pi r^2$

$4\pi r^2 = 154$

$4 \times \frac{22}{7} \times r^2 = 154$

$r^2 = \frac{154 \times 7}{4 \times 22}$

$r^2 = 12.25$

$r = \sqrt{12.25}$

$r = 3.5 cm$

7. Let the diameter of earth be x unit

diameter of moon be $\frac{1}{4} x$ unit

Radius of earth $(R) = \frac{x}{2}$

Radius of moon $(r) = \frac{x}{4} \times \frac{1}{2}$

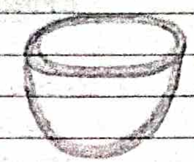
$= \frac{x}{8}$

$\frac{4\pi r^2}{4\pi R^2} = \frac{4\cancel{\pi} r^2}{4\cancel{\pi} R^2}$

$\frac{\left(\frac{x}{8}\right)^2}{\left(\frac{x}{2}\right)^2} = \frac{\cancel{x^2} \times \cancel{4}1}{\cancel{64}16\, \cancel{x^2}} = \frac{1}{16}$

1:16

8. Thickness of hemispherical bowl = 0.25cm



Inner radius (r) = 5 cm

Outer Radius (R) = Inner Radius + thickness

5 + 0.25

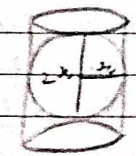
= 5.25 cm

CSA of hemisphere $= 2\pi r^2$

$= 2 \times \frac{22}{7} \times 5.25 \times 5.25$

$= 173.25 cm^2$

9. A right circular cylinder just encloses a sphere of radius (r units)



$\therefore$ Radius of cylinder = r unit

Height of cylinder = diameter of sphere

= $2r$ units

(i) S.A of sphere = $4\pi r^2$

(ii) C.S.A of cylinder = $2\pi rh$

$= 2\pi r(2r)$

$= 4\pi r^2$

(iii) Req. Ratio $= \frac{4\pi r^2}{4\pi r^2}$

= 1:1

Volume of cone $= \frac{1}{3}\pi r^2 h$

Volume of sphere $= \frac{4}{3}\pi r^2$

Volume of hemisphere $= \frac{2}{3}\pi r^2$

Page 134

Exercise 11.3

1. (i) For right circular cone

$r = 6\,cm \qquad h = 7\,cm$

$V = \frac{1}{3}\pi r^2 h$

$\frac{1}{\cancel{3}_1} \times \frac{22}{\cancel{7}} \times 6 \times \cancel{6}^2 \times \cancel{7}$

$= 264\ cm^3$

2 (i) $r = 7\,cm$

$l = 25\,cm$

$h = \sqrt{l^2 - r^2}$

$h = \sqrt{(25)^2 - (7)^2}$

$h = \sqrt{625 - 49}$

$h = \sqrt{576}$

$h = 24\,cm$

V. of cone $= \frac{1}{3}\pi r^2 h$

$\frac{1}{\cancel{3}_1} \times \frac{22}{\cancel{7}} \times \cancel{7} \times 7 \times \cancel{24}8$

$= 1232\ cm^3$

Page 135

3. $h = 15\,cm$

$\pi = 3.14$

V. of cone $= 1570\ cm^3$

$\frac{1}{3}\pi r^2 h = 1570$

$r^2 = \frac{1570 \times 3}{\pi h}$

$r^2 = \frac{1570 \times 3}{3.14 \times 15}$

$r = 10\,cm$

4. $h = 9\,cm$

V. of cone $= \frac{1}{3}\pi r^2 h = 48\pi$

$\frac{1}{3} \times \frac{22}{7} \times r^2 \times 9 = 48 \times \frac{22}{7}$

$\frac{1}{3} \times r^2 \times 9 = 48$

$r^2 = \frac{\cancel{48}\,16}{\cancel{3}\,1}$

$r = 4\,cm$

diameter $= 2 \times 4$

$= 8\,cm$

5. diameter of conical pit $= 3.5\,m$

$r = \frac{3.5}{2}$

$= \frac{35}{20} = \frac{7}{4}$

$h = 12\,m$

V. of cone $= \frac{1}{3}\pi r^2 h$

Page 136

$\frac{1}{3} \times \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times 12$

$= \frac{77}{2}$

$= 38.5 \, m^2$

$= 38.5 \, kl \quad | \; 1m^3 = 1kl$

6. Diameter of cone = 28 cm

$r = 14 cm$

V. of cone = $9856 \, cm^3$

(i) $\frac{1}{3}\pi r^2 h = 9856$

$\frac{1}{3} \times \frac{22}{7} \times 14 \times 14 \times h = 9856$

$h = \frac{9856 \times 3}{22 \times 2 \times 14}$

$h = 48 cm$

(ii) $l = \sqrt{h^2 + r^2}$

$= \sqrt{48^2 + 14^2}$

$= \sqrt{2304 + 196}$

$= 50 \, cm$

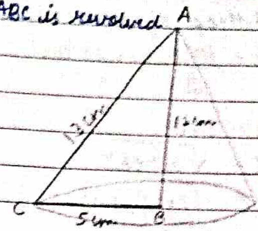
(iii) CSA of cone = $\pi r l$

$= \frac{22}{7} \times 14 \times 50$

$= 2200 \, cm^2$

Page 137

7. In fig a right angled triangle ABC is revolved about side 12 cm.



We get a cone of :-

$r = 5 cm$

$h = 12 cm$

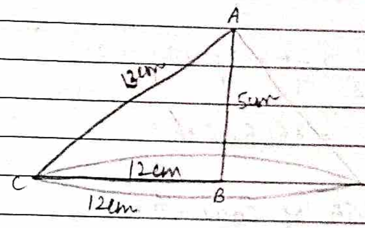
$l = 13 cm$

Volume of the cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \, 5 \times 5 \times 12$

$= 100\pi \, cm^3$

8. When a right angled triangle ABC whose sides are 12 cm, 13 cm & 5 cm is revolved about a side of 5 cm.



We get a cone of :-

$r = 12 cm$

$h = 5 cm$

$l = 13 cm$

V. of cone = $\frac{1}{3}\pi r^2 h$

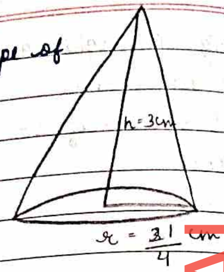
$= \frac{1}{3} \times \pi \times 12 \times 12 \times 5$

$= 240\pi \, cm^3$

Req. Ratio = $\frac{100\pi}{240\pi}$

$= 5:12$

9. A Heap of wheat is in conical shape of



$r = \frac{10.5}{2} = \frac{21}{4}$ cm

$h = 3m$

$l = \sqrt{r^2+h^2}$

$= \sqrt{\left(\frac{21}{4}\right)^2+(3)^2}$

$= \sqrt{\frac{441}{16}+9}$

$= \sqrt{\frac{441+144}{16}} = \sqrt{\frac{585}{16}}$

$= \sqrt{36.5625}$

$= 6.05$

		6.046
6		36.562500
		36
120		× 56
		-00
1204		5625
		-4816
12086		80900
		-72516

V. of cone $= \frac{1}{3}\pi r^2 h$

$\frac{1}{3} \times \frac{22}{7} \times \frac{21}{4} \times \frac{21}{4} \times 3$

$= 86.625\,m^3$

CSA of Cone $= \pi r l$

$\frac{22}{7} \times \frac{21}{4} \times 6.05$

$= 99.825\,m^2$

Volume of sphere $= \frac{4}{3}\pi r^3$

Volume of Hemisphere $= \frac{2}{3}\pi r^3$

Exercise 13.4

1. $r = 7$ cm

V. of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$

$= 1437\frac{1}{3}\ cm^3$

2. Diameter = 28 cm

$r = 14$ cm

Amt. of water displaced = V. of sphere

$= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \frac{63}{10} \times \frac{63}{10} \times \frac{63}{10}$

$1047\ cm^3$

3. Diameter of metallic ball = 4.2 cm

$r = 2.1$ cm

$= \frac{21}{10}$ cm

density $= 8.4\ g/cm^3$

vol. of ball $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \frac{21}{10} \times \frac{21}{10} \times \frac{21}{10}$

$= 38.808\ cm^3$

4. Let diameter of earth be x unit
So diameter of moon $= \frac{x}{4}$ unit

Radius of moon $(r) = \frac{x}{8}$ units

Radius of earth $(R) = \frac{x}{2}$ units

$$\frac{V.\ of\ moon}{V.\ of\ earth} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3}$$

$$\frac{V. Moon}{V.\ earth} = \frac{\left(\frac{x}{8}\right)^3}{\left(\frac{x}{2}\right)^3}$$

$$\frac{V.\ of\ Moon}{V.\ of\ earth} = \frac{\cancel{x^3}}{8\times8\times8} \times \frac{2\times2\times2}{\cancel{x^3}}$$

$$\frac{V.\ of\ moon}{V.\ of\ earth} = \frac{1}{64}$$

So. V. of moon $= \frac{1}{64}$ V. of earth

5. $d = 10.5$ cm
radius $= \frac{10.5}{2} = \frac{21}{4}$ cm

V. of hemisphere $= \frac{2}{3}\pi r^3$

$$\frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{22}}{7} \times \frac{\cancel{21}}{\cancel{4}} \times \frac{21}{4} \times \frac{21}{4}$$

$= 303.1875$ cm^3

$= \frac{303.1875}{1000}$ l $\quad$ 1 l $= 1000$ cm^3

$= 0.303$ l (approx)

6. Thickness of hemispherical tent $=$ 1cm
$= 0.01$m
Inner radius $=$ 1m
Outer radius $= 1+0.01$m
$= 1.01$m
V. of inner radius $-$ V. of outer radius
Outer Radius $-$ Inner Radius
$= \frac{2}{3}\pi R^3 - \frac{2}{3}\pi r^3$

$\frac{2}{3}\pi(R^3 - r^3)$

$\frac{2}{3} \times \frac{22}{7} \times (1.01^3 - 1^3)$

$= \frac{2}{3} \times \frac{22}{7} \times (1.03 - 1) = \frac{2}{3} \times \frac{22}{7} \times 0.03$

0.06348 m^3

7. S.A. of sphere $= 154$ cm^2
$4\pi r^2 = 154$
$r^2 = \frac{154}{4\pi}$
$r^2 = \frac{154 \times 7}{4 \times 22}$
$r^2 = \frac{49}{4}$
$r = \frac{7}{2}$ or 3.5cm

V. of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$= \frac{539}{3}$

$= 179\frac{2}{3}\ cm^3$

8. Hemispherical Dome

Total cost $= ₹\,4989.60$

Rate $= ₹\,20/m^2$

C.S.A of dome $= \frac{\text{Total Cost}}{\text{Rate}}$

$2\pi r^2 = \frac{4989.60}{20} = \frac{498960}{20 \times 100}$

C.S.A $= 249.48\ m^2$

$2 \times \frac{22}{7} \times r^2 = 249.48$

$r^2 = \frac{249.48 \times 7}{2 \times 22}$

$r^2 = 39.69$

$r = 6.3\,m$

	6.3
6	39.69
	36 ↓
123	369
	369
	x

$V = \frac{2}{3}\pi r^3$

$\frac{2}{3} \times \frac{22}{7} \times \cancel{6.3} \times \cancel{6.3} \times 6.3$

$= 523.908\ m^3$

10. Diameter of hemispherical capsule $= 3.5\,mm$

$r = \frac{3.5}{2} = \frac{7}{4}\,mm$

Amount of medicine $=$ V. of sphere

$= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times \frac{7}{4}$

$= 22.46\,m^3$

9. Radius and S.A of small spheres are :-

r & s respectively and

Radius and S.A of big sphere are :-

r' & s' respectively

Since 27 small spheres are melted to form a big sphere

So Sum of V. of 27 small spheres $=$ V. of big sphere

(i) $27 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi r'^3$

$= 27r^3 = r'^3$

$3r = r'$

(ii) $\frac{S}{S'} = \frac{4\cancel{\pi} r^2}{4\cancel{\pi} r'^2}$

$= \frac{r^2}{(3r')^2} = \frac{1 r^2}{9 r'^2}$

$= 1:9$

23/11/23